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13 Problem sheet

13.1 IN CLASS: Killing form

a. Study Ludwig and Falter, pages 273 and 274.

b. Calculate the Killing form for $SO(3)$ and $SO(4)$.

c. Are $SO(3)$ and $SO(4)$ semisimple?

13.2 AT HOME: $SU(27)$

We don't know in which direction fundamental particle physics develops.

a. Be prepared and make a suggestion for the Cartan subalgebra of $SU(27)$.

b. In the lecture, a few properties of the matrices defining the subalgebra were stated such as real entries on the diagonal and the property to be traceless. Why do we find these properties?

13.3 AT HOME: group averages

We consider the group average of an integrable function f on $SO(2)$

$$M(f) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \; f(g(\phi)) \; , \tag{22}$$

where

$$g(\phi) = \begin{pmatrix} \cos\phi & -\sin\phi \\ \sin\phi & \cos\phi \end{pmatrix} \tag{23}$$

is a group element of $SO(2)$ with $\phi \in [0, 2\pi)$.
Show that the group average is invariant under left translations,

$$M(f) = M_l(f) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \; f(g(\psi)g(\phi)) \; , \tag{24}$$

as well as under right translations,

$$M(f) = M_r(f) = \frac{1}{2\pi}\int_0^{2\pi} \mathrm{d}\phi \; f(g(\phi)g(\psi)) \; . \tag{25}$$