

# Magnetism of the $N = 42$ kagome lattice antiferromagnet

Jürgen Schnack, Jörg Schulenburg, Johannes Richter

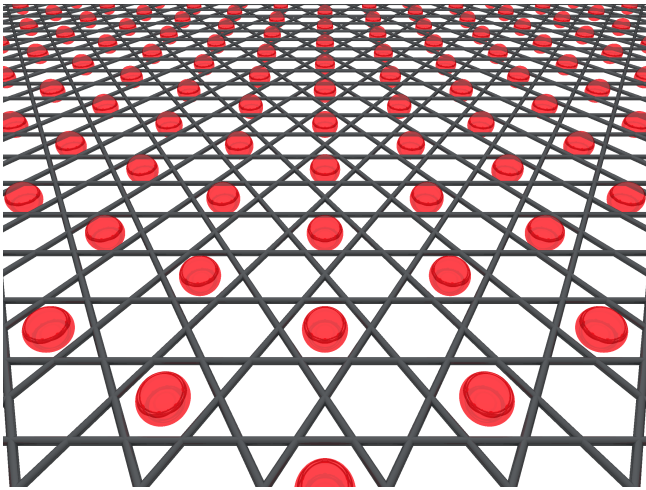
Department of Physics – University of Bielefeld – Germany

<http://obelix.physik.uni-bielefeld.de/~schnack/>

DPG Frühjahrstagung, TT 23.13,  
Regensburg, 02. 04. 2019



# Kagome lattice antiferromagnet – the problem



- Thermodynamic functions, in particular heat capacity and susceptibility.
- Magnetization curve, in particular thermal stability of plateau at  $\mathcal{M}_{\text{sat}}/3$ .
- Method: Finite-temperature Lanczos.
- Comparison with tensor-network calculations.

J. Schnack, J. Schulenburg, J. Richter, Phys. Rev. B **98** (2018) 094423

# Model Hamiltonian

$$\tilde{H} = J \sum_{i < j} \vec{\tilde{s}}_i \cdot \vec{\tilde{s}}_j + g \mu_B B \sum_i^N \tilde{s}_i^z$$

Heisenberg
Zeeman

# Finite-temperature Lanczos Method I

$$Z(T, B) = \sum_{\nu} \langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle$$

$$\langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle \approx \sum_n \langle \nu | n(\nu) \rangle \exp \left\{ -\beta \epsilon_n \right\} \langle n(\nu) | \nu \rangle$$

$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp \left\{ -\beta \epsilon_n \right\} |\langle n(\nu) | \nu \rangle|^2$$

- $|n(\nu)\rangle$  n-th Lanczos eigenvector starting from  $|\nu\rangle$
- Partition function replaced by a small sum:  $R = 1 \dots 10, N_L \approx 100$ .

J. Jaklic and P. Prelovsek, Phys. Rev. B **49**, 5065 (1994).

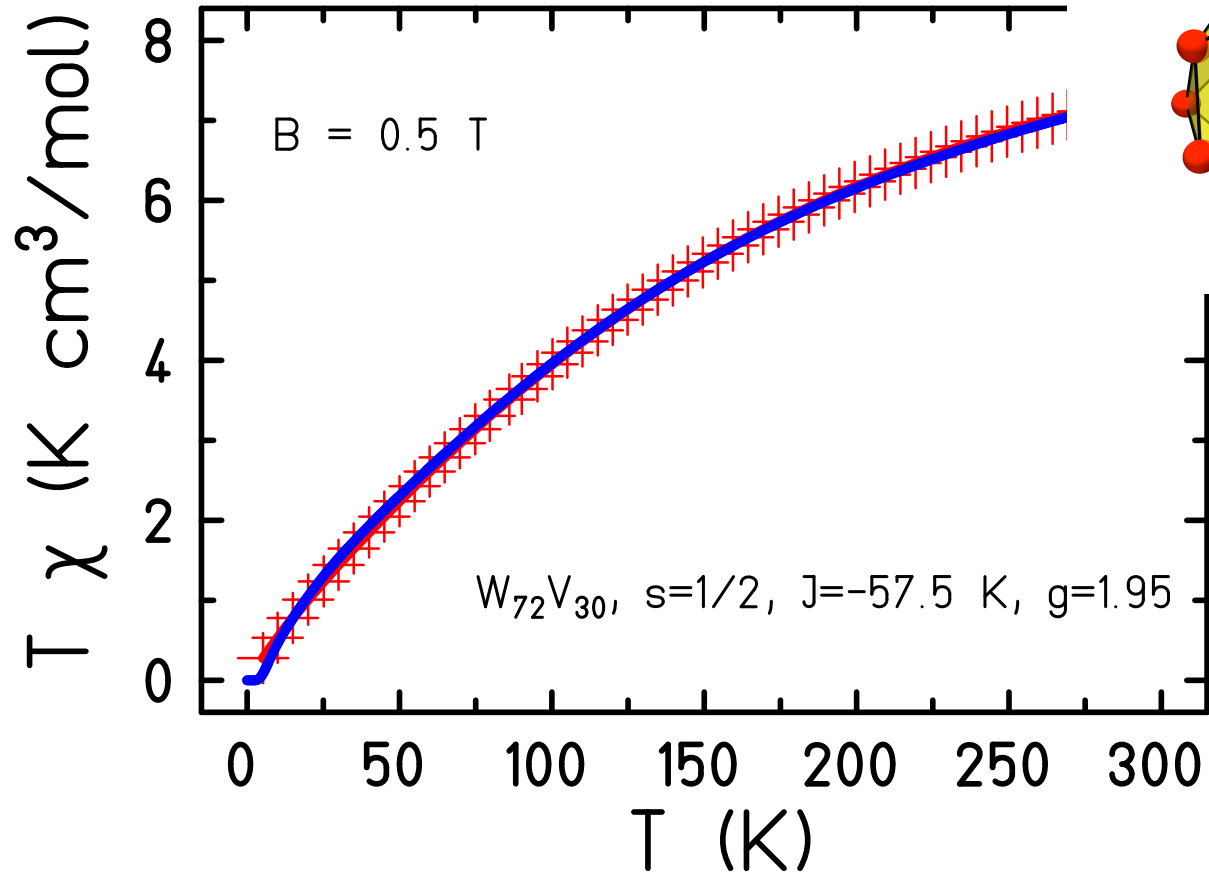
## Finite-temperature Lanczos Method II

$$Z(T, B) \approx \sum_{\Gamma} \frac{\dim(\mathcal{H}(\Gamma))}{R_{\Gamma}} \sum_{\nu=1}^{R_{\Gamma}} \sum_{n=1}^{N_L} \exp\{-\beta\epsilon_n\} |\langle n(\nu, \Gamma) | \nu, \Gamma \rangle|^2$$

- Approximation better if symmetries taken into account.
- $\Gamma$  denotes the used irreducible representations; often this is just the  $\mathcal{S}^z$  symmetry, i.e.  $\Gamma \equiv M$

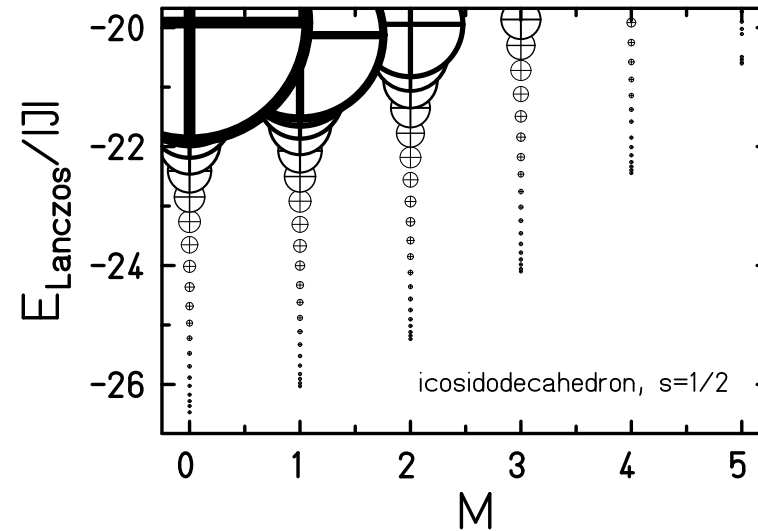
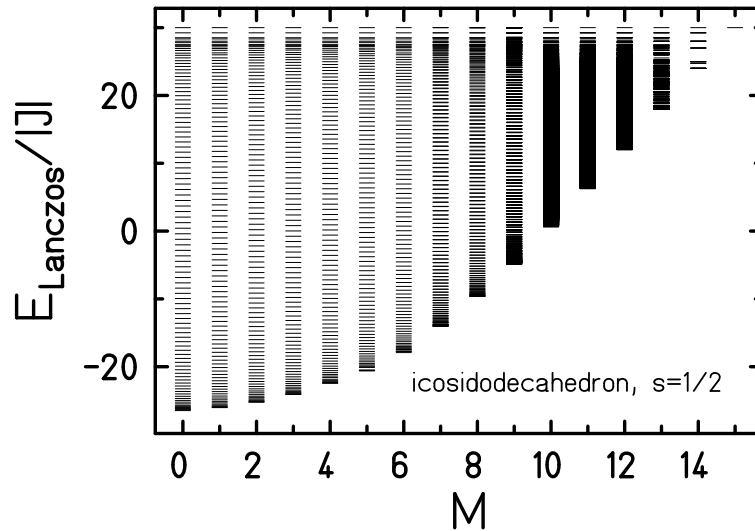
J. Schnack and O. Wendland, Eur. Phys. J. B **78** (2010) 535-541

# Icosidodecahedron $s = 1/2$



Exp. data: A. M. Todea, A. Merca, H. Bögge, T. Glaser, L. Engelhardt, R. Prozorov, M. Luban, A. Müller, Chem. Commun., 3351 (2009).

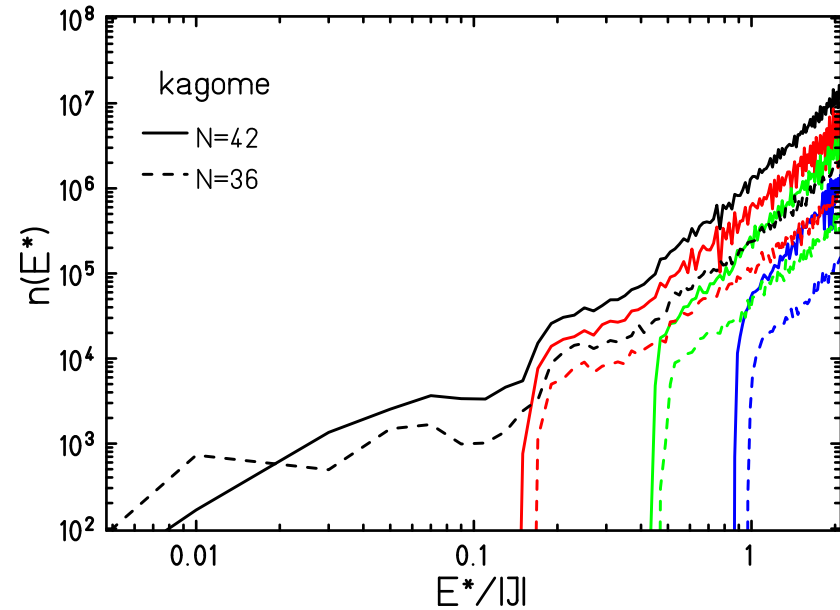
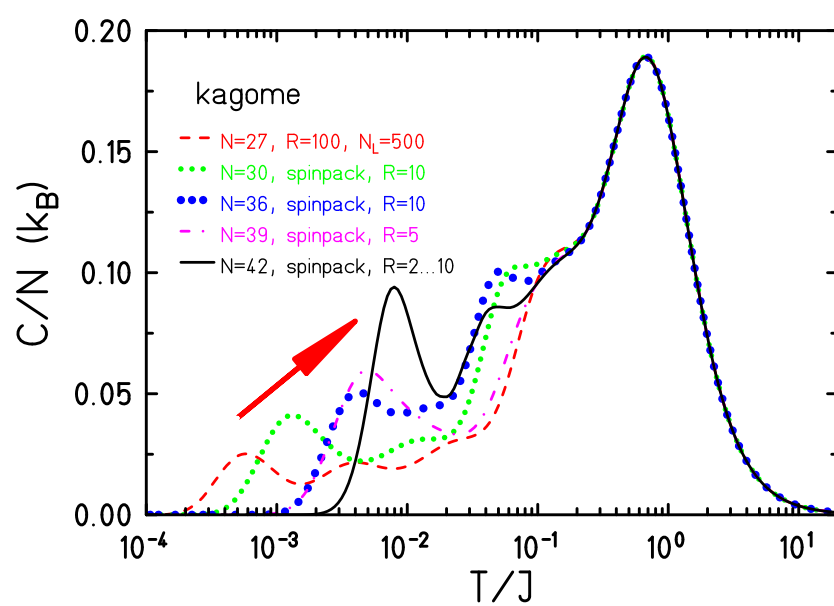
# Icosidodecahedron $s = 1/2$



- The true spectrum will be much denser. This is miraculously compensated for by the weights.

$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp \{ -\beta \epsilon_n \} |\langle n(\nu, \Gamma) | \nu, \Gamma \rangle|^2$$

# Kagome 42 – heat capacity

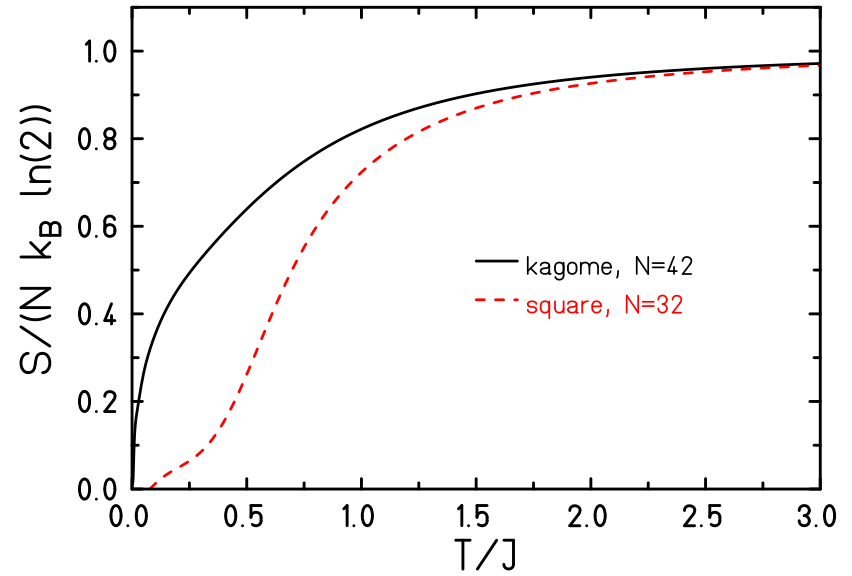
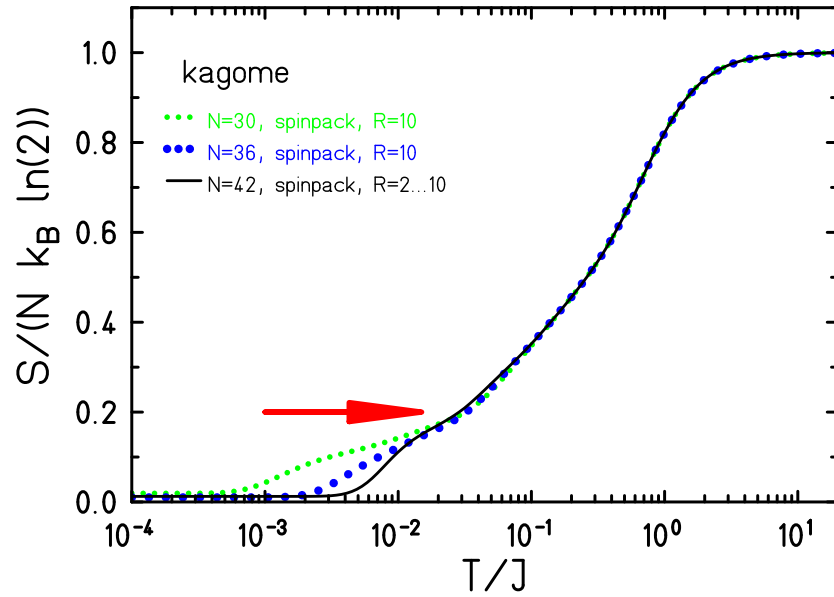


- Low- $T$  peak moves to higher  $T$  with increasing  $N$ .
- Density of low-lying singlets seems to move to higher excitation energies.

J. Schnack, J. Schulenburg, J. Richter, Phys. Rev. B **98** (2018) 094423



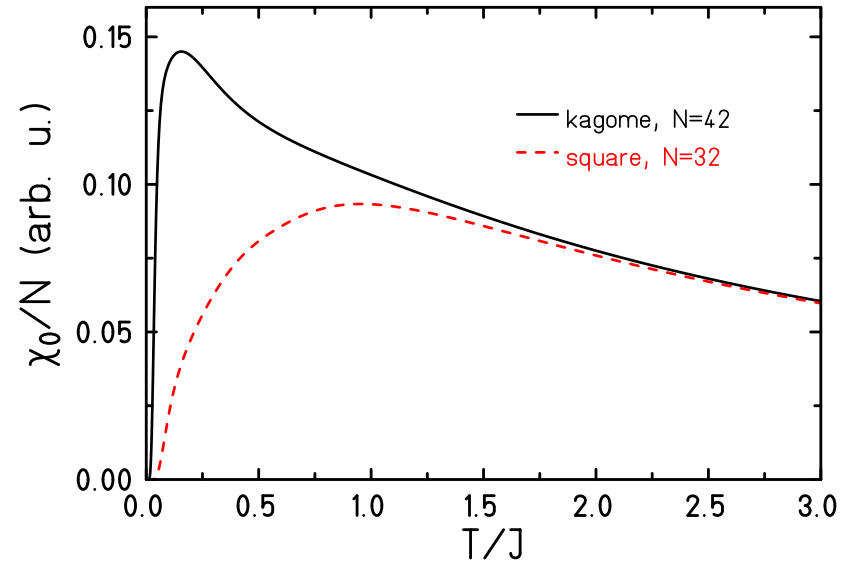
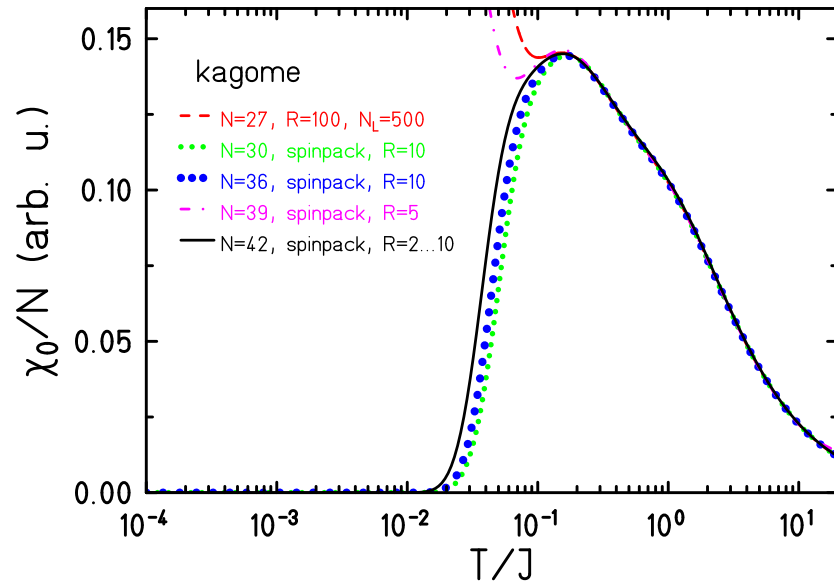
# Kagome 42 – entropy



- Rise of entropy for higher  $T$  with increasing  $N$ .

J. Schnack, J. Schulenburg, J. Richter, Phys. Rev. B **98** (2018) 094423

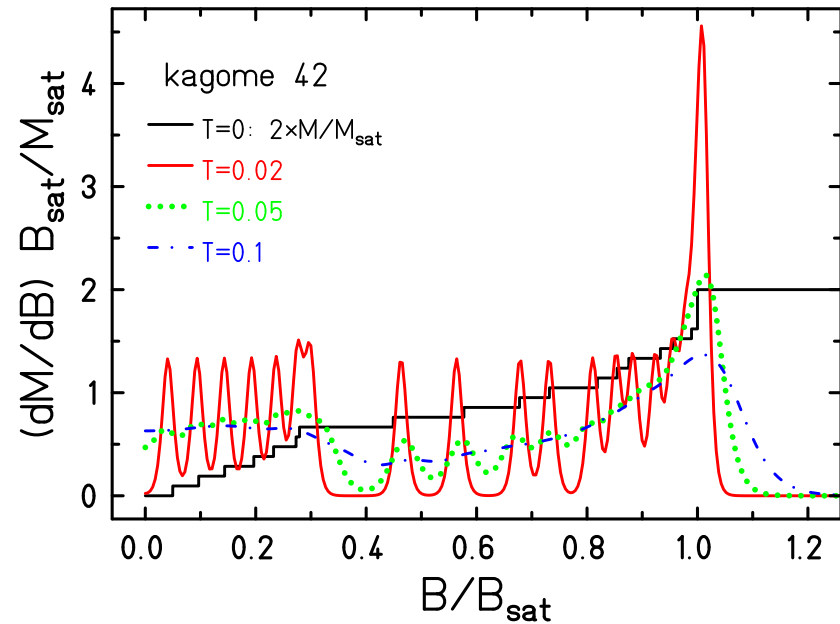
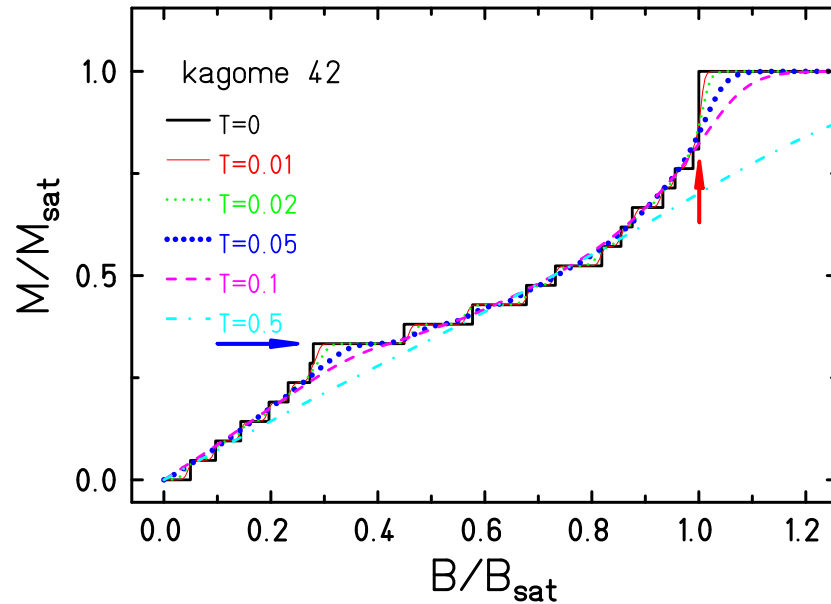
# Kagome 42 – susceptibility



- Singlet-triplet gap shrinks very slowly with increasing  $N$ .

(1) A. Laeuchli, J. Sudan, and R. Moessner, arXiv:1611.06990.  
 (2) J. Schnack, J. Schulenburg, J. Richter, Phys. Rev. B **98** (2018) 094423

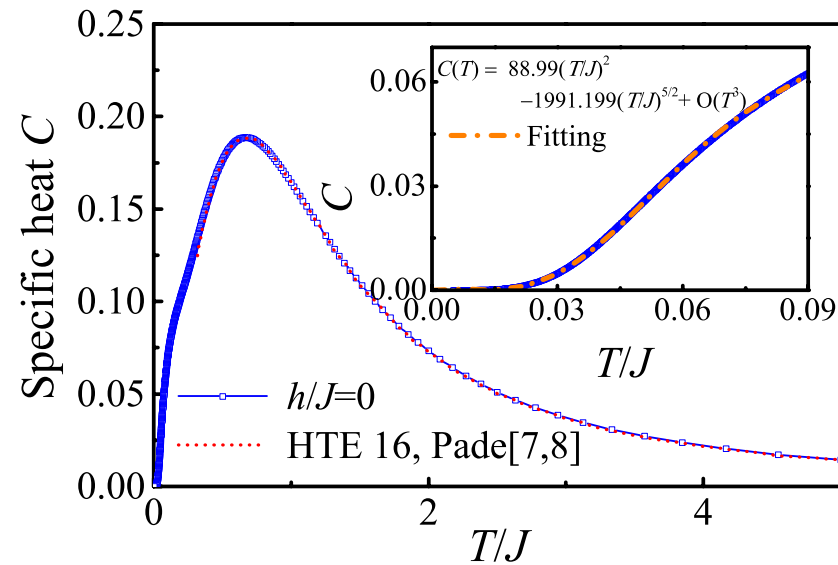
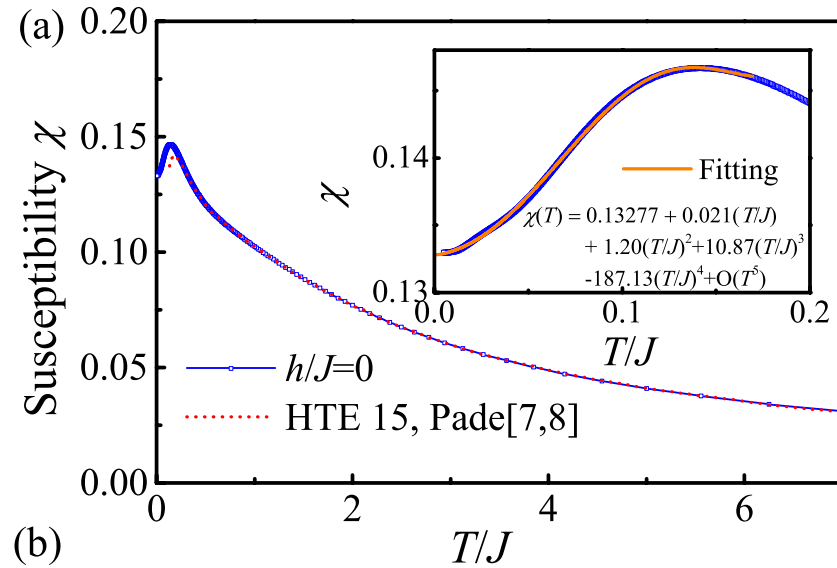
# Kagome 42 – magnetization



- Plateaus and jump; asymmetric melting of the plateau at  $\mathcal{M}_{\text{sat}}/3$ .

(1) S. Capponi, O. Derzhko, A. Honecker, A. M. Laeuchli, J. Richter, Phys. Rev. B **88**, 2 144416 (2013).  
 (2) J. Schulenburg, A. Honecker, J. Schnack, J. Richter, H.-J. Schmidt, Phys. Rev. Lett. **88**, 167207 (2002).  
 (3) H. Nakano and T. Sakai, J. Phys. Soc. Jpn. **83**, 104710 (2014).

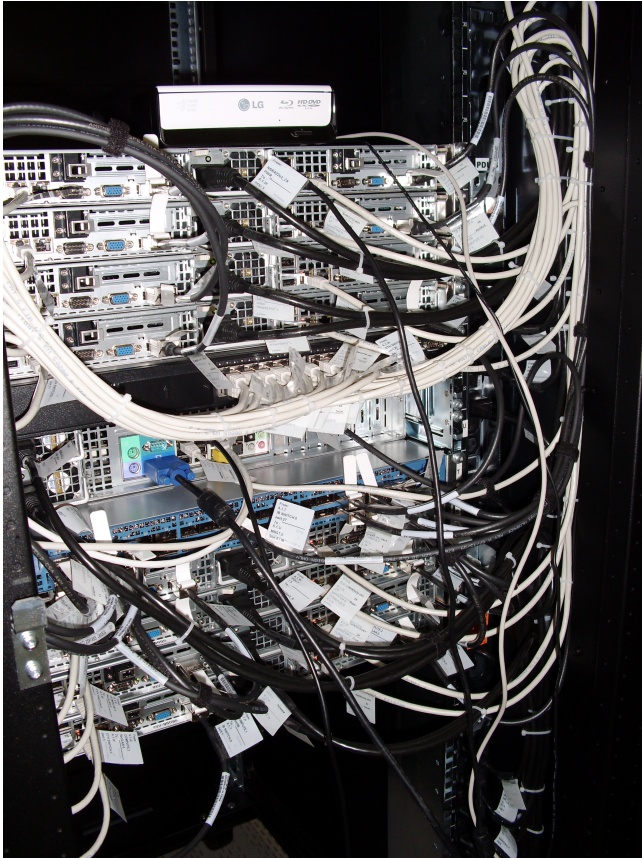
# Kagome – tensor network calculations



- Tensor network calculations for the infinite system (1).

(1) Xi Chen, Shi-Ju Ran, Tao Liu, Cheng Peng, Yi-Zhen Huang, Gang Su, Science Bulletin **63**, 1545 (2018).

# Summary



- Largest FTLM calculation for a spin system so far (5 Mio. core hours).
- Unexpected  $N$ -dependence of low- $T$  peak of heat capacity.
- $B$ -dependence of density of states leads to asymmetric melting of plateaus.

# Many thanks to my collaborators



- C. Beckmann, M. Czopnik, T. Glaser, O. Hanebaum, Chr. Heesing, M. Höck, N.B. Ivanov, H.-T. Langwald, A. Müller, R. Schnalle, Chr. Schröder, J. Ummethum (Bielefeld)
- **K. Bärwinkel, H.-J. Schmidt, M. Neumann (Osnabrück)**
- M. Luban (Ames Lab); P. Kögerler (Aachen, Jülich, Ames); D. Collison, R.E.P. Winpenny, E.J.L. McInnes, F. Tuna (Man U); L. Cronin, M. Murrie (Glasgow); E. Brechin (Edinburgh); H. Nojiri (Sendai, Japan); A. Postnikov (Metz); M. Evangelisti (Zaragosa); A. Honecker (U de Cergy-Pontoise); E. Garlatti, S. Carretta, G. Amoretti, P. Santini (Parma); A. Tennant (ORNL); Gopalan Rajaraman (Mumbai); M. Affronte (Modena)
- **J. Richter, J. Schulenburg** (Magdeburg); B. Lake (HMI Berlin); B. Büchner, V. Kataev, H.-H. Klauß (Dresden); A. Powell, W. Wernsdorfer (Karlsruhe); E. Rentschler (Mainz); J. Wosnitza (Dresden-Rossendorf); J. van Slageren (Stuttgart); R. Klingeler (Heidelberg); O. Waldmann (Freiburg)

Thank you very much for your  
attention.

The end.

Molecular Magnetism Web

[www.molmag.de](http://www.molmag.de)

Highlights. Tutorials. Who is who. Conferences.