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12 Problem sheet

12.1 IN CLASS: Jacobi identity

a. State the Jacobi identity.

b. Prove the Jacobi identity for angular momenta (spins, orbital angular momenta). This is actually somewhat trivial.

c. Therefore, prove the Jacobi identity for linear maps A, B, C on some vector space V.

12.2 AT HOME: $O(2)$

We consider the group $O(2)$. This group contains matrices of two types

$$A_1(\alpha) = \begin{pmatrix} \cos\alpha & -\sin\alpha \\ \sin\alpha & \cos\alpha \end{pmatrix} , \tag{19}$$

$$A_2(\alpha) = \begin{pmatrix} \cos\alpha & \sin\alpha \\ \sin\alpha & -\cos\alpha \end{pmatrix} . \tag{20}$$

a. Show that both matrices fulfill the definition of $O(2)$.

b. What is the difference between the two types in terms of what they do and mathematically?

c. Calculate the distance of both types to $\mathbf{1}$. Repeat the matrix definition of distance.

d. Determine the generators? How many do you get? Give a geometrical interpretation.

e. **extra:** How do you like the equations (4.34a), (4.34b) und (4.35b) in the book of Böhm?

12.3 AT HOME: Heisenberg group

We consider the Heisenberg group.

a. The three operators $\underset{\sim}{x}$, $\underset{\sim}{p}$, and $i\hbar\mathbf{1}$ form a real Lie algebra (the Heisenberg algebra) which is closed under taking commutators. Give the commutation relations.

b. The Heisenberg group has a three-dimensional representation by upper triangular matrices of the form

$$\begin{pmatrix} 1 & a & c \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} , \tag{21}$$

where $a, b, c \in \mathbb{R}$. Show that this matrix group is a Lie group with Lie algebra isomorphic to the one considered in (a).

c. Is the Heisenberg group a simple or semi-simple Lie group? This is a real challenge!!! Look into the book of Ludwig and Falter pages 273 and 274.