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11 Problem sheet

11.1 IN CLASS: $U(1)$ and $SO(2)$

a. Repeat what $U(1)$ and $SO(2)$ are. Write down the definitions. Are they Abelian?

b. Show that $U(1)$ and $SO(2)$ are isomorphic.

11.2 AT HOME: Repeat a bit on manifolds and topology

We need a bit of knowledge on manifolds. Please repeat or look up the following terms or concepts, respectively:

a. Manifold, chart, and atlas.

b. How many charts do we need to cover a two-dimensional sphere? Provide an example.

c. Compact.

d. Connected and simply connected.

e. Is $U(1)$ compact, connected, or simply connected?

f. Can you define the tangent space of $U(1)$? There is a simple connection to radial and tangential normal vectors used in circular motion.

Remark: It is clear that we don't intend to repeat the full abstract mathematical theory, and that we only aim at some reasonable understanding of these terms. Fell free to know more.

11.3 AT HOME: Dimensions of groups

Determine the dimensions of the groups $U(n)$, $SU(n)$, $O(n)$, and $SO(n)$.