

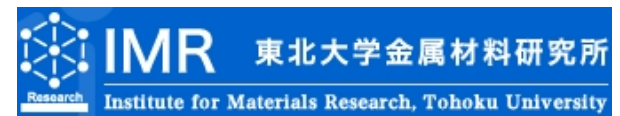
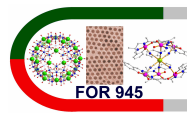
Evaluation of the local magnetization of Kondo-screened deposited magnetic molecules

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<http://obelix.physik.uni-bielefeld.de/~schnack/>

14th International Conference on Molecule-based Magnets
5-10 July 2014, St. Petersburg, Russia



Contents for you today

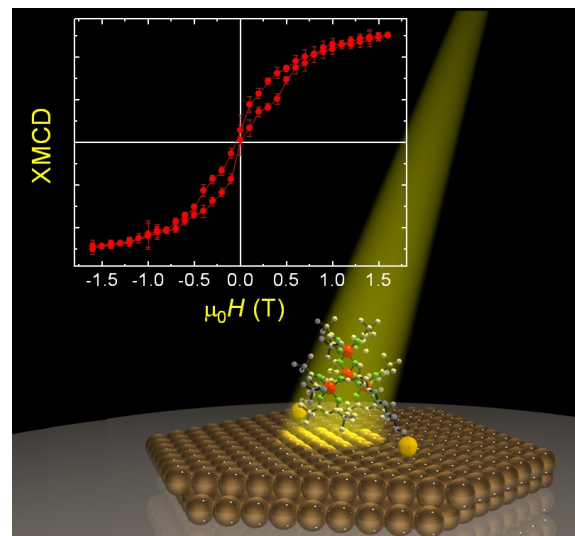
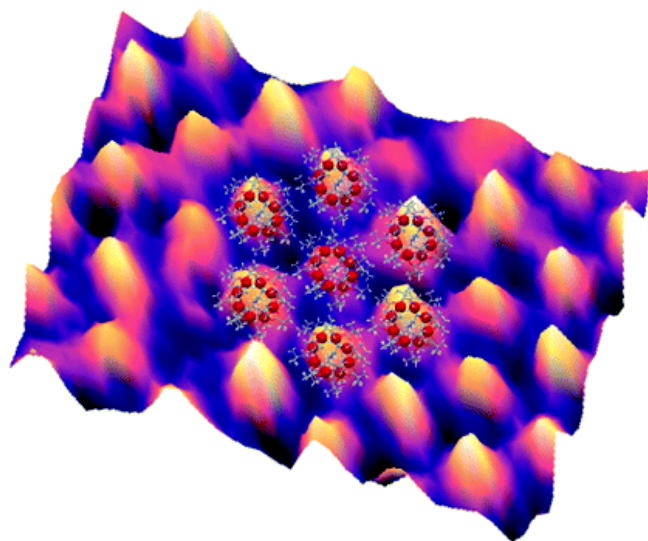
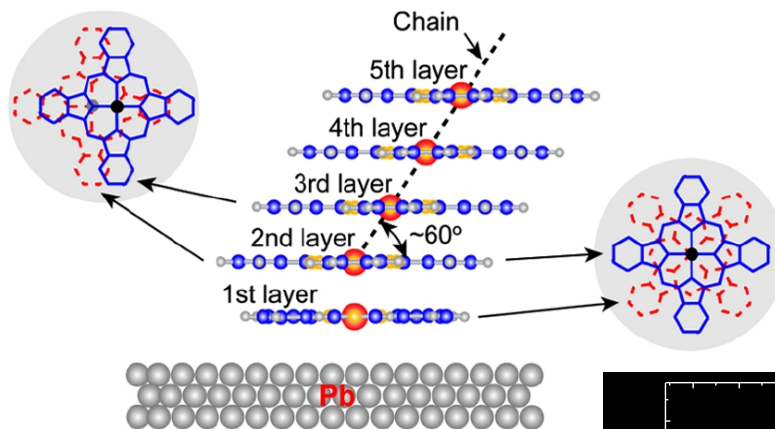
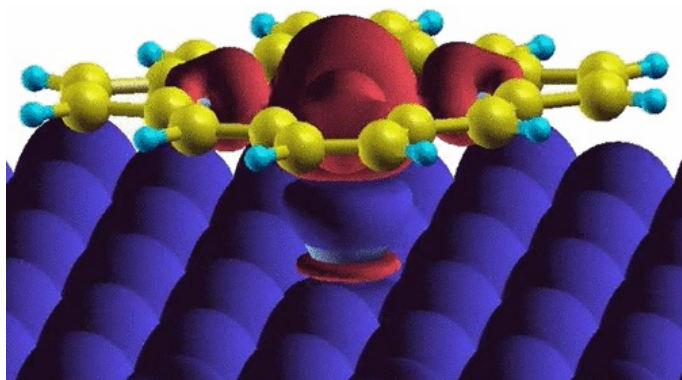


$$\begin{pmatrix} 3 & 42 & 4711 \\ 42 & 0 & 3.14 \\ 4711 & 3.14 & 8 \\ -17 & 007 & 13 \\ 1.8 & 15 & 081 \end{pmatrix}$$

1. Deposited magnetic molecules:
Numerical Renormalization Group
calculations
2. Size matters:
Finite-Temperature Lanczos Method

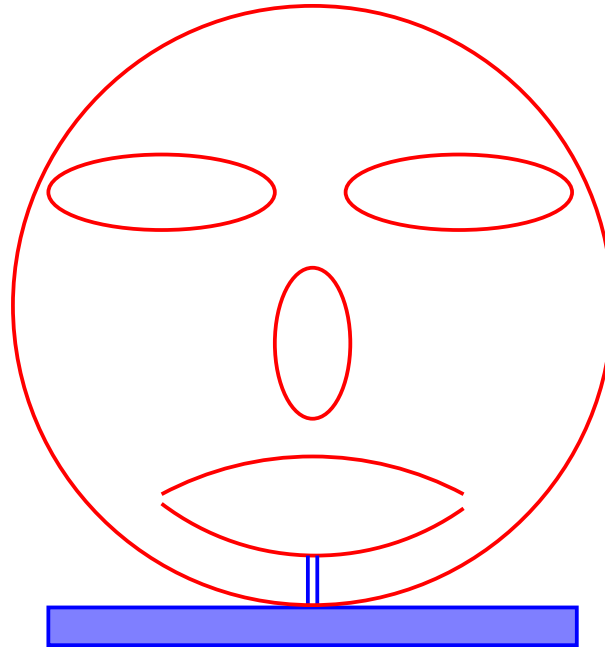
We are the sledgehammer team of matrix diagonalization.
Please send inquiries to jschnack@uni-bielefeld.de!

You want to deposit a molecule



M. Bernien *et al.*, Phys. Rev. Lett. **102**, 047202 (2009); A. Ghirriet *et al.*, ACS Nano, **5**, 7090-7099 (2011); X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008); M. Mannini *et al.*, Nature Materials **8**, 194 - 197 (2009).

From a bloody physicist's perspective ...



Molecule with nice properties deposited on non-magnetic metal substrate;
Exchange coupled to metal spins;
Kondo screening may ...

Properties may change drastically

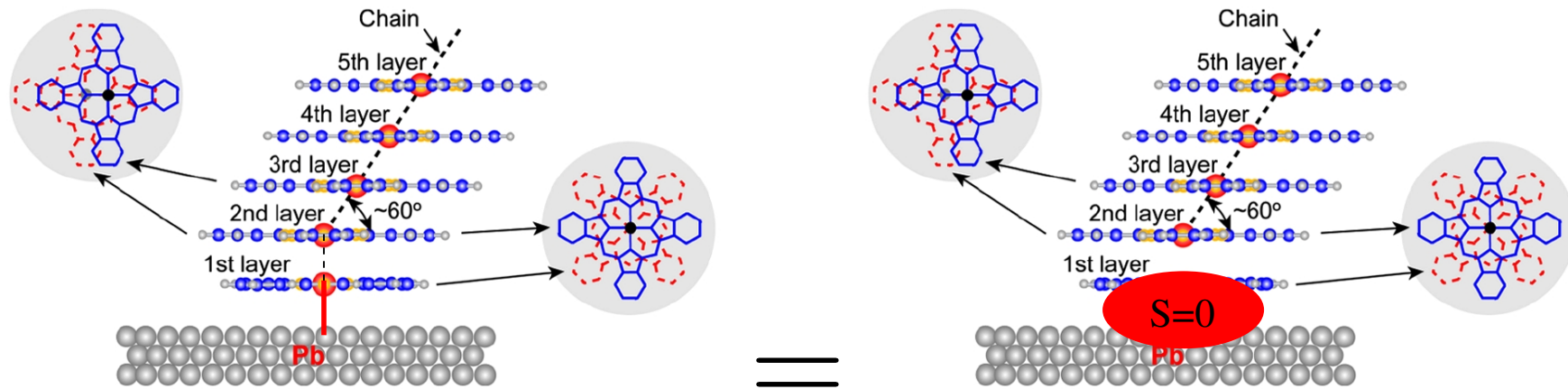


Kondo screening may improve or worsen the magnetic properties;

How does the exchange coupling to the metal influence the magnetic properties?

How to calculate such things?

Physical example (ICMM 2010)



Stack of deposited Cobalt phthalocyanine (CoPc) molecules;
 Co^{2+} with spin $s = 1/2$.

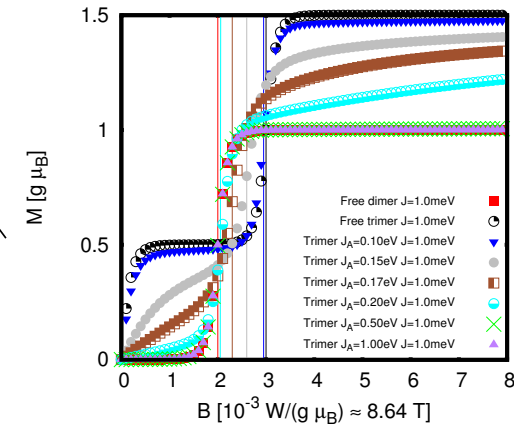
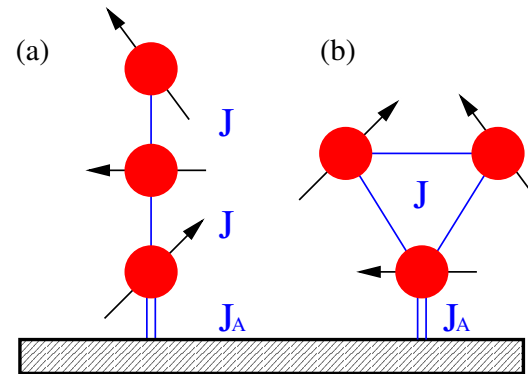
Under which circumstances is the picture of total screening correct?

X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008).

Numerical Renormalization Group calculations

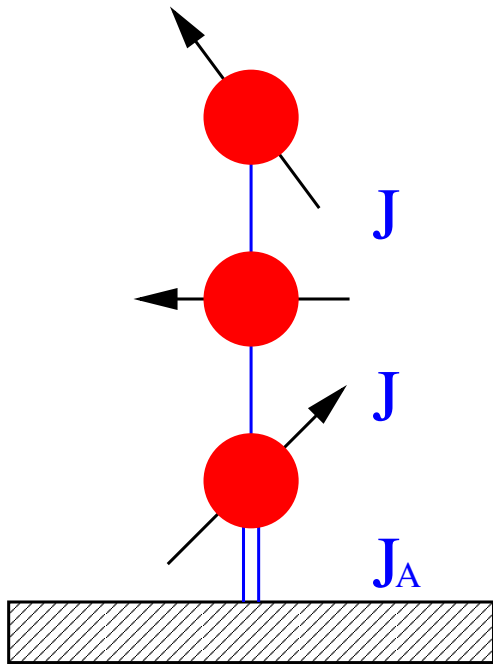
(Good for deposited molecules.)

Numerical Renormalization Group (Wilson)



- Magnetic properties of deposited spin systems;
- Martin Höck (until 07/2013): anisotropic single spins (PRB **87**, 184408 (2013));
- Henning-Timm Langwald: deposited Heisenberg systems.

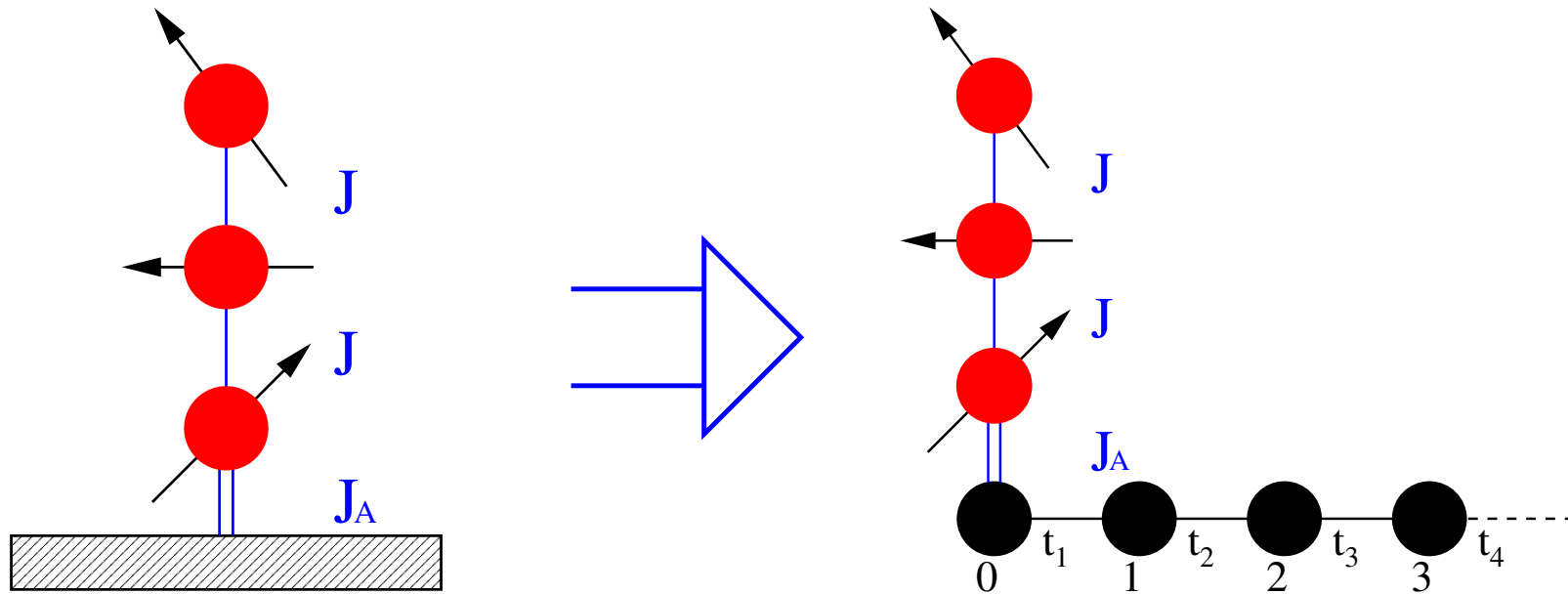
NRG – minimal model (already an approximation!)



- $\tilde{H} = \tilde{H}_{\text{electrons}} + \tilde{H}_{\text{coupling}} + \tilde{H}_{\text{impurity}}$
- $\tilde{H}_{\text{electrons}} = \sum_{i \neq j, \sigma} t_{ij} d_{i\sigma}^\dagger d_{j\sigma} + g_e \mu_B B \mathcal{S}^z$
- $\tilde{H}_{\text{coupling}} = -2J_A \mathcal{S} \cdot \underline{s}_0$, \underline{s}_0 – spin density at contact
- $\tilde{H}_{\text{impurity}} =$ Hamiltonian of your molecule!
- NRG \equiv construction of a small (!) effective model in order to evaluate properties of the deposited cluster, the impurity (3).

(1) K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975)
 (2) M. Höck, J. Schnack, Phys. Rev. B **87**, 184408 (2013)
 (3) *Impurity* is a technical term in this context and not an insult to chemists.

NRG in a cartoon



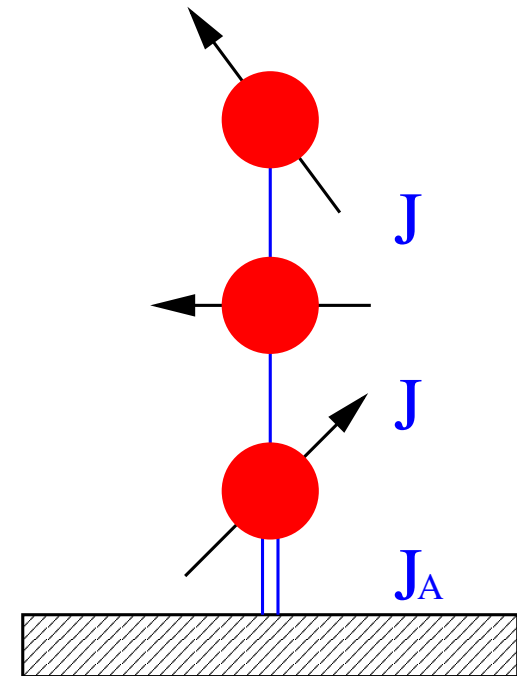
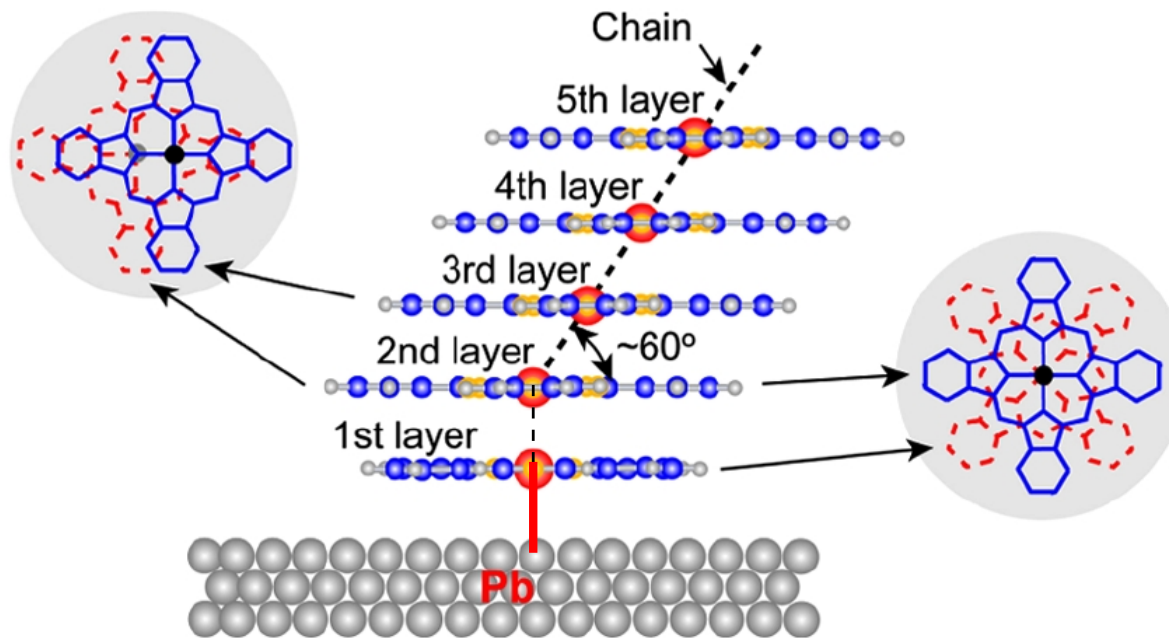
Metallic surface is replaced by semi-infinite Hubbard chain;

Parameters of the chain: hopping matrix elements and on-site energies;

Stepwise enlargement of the chain ($t_1 > t_2 > t_3 \dots$);

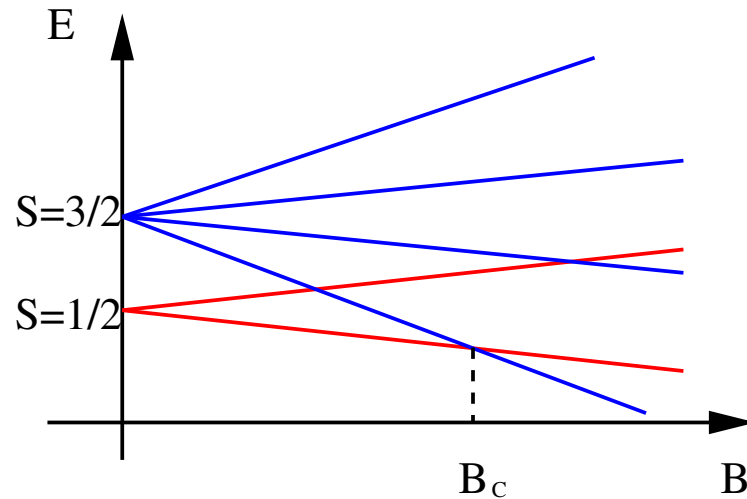
Truncation of basis set when matrices grow too big.

Once more: deposited chain

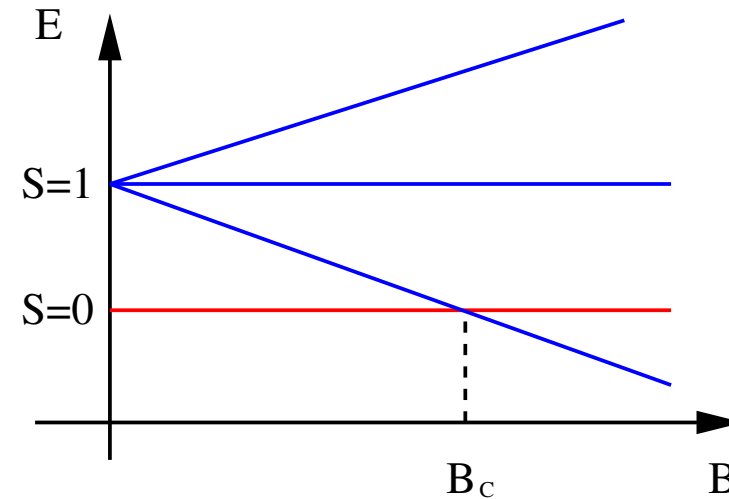


X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008).

Energy levels of limiting cases for deposited trimer



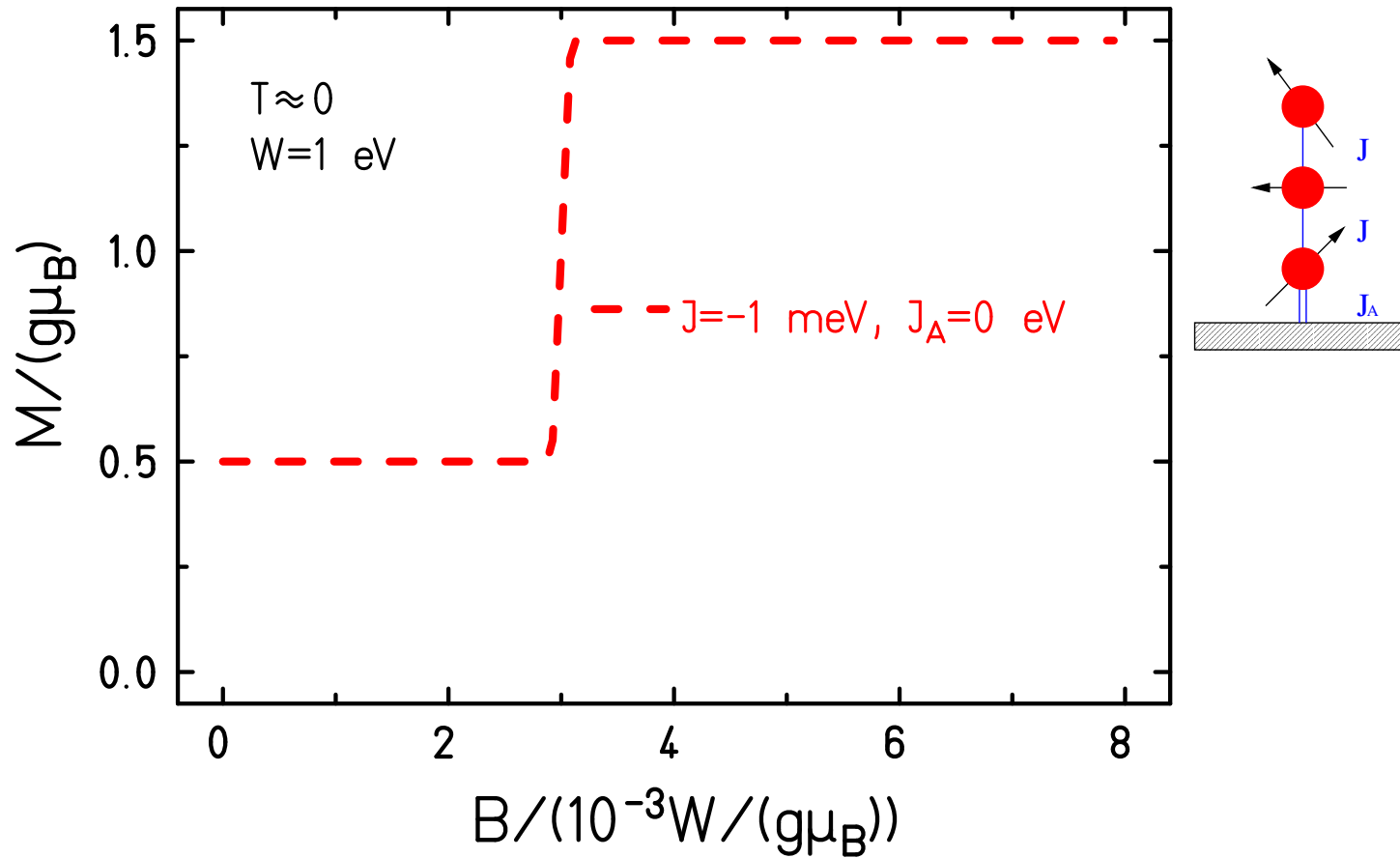
• energy levels of a trimer



• energy levels of a dimer

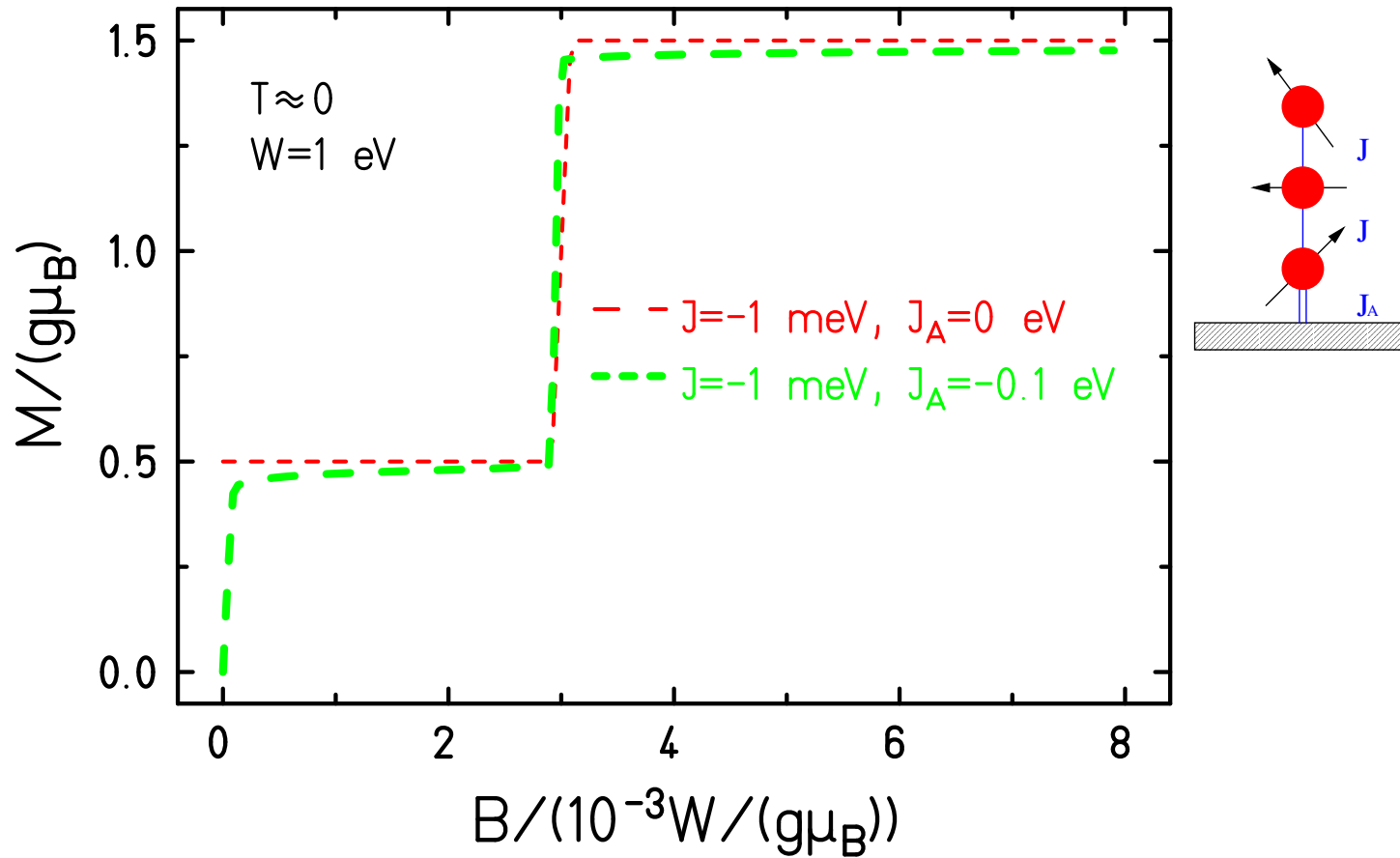
Magnetization curves different; could be seen in XMCD.
 NRG calculates observables also between limiting cases
 and can thus tell under which circumstances a limiting case applies.

Increasing coupling to the substrate



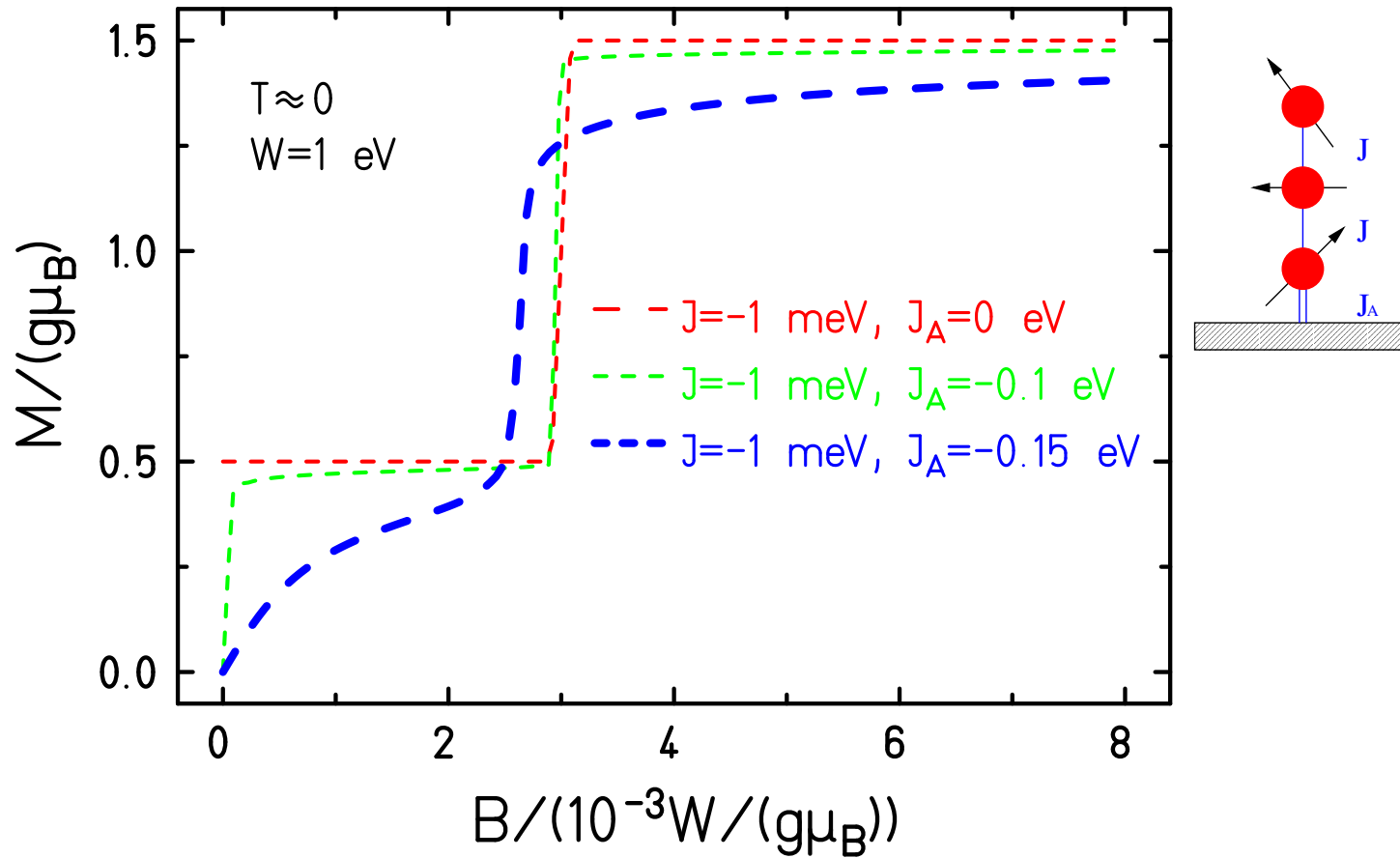
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

Increasing coupling to the substrate



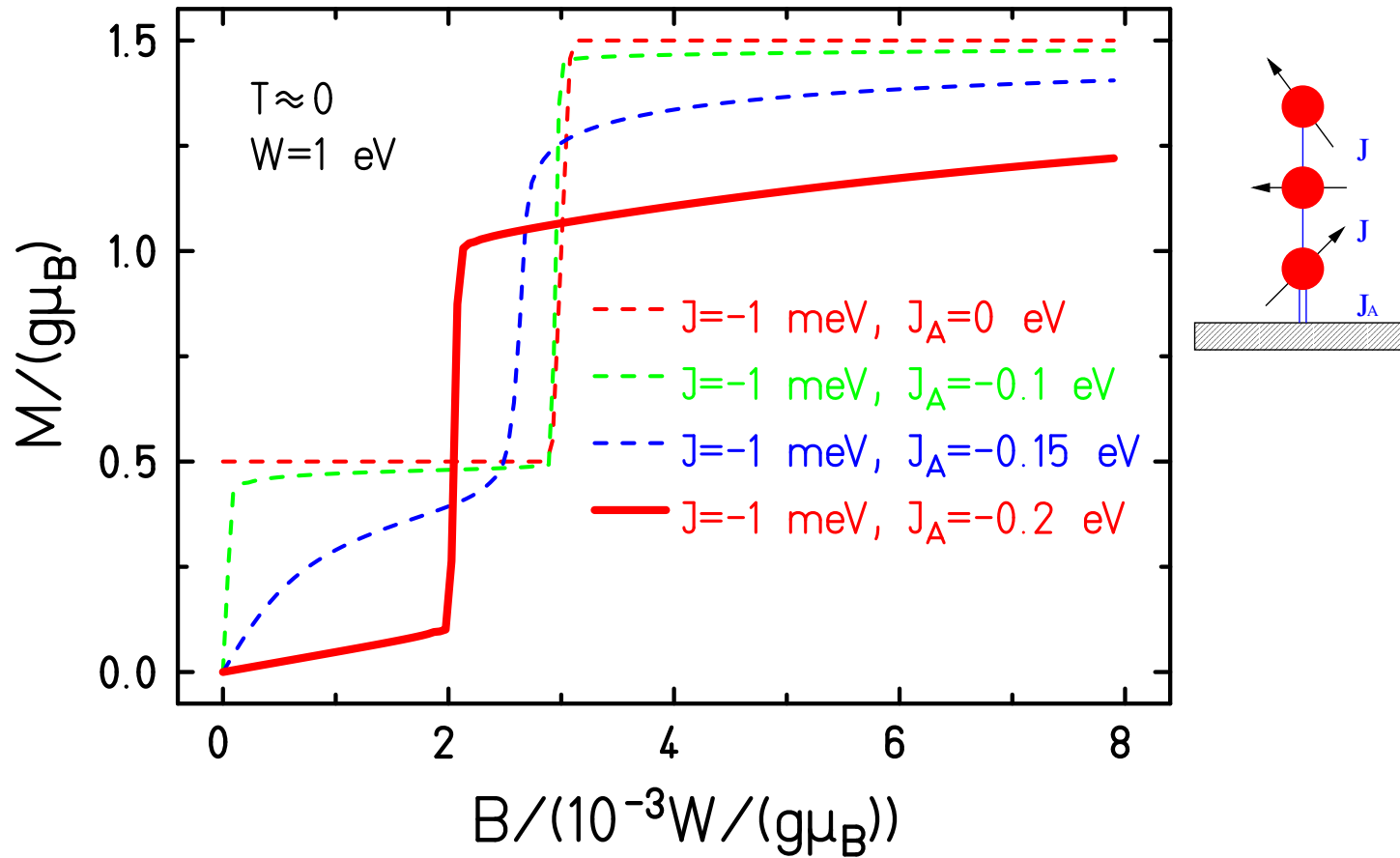
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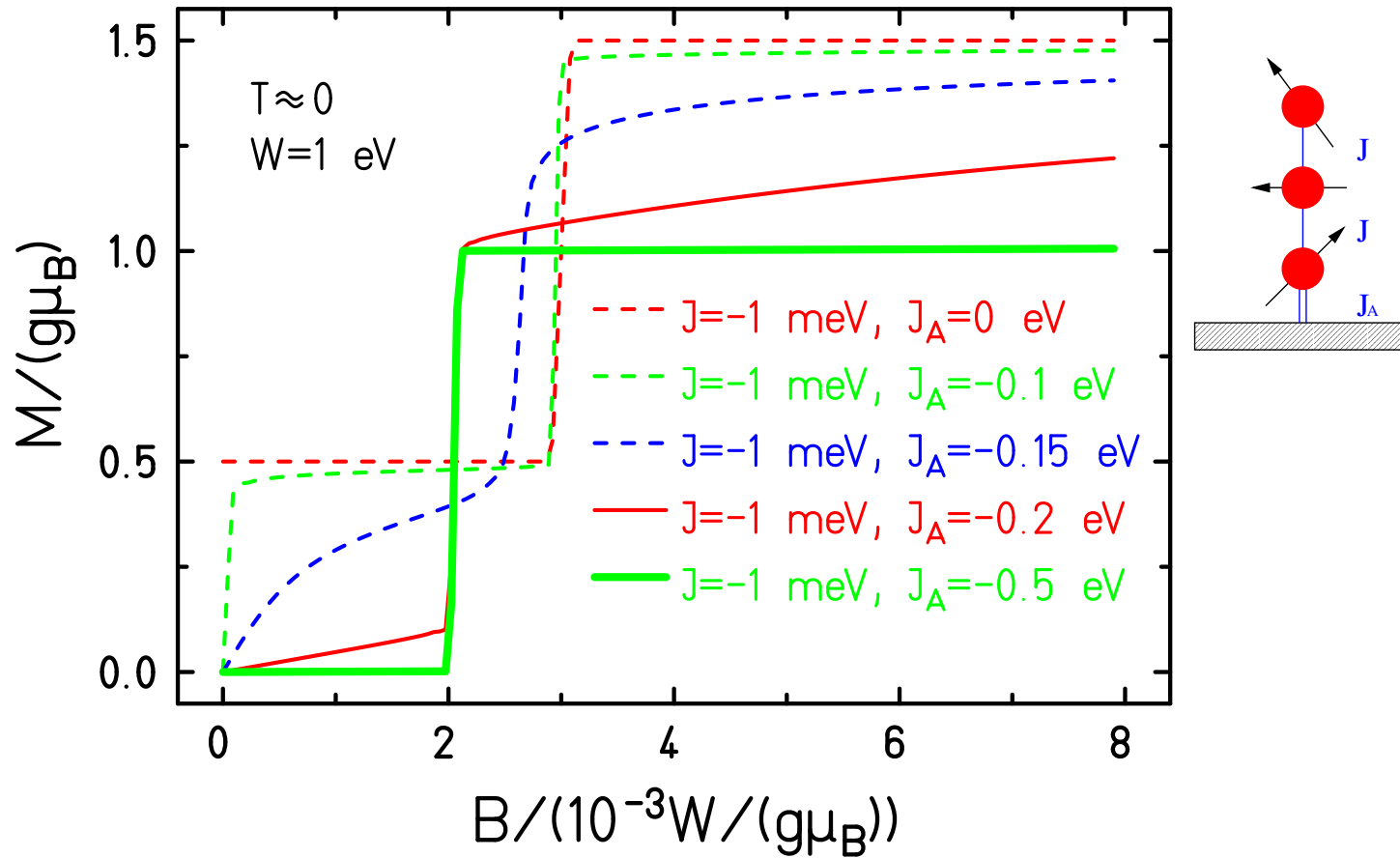
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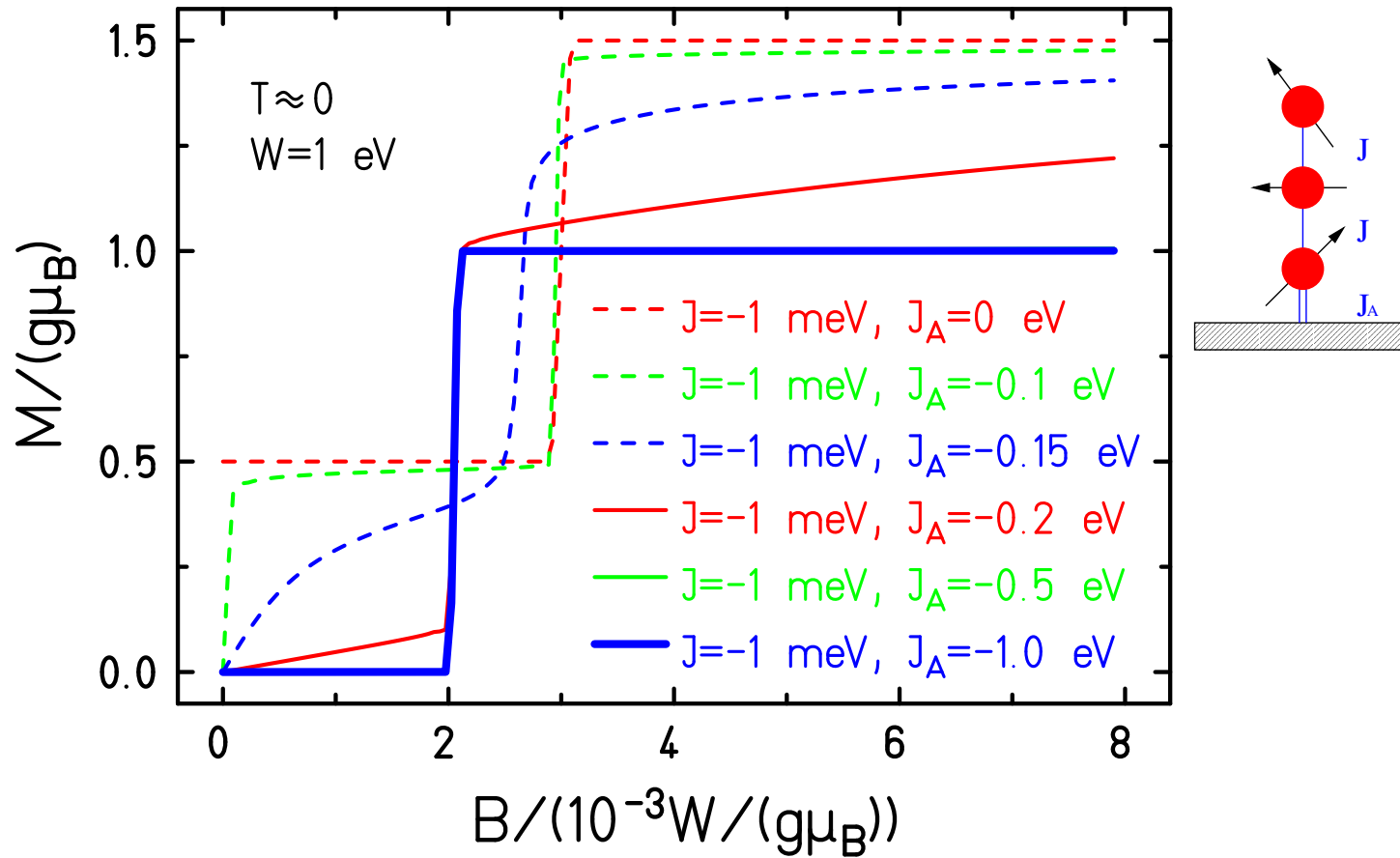
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Increasing coupling to the substrate



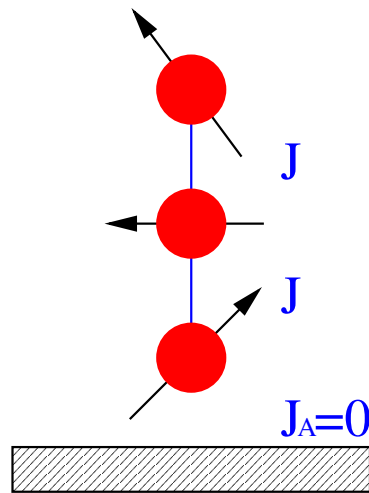
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

Increasing coupling to the substrate



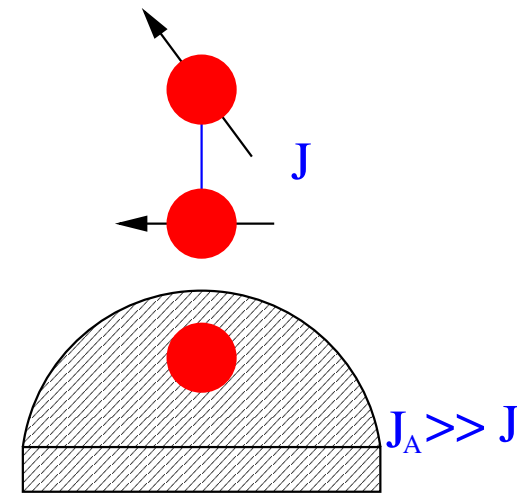
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

Weak vs. strong coupling



- weak coupling limit:
unperturbed molecule (trimer)

- $|J_A| \lesssim 0.1W$



- strong coupling limit:
effective remainder (dimer)

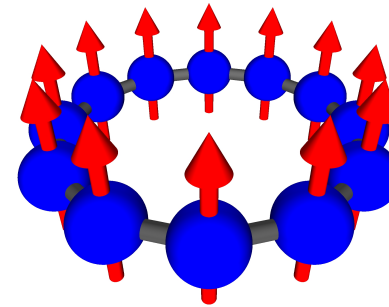
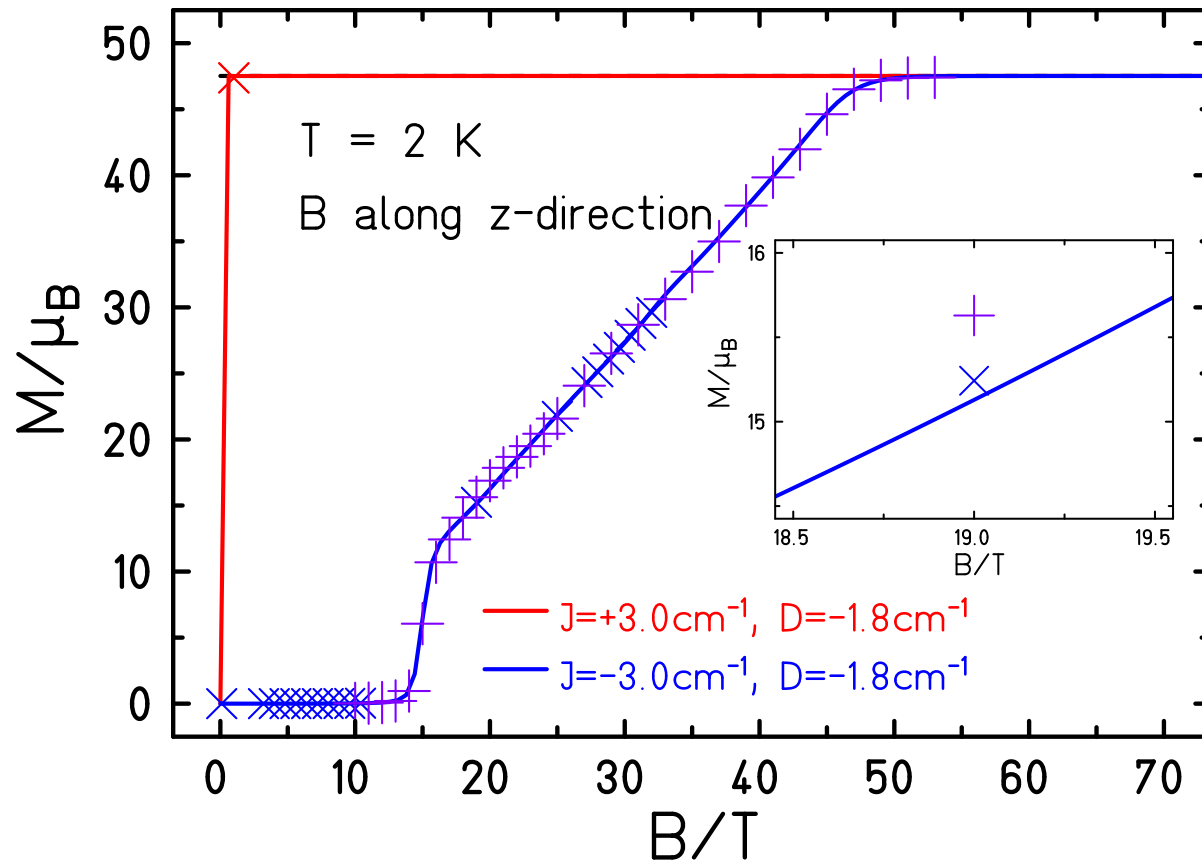
- $|J_A| \gtrsim 0.5W$

Inbetween: no simple characterization + further sequential screening possible

Finite-Temperature Lanczos Method

(Good for dimensions up to 10^{10} .)

A fictitious $\text{Mn}_{12}^{\text{III}}$ – M_z vs B_z



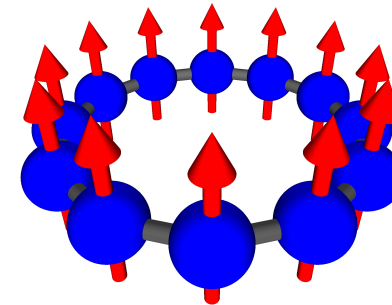
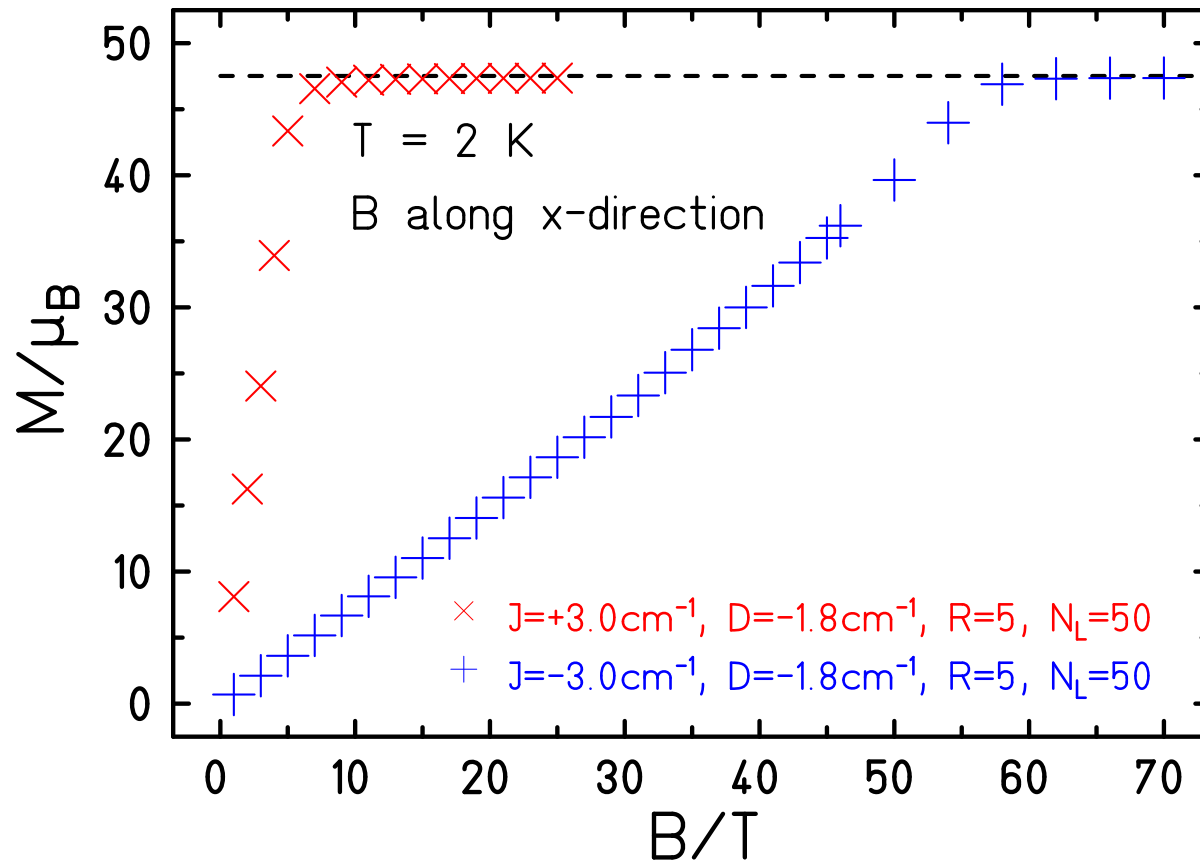
$s = 2$

$\dim(\mathcal{H}) = 244, 140, 625$
 collinear easy axes

A few days compared to *impossible!*

O. Hanebaum, J. Schnack, submitted; arXiv:1405.3068.

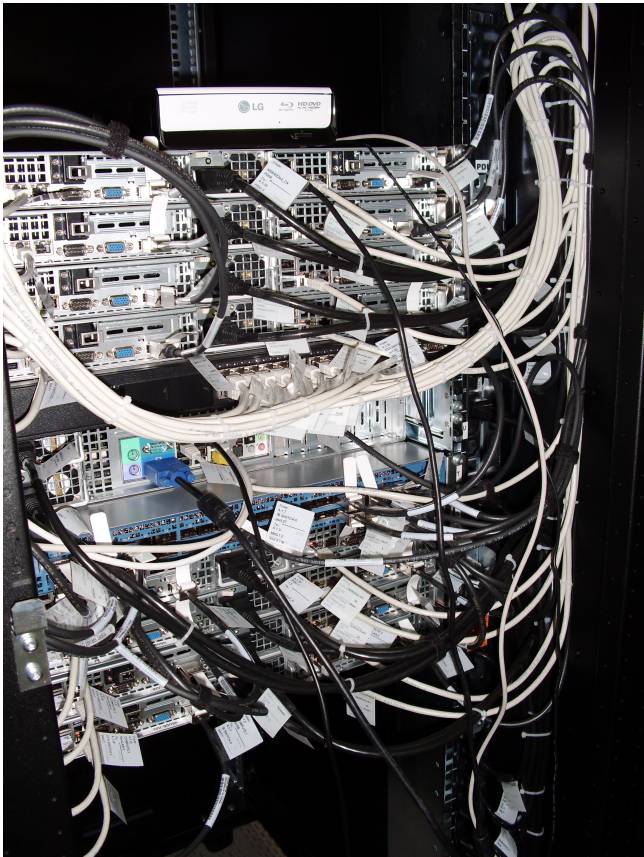
A fictitious $\text{Mn}_{12}^{\text{III}}$ – M_x vs B_x



No other method can deliver these curves!

O. Hanebaum, J. Schnack, submitted; arXiv:1405.3068.

Summary



- Magnetic molecules change their properties on metallic surfaces.
- Question: appropriate model? NRG deals with molecules that are exchange-coupled to the substrate.
- NRG delivers local observables, such as magnetization, which can be compared with XMCD results.
- Screening can lead to interesting limiting cases, which might show different (worse/better) behavior compared to the free molecule.
- We can treat anisotropic spin Hamiltonians for Hilbert space dimensions up to 10^{10} !

Many thanks to my collaborators worldwide

- M. Czopnik, T. Glaser, O. Hanebaum, Chr. Heesing, N.B. Ivanov, F. Kaiser, H.-T. Langwald, A. Müller, Chr. Schröder (Bielefeld)
- K. Bärwinkel, H.-J. Schmidt, M. Neumann (Osnabrück)
- M. Luban (Ames Lab, USA); P. Kögerler (Aachen, Jülich, Ames); R.E.P. Wimpenny, E.J.L. McInnes (Man U, UK); L. Cronin, M. Murrie (Glasgow, UK); E. Brechin (Edinburgh, UK); H. Nojiri (Sendai, Japan); A. Postnikov (Metz, France); M. Evangelisti (Zaragoza), A. Tennant (Oak Ridge, USA)
- J. Richter, J. Schulenburg (Magdeburg); A. Honecker (Göttingen); U. Kortz (Bremen); B. Lake (HMI Berlin); B. Büchner, V. Kataev, H.-H. Klauß (Dresden); P. Chaudhuri (Mühlheim); J. Wosnitza (Dresden-Rossendorf); J. van Slageren (Stuttgart); R. Klingeler (Heidelberg); O. Waldmann (Freiburg)

Thank you very much for your
attention.

The end.

Molecular Magnetism Web

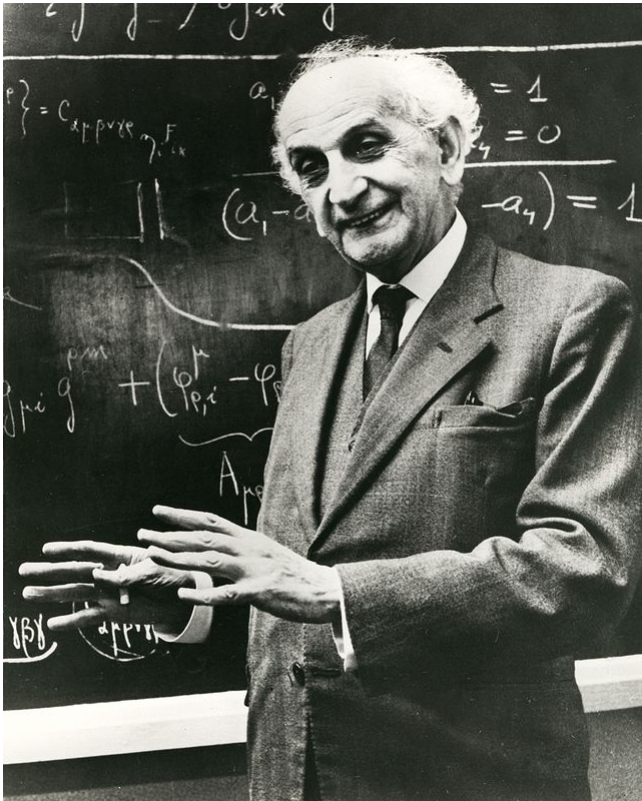
www.molmag.de

Highlights. Tutorials. Who is who. Conferences.

Finite-Temperature Lanczos Method

(Good for dimensions up to 10^{10} .)

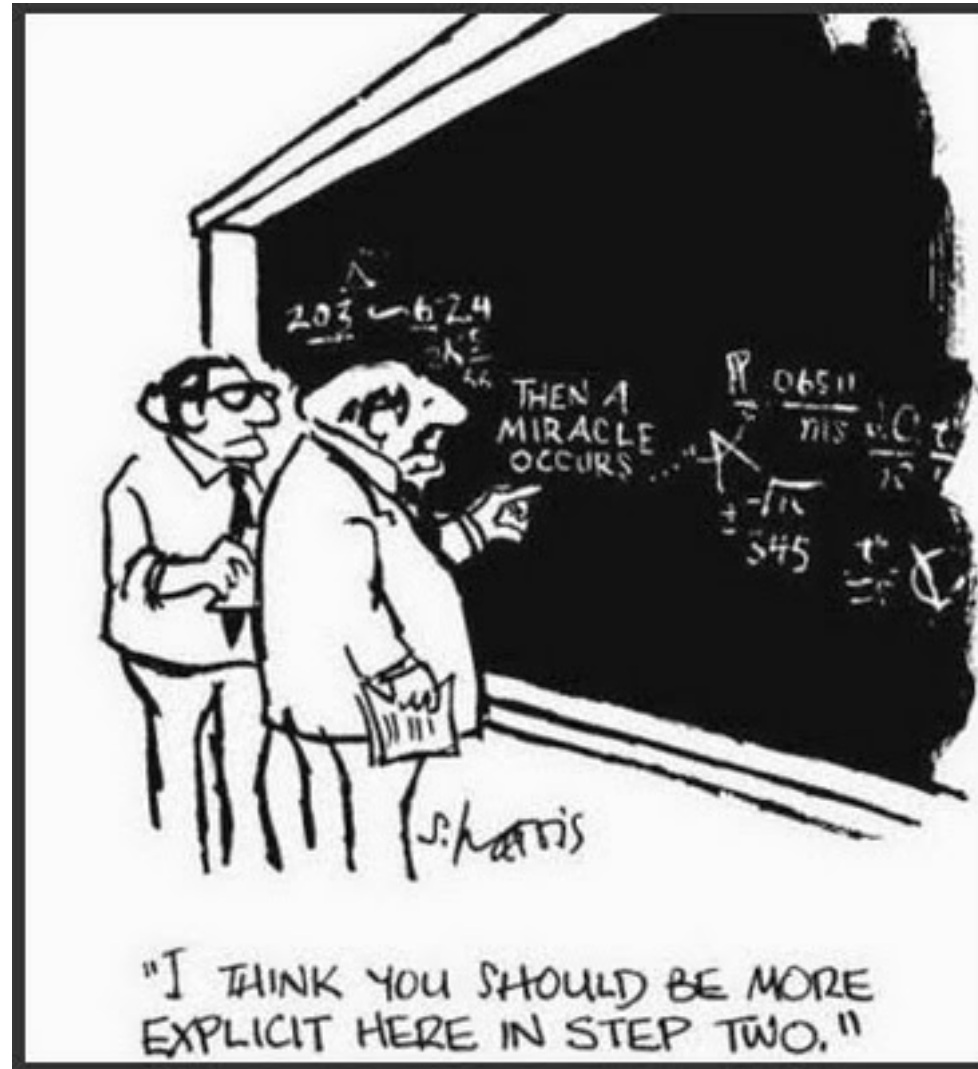
Lanczos – a Krylov space method



Cornelius Lanczos
(1893-1974)

- You do know exact diagonalization. What about diagonalization in reduced basis sets?!
Full matrix \implies small matrix!
- But which set to choose???
- Idea: generate the basis set with the operator you want to diagonalize:
 $\{ |\phi\rangle, \tilde{H}|\phi\rangle, \tilde{H}^2|\phi\rangle, \tilde{H}^3|\phi\rangle, \dots \}$
 Hamiltonian creates its own relevant states!
- But which starting vector to choose???
- Idea: almost any will do!

(1) C. Lanczos, J. Res. Nat. Bur. Stand. **45**, 255 (1950).



Finite-temperature Lanczos Method I

$$Z(T, B) = \sum_{\nu} \langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle$$

$$\langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle \approx \sum_n \langle \nu | n(\nu) \rangle \exp \left\{ -\beta \epsilon_n \right\} \langle n(\nu) | \nu \rangle$$

$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp \left\{ -\beta \epsilon_n \right\} |\langle n(\nu) | \nu \rangle|^2$$

- $|n(\nu)\rangle$ n-th Lanczos eigenvector starting from $|\nu\rangle$
- Partition function replaced by a small sum: $R = 1 \dots 100, N_L \approx 100$.

J. Jaklič and P. Prelovšek, Phys. Rev. B **49**, 5065 (1994).

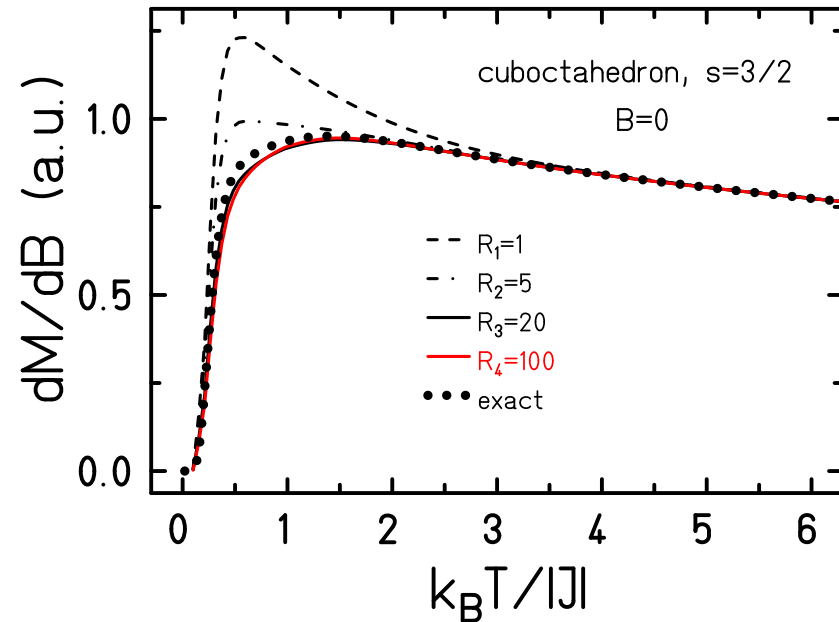
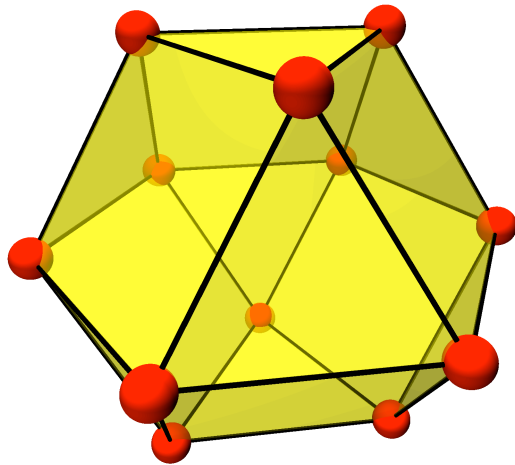
Finite-temperature Lanczos Method II

$$Z(T, B) \approx \sum_{\Gamma} \frac{\dim(\mathcal{H}(\Gamma))}{R_{\Gamma}} \sum_{\nu=1}^{R_{\Gamma}} \sum_{n=1}^{N_L} \exp\{-\beta\epsilon_n\} |\langle n(\nu, \Gamma) | \nu, \Gamma \rangle|^2$$

- Approximation better if symmetries are taken into account.
- Γ denotes the used irreducible representations.

J. Schnack and O. Wendland, Eur. Phys. J. B **78** (2010) 535-541

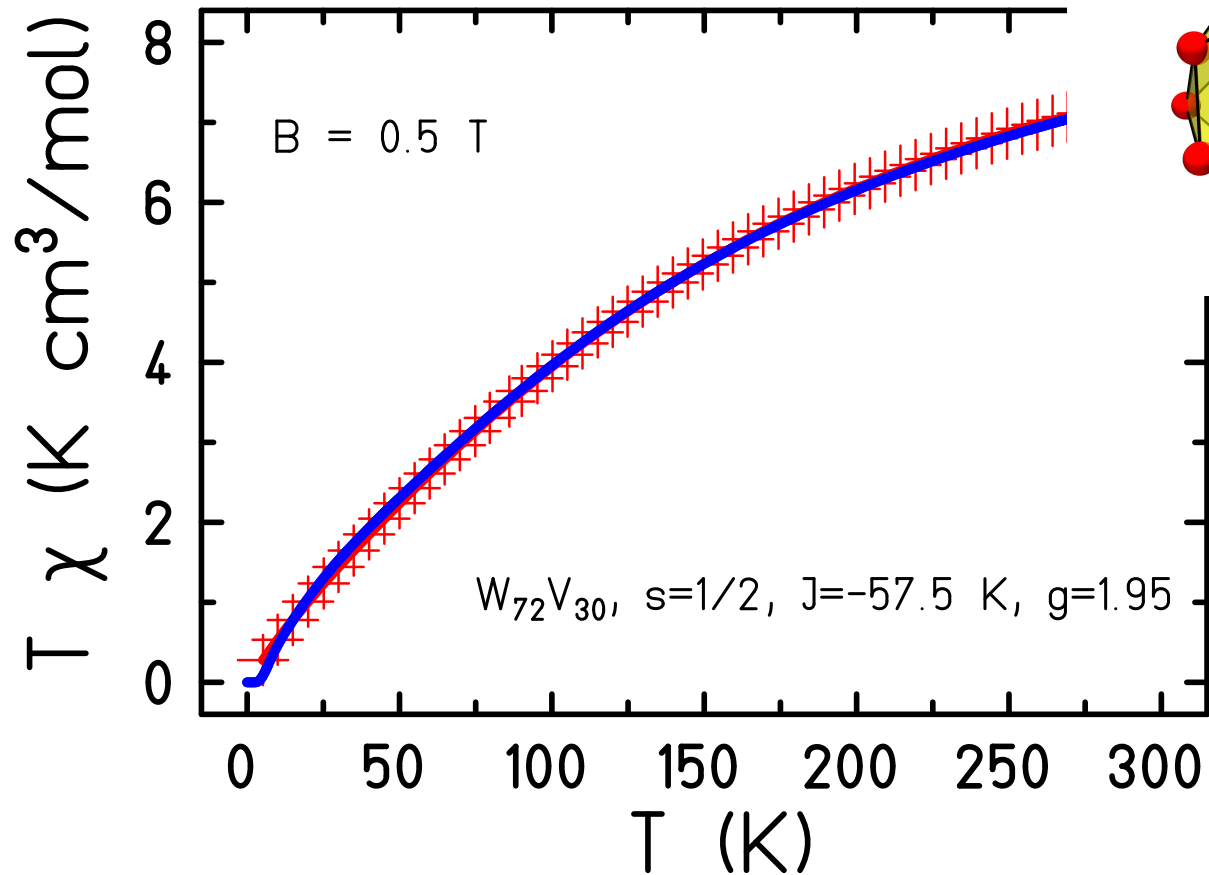
How good is finite-temperature Lanczos?



- Works very well: compare frustrated cuboctahedron.
- $N = 12, s = 3/2$: Considered $< 100,000$ states instead of $16,777,216$.

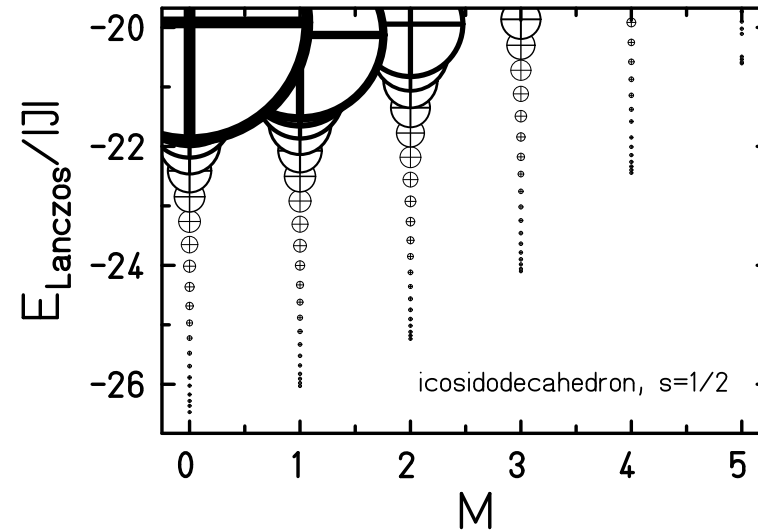
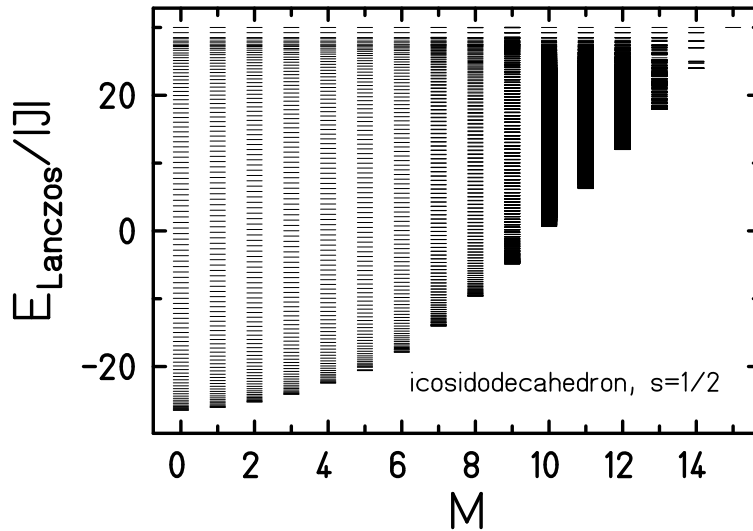
Exact results: R. Schnalle and J. Schnack, *Int. Rev. Phys. Chem.* **29**, 403-452 (2010).
 FTLM: J. Schnack and O. Wendland, *Eur. Phys. J. B* **78**, 535-541 (2010).

Icosidodecahedron $s = 1/2$



Exp. data: A. M. Todea, A. Merca, H. Bögge, T. Glaser, L. Engelhardt, R. Prozorov, M. Luban, A. Müller, Chem. Commun., 3351 (2009).

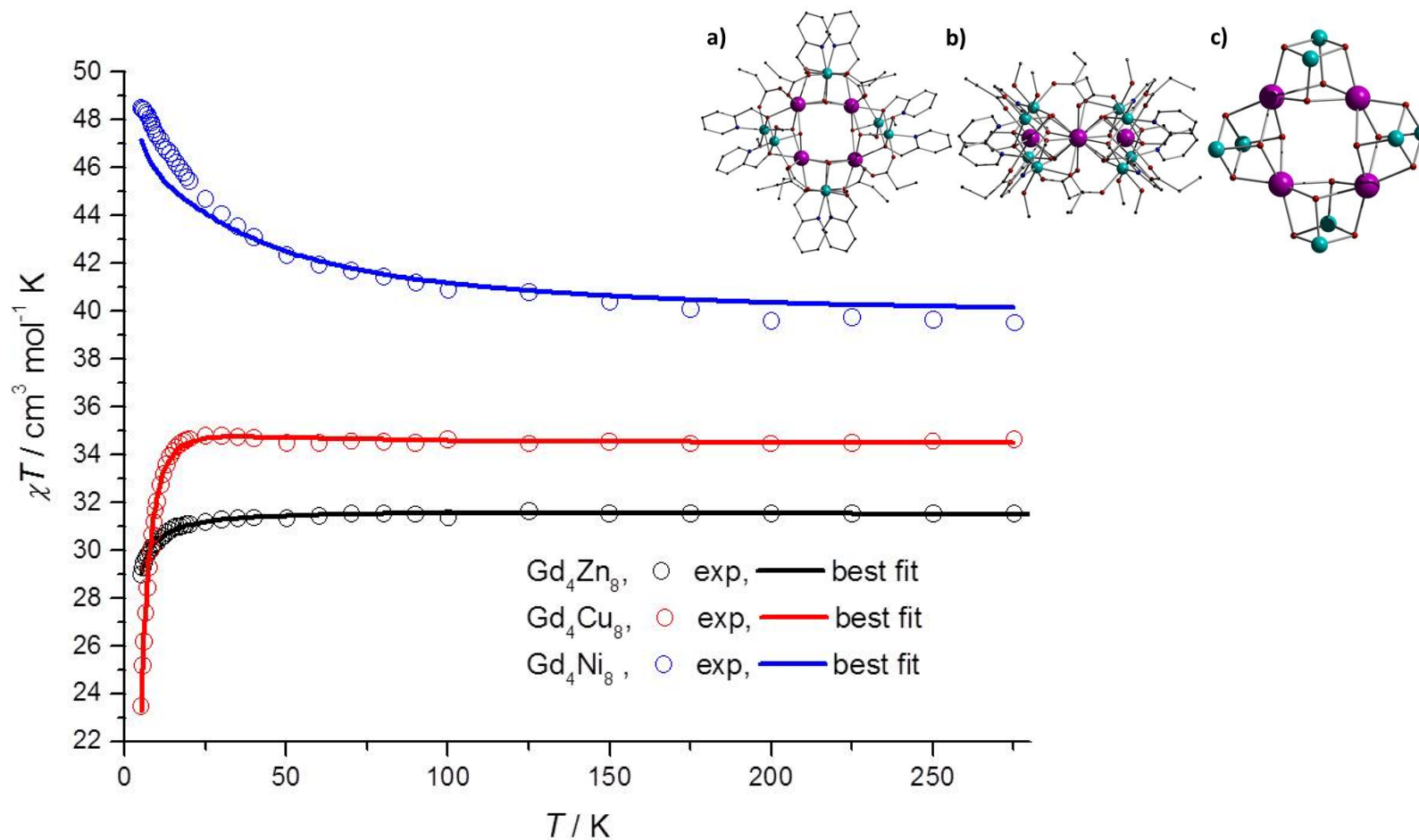
Icosidodecahedron $s = 1/2$



- The true spectrum will be much denser. This is miraculously compensated for by the weights. (Exact at low T , coarse grained at high T .)



$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp\{-\beta\epsilon_n\} |\langle n(\nu, \Gamma) | \nu, \Gamma \rangle|^2$$

Gd₄M₈ – Susceptibility

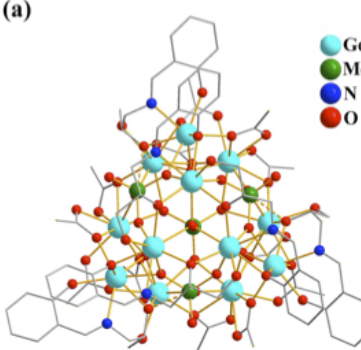


T. N. Hooper, J. Schnack, St. Piligkos, M. Evangelisti, E. K. Brechin, *Angew. Chem. Int. Ed.* **51** (2012) 4633-4636.

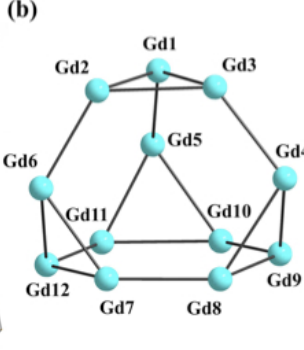
Recent developments

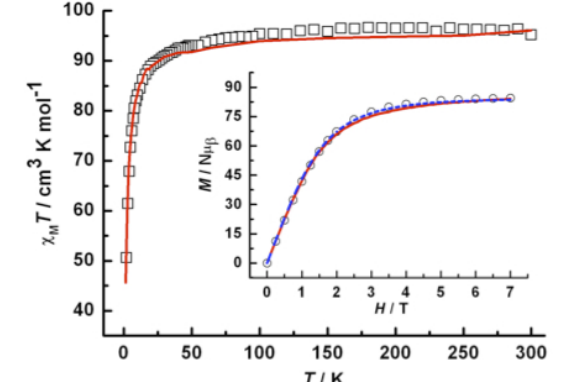



(a)



(b)





$12 \times s = 7/2$, dimension = 68,719,476,736

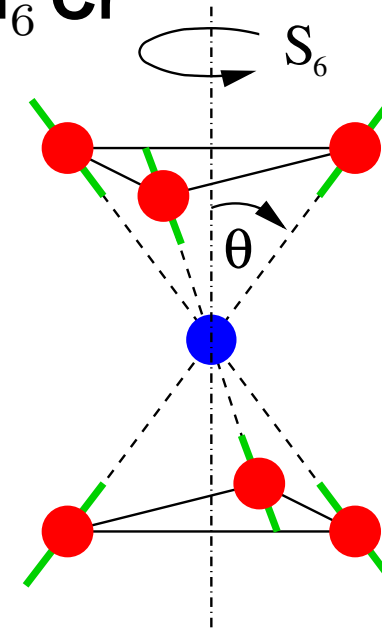
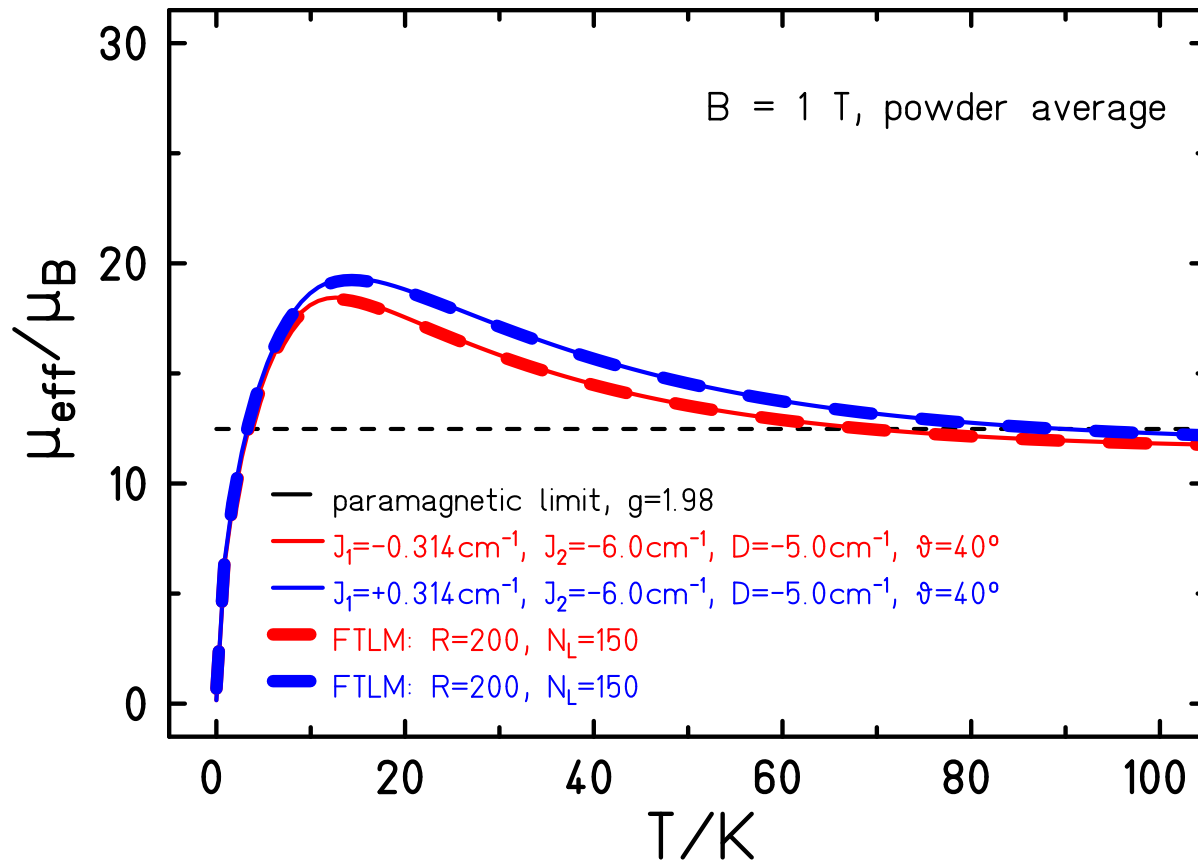
- Goal: magnetic properties of anisotropic systems;
- Oliver Hanebaum: single-ion anisotropy;
- Christian Heesing: Dzyaloshinskii-Moriya & anisotropic exchange.

Hamiltonian with single-ion anisotropy

$$\tilde{H}(\vec{B}) = -2 \sum_{i < j} J_{ij} \vec{\tilde{S}}_i \cdot \vec{\tilde{S}}_j + \sum_i d_i (\vec{e}_i \cdot \vec{\tilde{S}}_i)^2 + \mu_B \vec{B} \cdot \sum_i^N g_i \vec{\tilde{S}}_i$$

- $[\tilde{H}, \vec{\tilde{S}}^2] \neq 0, [\tilde{H}, \tilde{S}_z] \neq 0; \Rightarrow$ **MAGPACK does not work!**
- You have to diagonalize $\tilde{H}(\vec{B})$ for every field (direction and strength)!
- Orientational average for powder samples.

Glaser-type molecules: $\text{Mn}_6\text{Cr}^{\text{III}}$

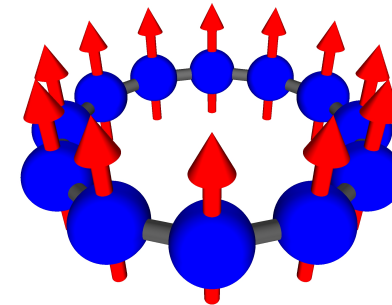
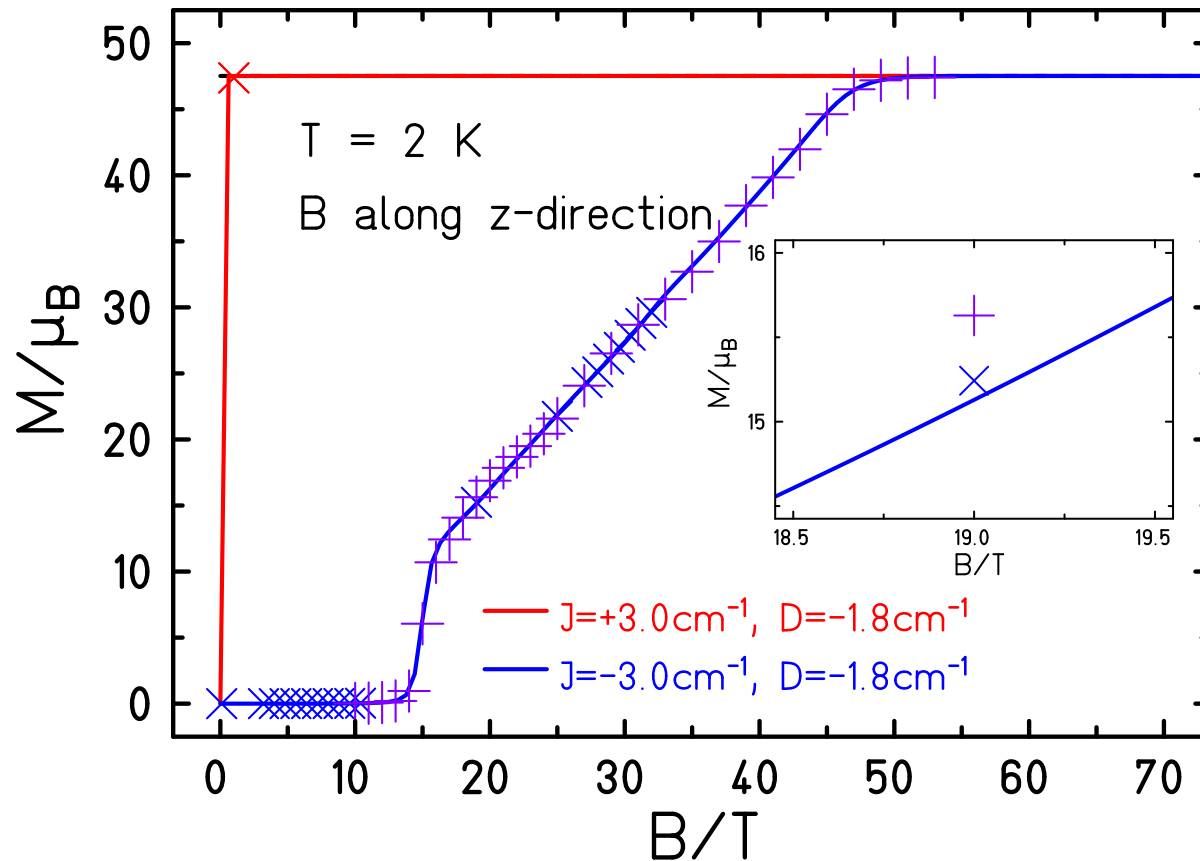


$s = 2, s = 3/2$
 $\dim(\mathcal{H}) = 62,500$
 non-collinear easy axes

Hours compared to days, notebook compared to supercomputer!

O. Hanebaum, J. Schnack, submitted; arXiv:1405.3068.

A fictitious $\text{Mn}_{12}^{\text{III}}$ – M_z vs B_z



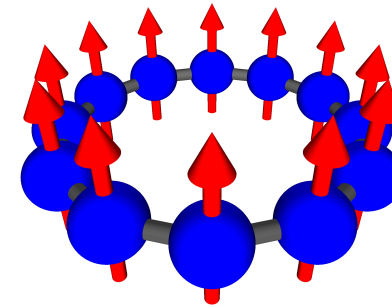
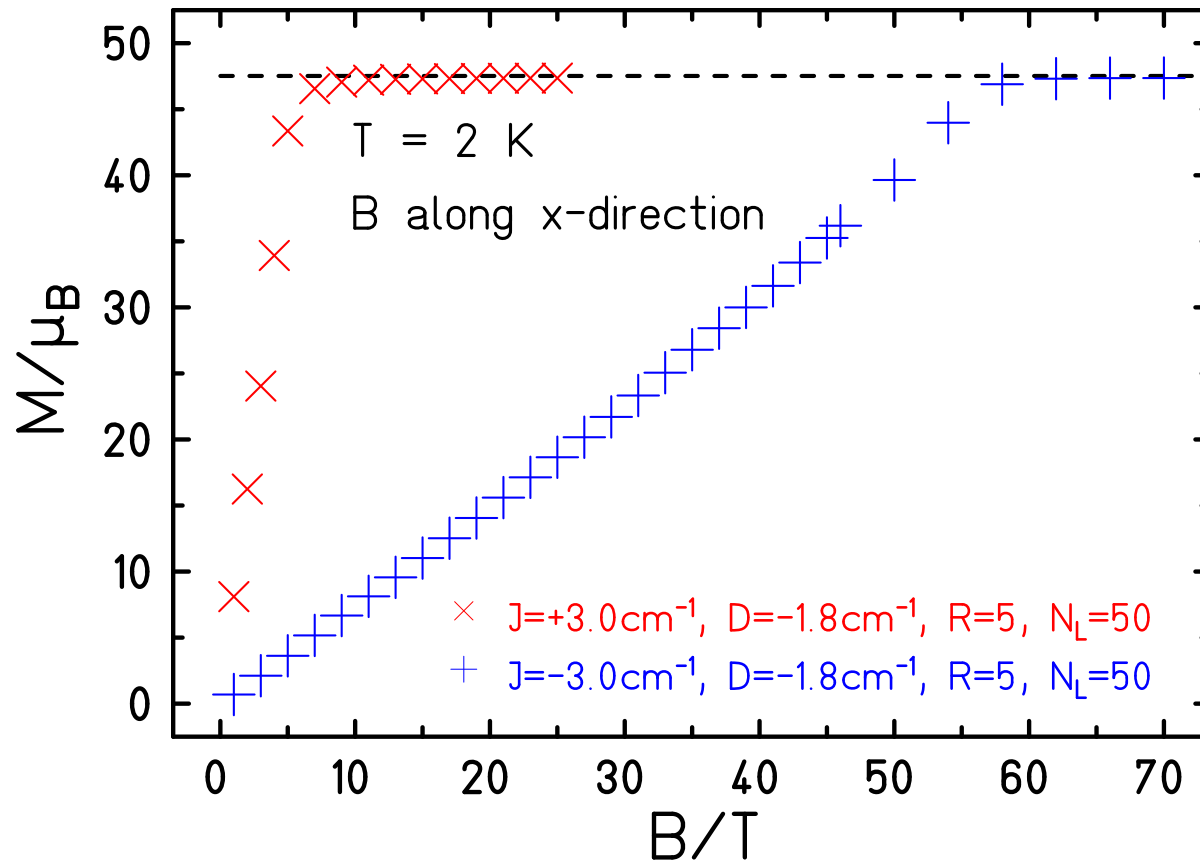
$s = 2$

$\dim(\mathcal{H}) = 244, 140, 625$
 collinear easy axes

A few days compared to *impossible!*

O. Hanebaum, J. Schnack, submitted; arXiv:1405.3068.

A fictitious $\text{Mn}_{12}^{\text{III}}$ – M_x vs B_x



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