

Bielefeld University Faculty of Physics	Symmetries in Physics WS 2025/2026	Prof. Dr. Jürgen Schnack jschnack@uni-bielefeld.de
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## 4 Problem sheet

### 4.1 IN CLASS: Subgroup of the symmetric group

Let  $f \in \mathcal{S}_6$  be the permutation

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 3 & 1 & 2 & 4 & 6 & 5 \end{pmatrix} = (132)(56) . \quad (3)$$

- What is the smallest subgroup  $G$  of  $\mathcal{S}_6$  containing  $f$ ?
- What is the order of  $G$ ?
- What is the inverse of  $f$ ?

### 4.2 AT HOME: Unit circle

The group  $O(2)$  consists of rotations  $r_\alpha$  and reflections  $m_\alpha$  which act on  $S = \{Z \in \mathbf{C}, |z| = 1\}$  as

$$r_\alpha(e^{i\phi}) = e^{i(\phi+\alpha)} \quad , \quad m_\alpha(e^{i\phi}) = e^{i(2\alpha-\phi)} . \quad (4)$$

- Explain in words what these transformations do.
- What would be a sensible range for  $\alpha$ ?
- Express  $r_\alpha m_\beta$ ,  $m_\beta r_\alpha$ , and  $m_\alpha m_\beta$  as a single rotation or reflection.
- Express the conjugate elements  $r_\alpha^{-1} m_\beta r_\alpha$ ,  $m_\beta^{-1} r_\alpha m_\beta$ , and  $m_\beta^{-1} m_\alpha m_\beta$  as a single rotation or reflection.
- Let  $D_k$  be the group generated by  $r_{2\pi/k}$  and  $m_0$ , i.e., the smallest group containing these elements. What are the elements of  $D_k$  and what is the order of the group?

### 4.3 AT HOME: A representation for $C_{3v}$

Plot an equilateral triangle into a two-dimensional coordinate system and derive a faithful representation for this group.