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2 Problem sheet

2.1 IN CLASS: Angular momenta

a. Angular momentum operators, here in short spin, are defined by their commutation relations. Please write down these commutation relations.

b. Write down the commutator of each component with the square of the spin.

c. Two eigenvalue equations hold for spin. Please write them down and explain the details.

d. Explain the operators $\underset{\sim}{s}^+$ and $\underset{\sim}{s}^-$ (raising and lowering operators) as well as their action on the spin eigenstates.

2.2 AT HOME: One-magnon space of the Heisenberg spin ring

We will study the Heisenberg model on a ring. More precisely, let there be N spins $s = 1/2$ interacting according to the Hamiltonian

$$\underset{\sim}{H} = J \sum_{i=1}^{N} \underset{\sim}{\vec{s}}_i \cdot \underset{\sim}{\vec{s}}_{i+1} \ , \tag{1}$$

with $\underset{\sim}{\vec{s}}_{N+1} \equiv \underset{\sim}{\vec{s}}_1$ (periodic boundary conditions, pbc).

a. Show that

$$\underset{\sim}{\vec{s}}_i \cdot \underset{\sim}{\vec{s}}_j = \underset{\sim}{s}_i^z \underset{\sim}{s}_j^z + \frac{1}{2} \left(\underset{\sim}{s}_i^+ \underset{\sim}{s}_j^- + \underset{\sim}{s}_i^- \underset{\sim}{s}_j^+ \right) \ . \tag{2}$$

b. Compare with the introduction and define the product basis, the appropriate operators, and one-magnon space. You have some freedom in your definitions.

c. Calculate the energy eigenvalues $E(k) = \langle\, k \,|\, \underset{\sim}{H} \,|\, k \,\rangle$, $k = 0, \dots, N-1$ in one-magnon space.

d. Derive $\langle\, k \,|\, k' \,\rangle$.