8 Problem sheet

8.1 IN CLASS (finish at home): Multiplicities for coupling two spins

We consider two angular momentum operators (jargon "spin" in the following) of quantum number l and s with $l \geq s$. The product representation can be decomposed as follows:

$$D^{(l)} \otimes D^{(s)} = D^{(l \otimes s)} = \bigoplus_{j=|l-s|}^{l+s} n_j D^{(j)} \tag{12}$$

$$n_j = (l,s|j) = \langle\, \chi^{(l)} \chi^{(s)} \,|\, \chi^{(j)} \,\rangle \; . \tag{13}$$

For the scalar product we have to do the following replacement

$$\frac{1}{|G|} \sum_g \Rightarrow \int \frac{\mathrm{d}\phi}{2\pi} (1 - \cos(\phi)) \; , \tag{14}$$

compare H.F. Jones, *Groups, representations and physics*, Taylor & Francis, New York (1998).

a. Show that $\langle\, \chi^{(j)} \,|\, \chi^{(j')} \,\rangle = \delta_{jj'}$.

b. We want to show that all possible n_j in (12) are $n_j = 1$. We can accomplish this task by equivalently showing

$$\chi^{(l)} \chi^{(s)} = \sum_{j=|l-s|}^{l+s} \chi^{(j)} \; . \tag{15}$$