

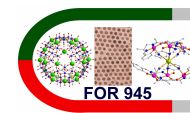
# Advanced quantum many-body methods for magnetic molecules: what theory can do for you

Jürgen Schnack

Department of Physics – University of Bielefeld – Germany

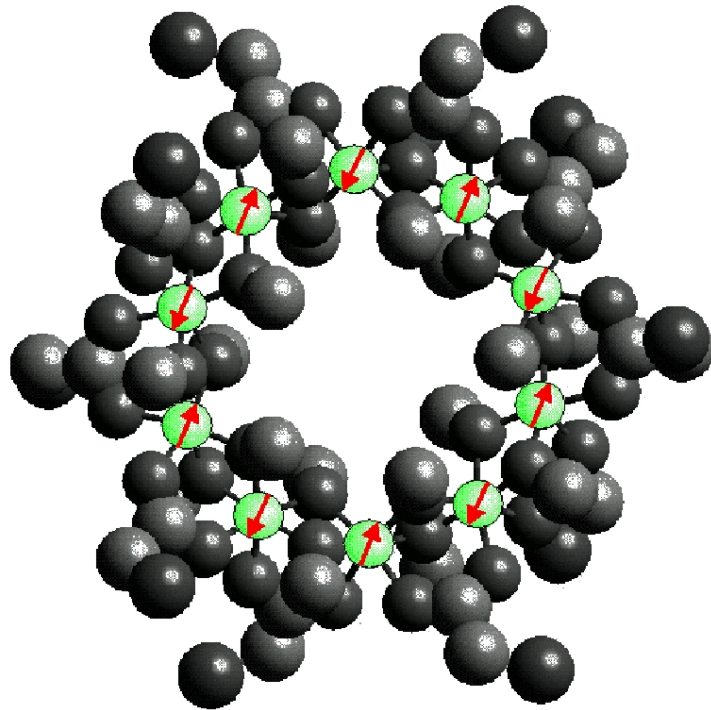
<http://obelix.physik.uni-bielefeld.de/~schnack/>

Seminar, Prof. Yamashita's group  
11. 06. 2015, Sendai, Japan



# The problem

# You have got a molecule!



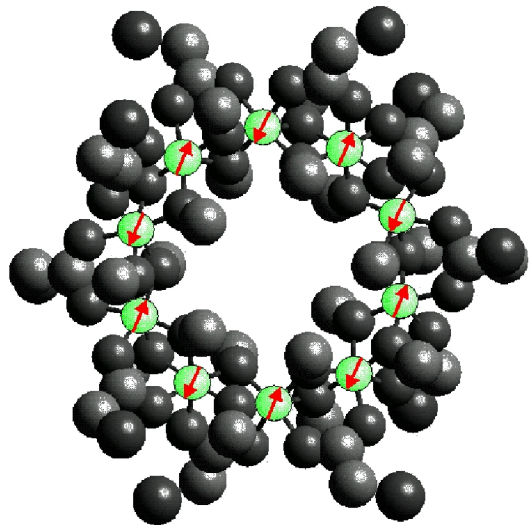
Congratulations!

You have got an idea about the modeling!

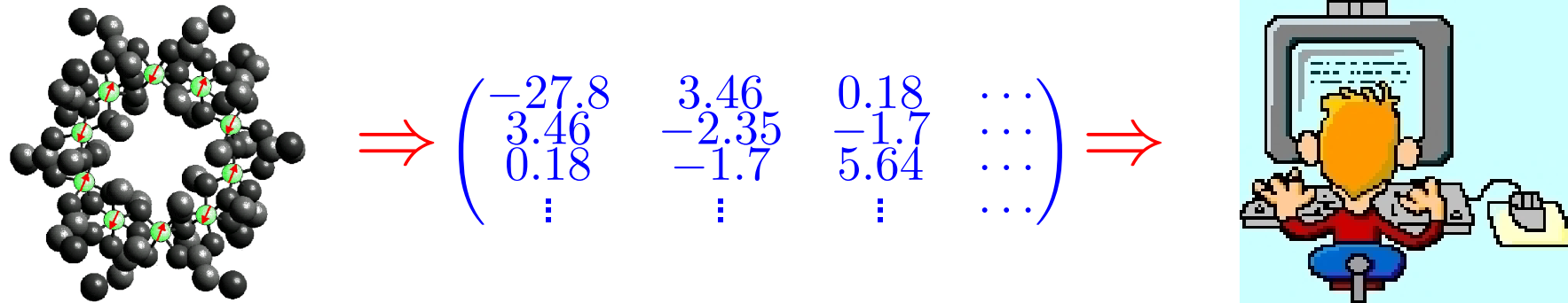
$$\tilde{H} = -2 \sum_{i < j} J_{ij} \vec{s}_{\sim}(i) \cdot \vec{s}_{\sim}(j) + g \mu_B B \sum_i^N s_{z}(i)$$

Heisenberg

Zeeman



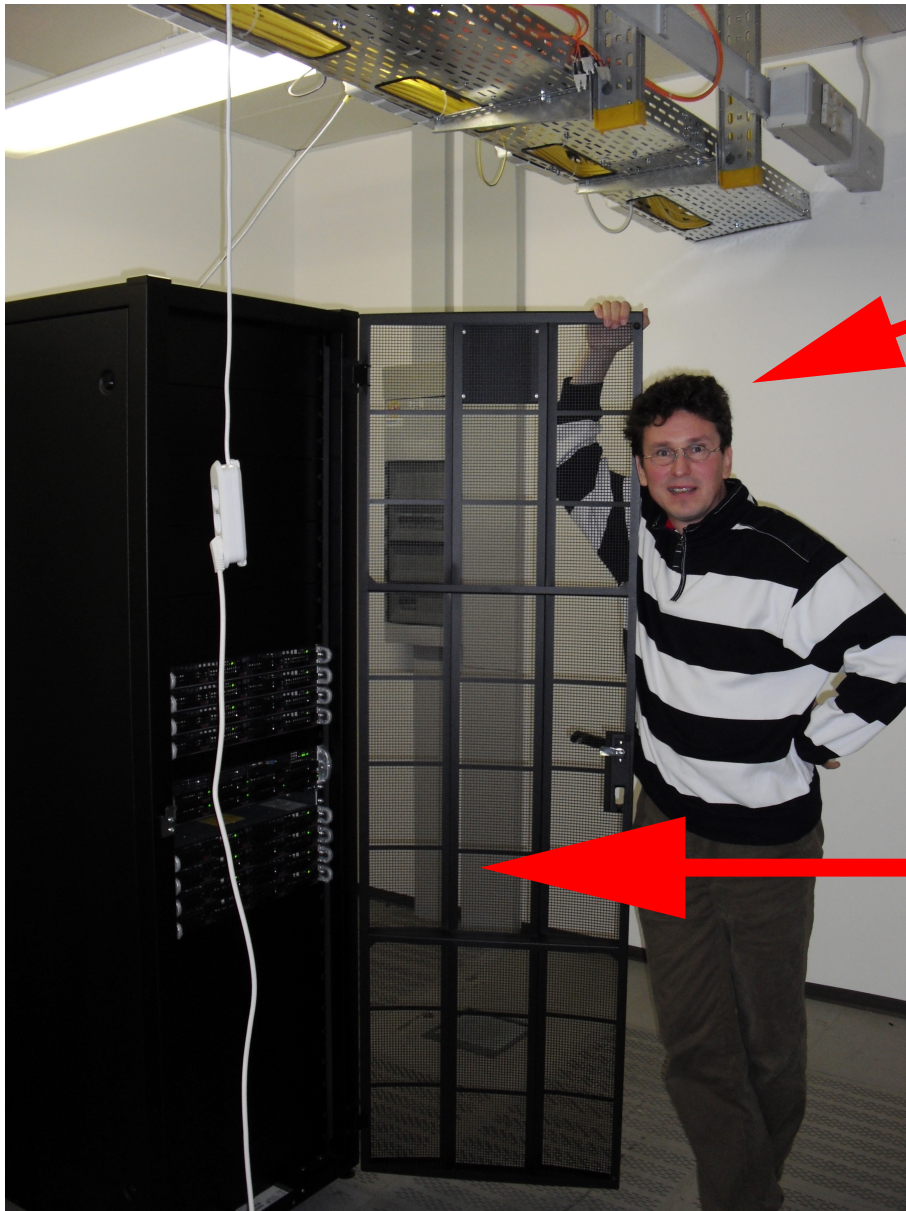
In the end it's always a big matrix!



$$\text{Fe}_{10}^{\text{III}}: N = 10, s = 5/2$$

Dimension=**60,466,176**. Maybe **too** big?

# Thank God, we have computers



“Espresso-doped multi-core”

128 cores, 384 GB RAM

... but that's not enough!

# Contents for you today

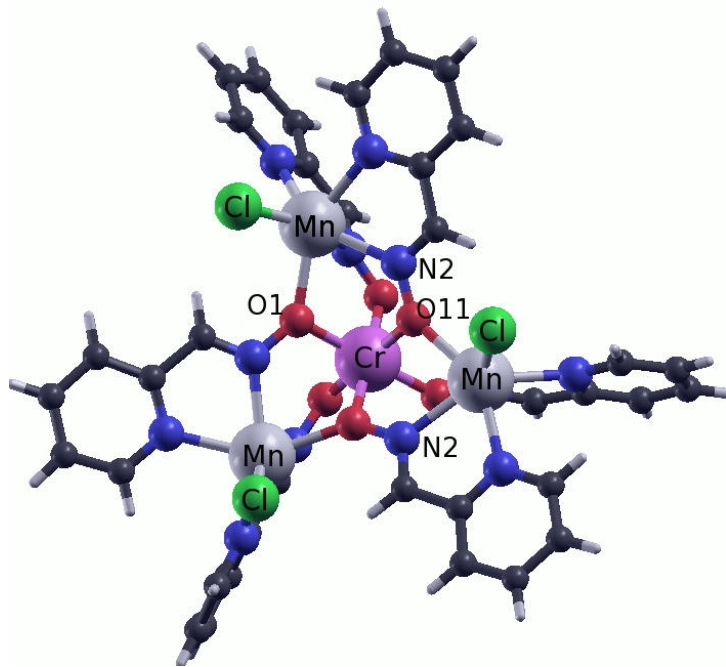


$$\begin{pmatrix} 3 & 42 & 4711 \\ 42 & 0 & 3.14 \\ 4711 & 3.14 & 8 \\ -17 & 007 & 13 \\ 1.8 & 15 & 081 \end{pmatrix}$$

1. Small: Complete diagonalization
2. **Medium: FTLM**
3. Big: DDMRG
4. **Deposited: NRG**

We are the sledgehammer team of matrix diagonalization.  
Please send inquiries to [jschnack@uni-bielefeld.de](mailto:jschnack@uni-bielefeld.de)!

# Magnetic molecules in one slide



- Crystals of inorganic or organic macro molecules, e.g. polyoxometalates, where paramagnetic ions such as Iron (Fe), Chromium (Cr), Copper (Cu), Nickel (Ni), Vanadium (V), Manganese (Mn), or rare earth ions are embedded in a host matrix;
- Pure organic magnetic molecules: magnetic coupling between high spin units (e.g. free radicals);
- Single spin quantum number  $1/2 \leq s \leq 7/2$ ;
- Intermolecular interaction relatively small, therefore measurements reflect the thermal behaviour of a single molecule.

*Magnetism goes Nano*, Ed. Stefan Blügel, Thomas Brückel, and Claus M. Schneider, FZ Jülich, Institute of Solid State Research, Lecture Notes **36** Jülich 2005

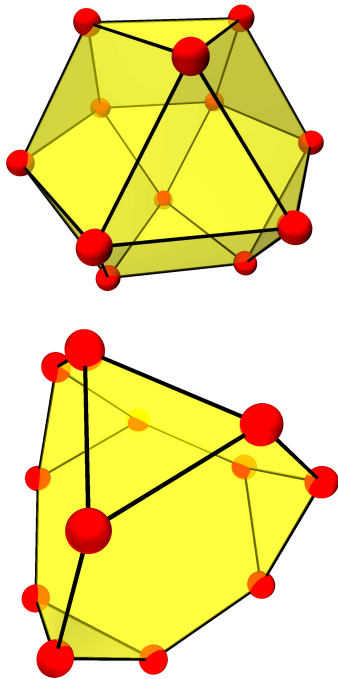


# Reminder on complete diagonalization: SU(2) & point group symmetry

Quantum chemists need to be much smarter since they have smaller computers!

- (1) D. Gatteschi and L. Pardi, *Gazz. Chim. Ital.* **123**, 231 (1993).
- (2) J. J. Borrás-Almenar, J. M. Clemente-Juan, E. Coronado, and B. S. Tsukerblat, *Inorg. Chem.* **38**, 6081 (1999).
- (3) B. S. Tsukerblat, *Group theory in chemistry and spectroscopy: a simple guide to advanced usage*, 2nd ed. (Dover Publications, Mineola, New York, 2006).

# Irreducible Tensor Operator approach



## Spin rotational symmetry SU(2):

- $\underline{H} = -2 \sum_{i < j} J_{ij} \underline{\tilde{S}}_i \cdot \underline{\tilde{S}}_j + g\mu_B \underline{\tilde{S}} \cdot \underline{\tilde{B}} ;$

- Physicists employ:  $[\underline{H}, \underline{S}_z] = 0;$

- Chemists employ:  $[\underline{H}, \underline{\tilde{S}}^2] = 0, [\underline{H}, \underline{S}_z] = 0;$

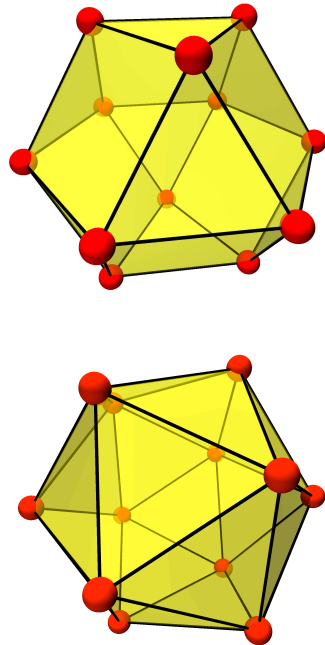
Irreducible Tensor Operator (ITO) approach;  
Free program MAGPACK (2) available.

(1) D. Gatteschi and L. Pardi, *Gazz. Chim. Ital.* **123**, 231 (1993).

(2) J. J. Borrás-Almenar, J. M. Clemente-Juan, E. Coronado, and B. S. Tsukerblat, *Inorg. Chem.* **38**, 6081 (1999).

(3) B. S. Tsukerblat, *Group theory in chemistry and spectroscopy: a simple guide to advanced usage*, 2nd ed. (Dover Publications, Mineola, New York, 2006).

# Point Group Symmetry

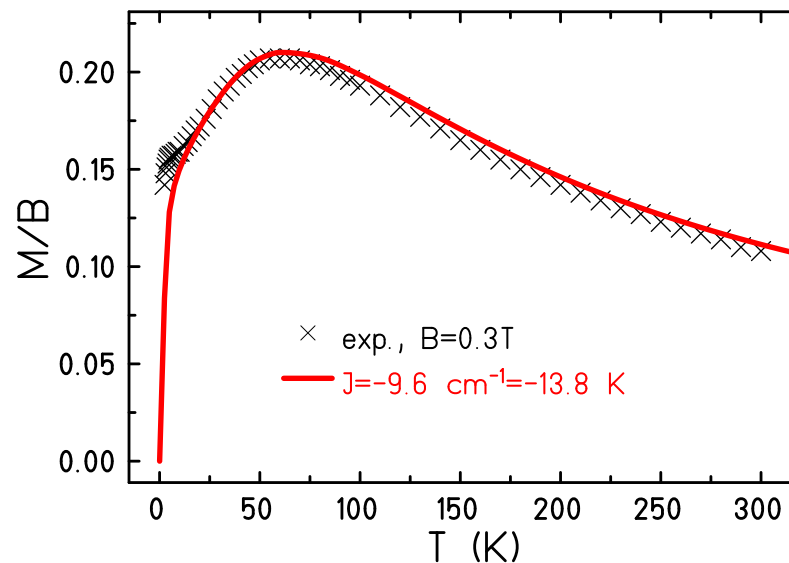
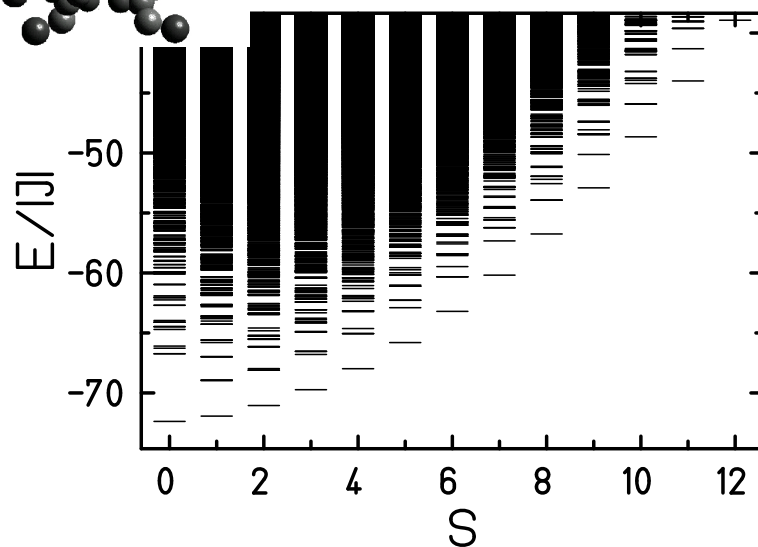
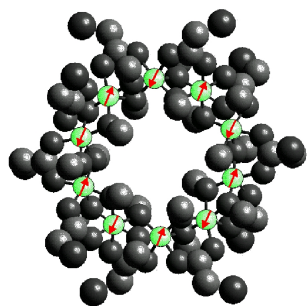


- Point groups, e.g.  $O_h$  or  $I_h$ , realized as permutations;
- Hamiltonian commutes with all group operations:  

$$[H, G_i] = 0;$$
- Construct irreducible representations and related Hamilton matrices;  
 No free program available (4).

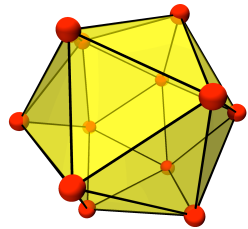
(1) M. Tinkham, *Group Theory and Quantum Mechanics*, Dover.  
 (2) D. Gatteschi and L. Pardi, *Gazz. Chim. Ital.* **123**, 231 (1993).  
 (3) O. Waldmann, *Phys. Rev. B* **61**, 6138 (2000).  
 (4) R. Schnalle and J. Schnack, *Int. Rev. Phys. Chem.* **29**, 403-452 (2010) ← contains EVERYTHING.

# Example: Fe<sub>10</sub> – SU(2) & D<sub>2</sub>

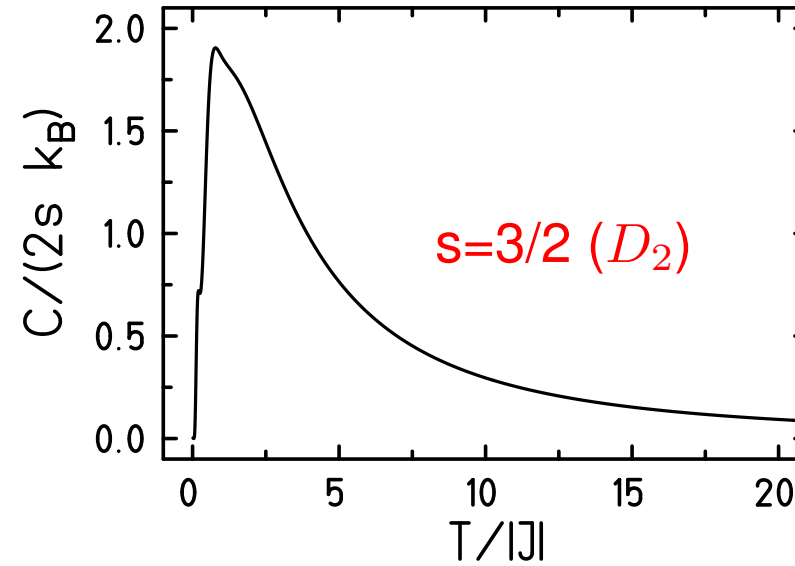
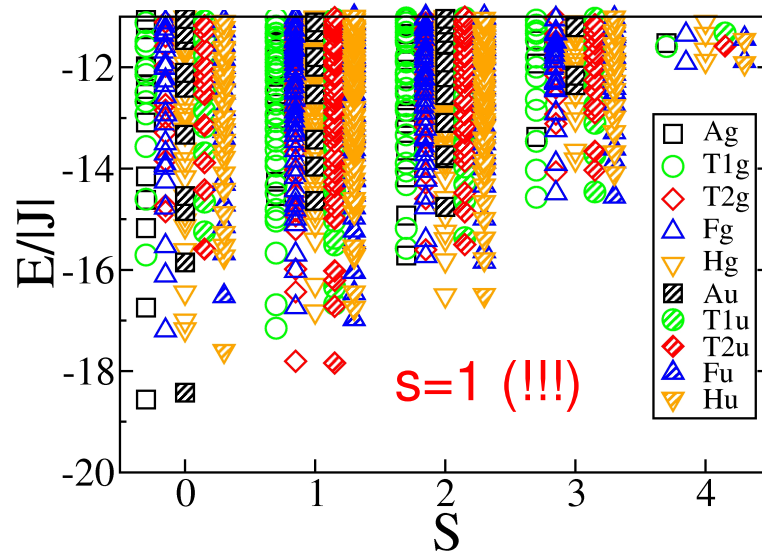


Spin ring,  $N = 10$ ,  $s = 5/2$ , Hilbert space dimension 60,466,176; symmetry  $D_2$  (1).

- (1) R. Schnalle and J. Schnack, *Int. Rev. Phys. Chem.* **29**, 403-452 (2010).
- (2) C. Delfs *et al.*, *Inorg. Chem.* **32**, 3099 (1993).



# Example: Icosahedron – SU(2) & $I_h$



Icosahedron,  $s = 3/2$ , Hilbert space dimension 16,777,216; symmetry  $I_h$ ;  
 Evaluation of recoupling coefficients for  $s = 3/2$  in  $I_h$  **practically impossible** (1).

(1) R. Schnalle and J. Schnack, Int. Rev. Phys. Chem. **29**, 403-452 (2010).

# What if your molecule is BIGGER?



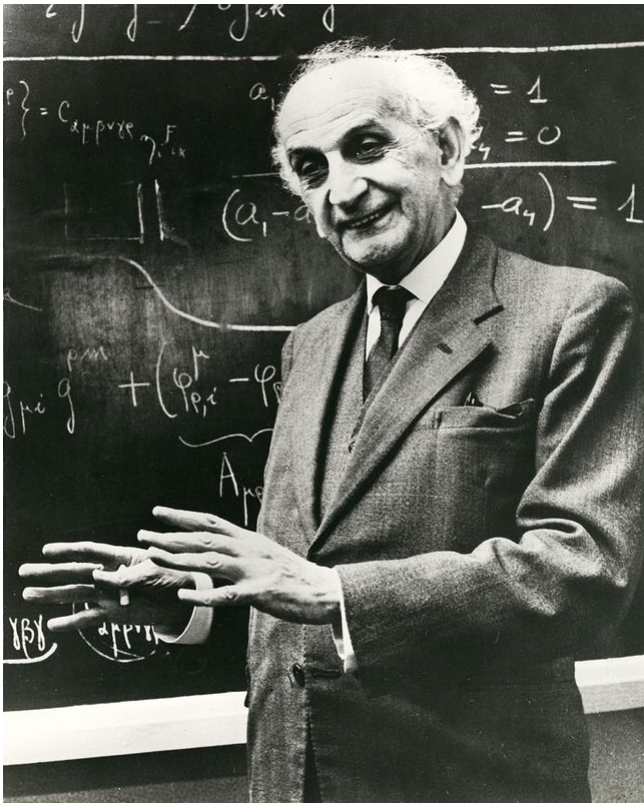
# K-Computer?

# Finite-temperature Lanczos Method

(Good for dimensions up to  $10^{10}$ .)



# Lanczos – a Krylov space method



- **Idea: exact diagonalization in reduced basis sets.**
- But which set to choose???
- **Idea: generate the basis set with the operator you want to diagonalize:**  
 $\{ |\phi\rangle, \tilde{H} |\phi\rangle, \tilde{H}^2 |\phi\rangle, \tilde{H}^3 |\phi\rangle, \dots \}$
- But which starting vector to choose???
- **Idea: almost any will do!**
- Cornelius Lanczos (Lánczos Kornél, 1893-1974)

(1) C. Lanczos, J. Res. Nat. Bur. Stand. **45**, 255 (1950).

# Finite-temperature Lanczos Method I

$$Z(T, B) = \sum_{\nu} \langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle$$

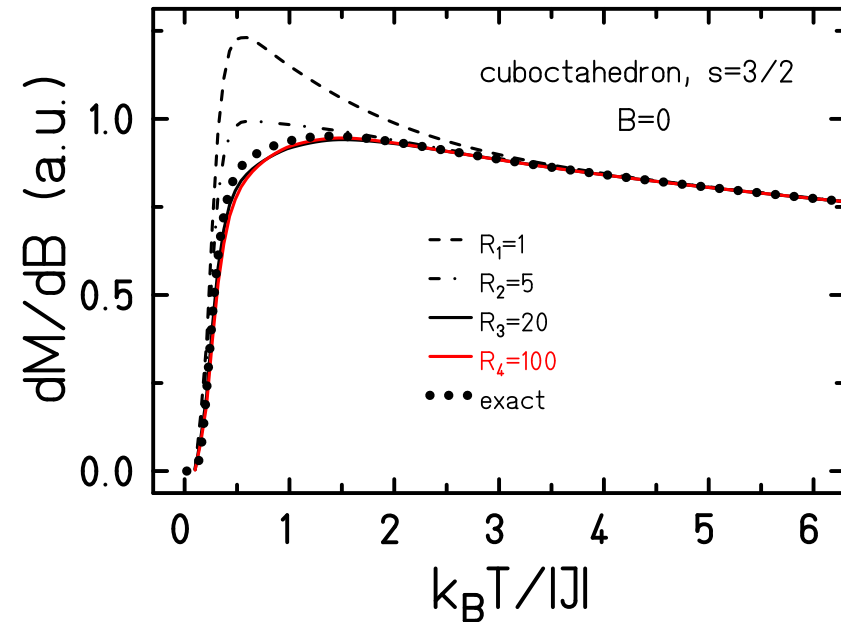
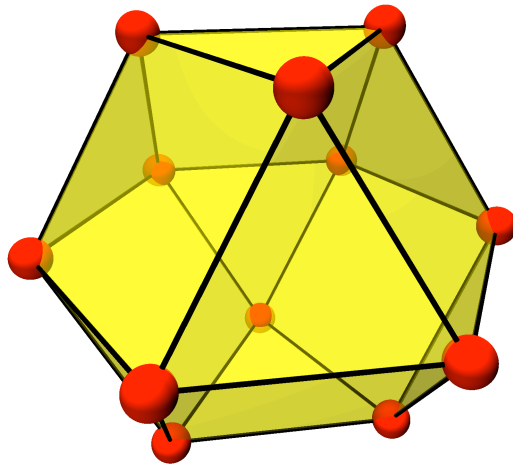
$$\langle \nu | \exp \left\{ -\beta \tilde{H} \right\} | \nu \rangle \approx \sum_n \langle \nu | n(\nu) \rangle \exp \left\{ -\beta \epsilon_n \right\} \langle n(\nu) | \nu \rangle$$

$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp \left\{ -\beta \epsilon_n \right\} |\langle n(\nu) | \nu \rangle|^2$$

- $|n(\nu)\rangle$  n-th Lanczos eigenvector starting from  $|\nu\rangle$
- Partition function replaced by a small sum:  $R = 1 \dots 10, N_L \approx 100$ .

J. Jaklic and P. Prelovsek, Phys. Rev. B **49**, 5065 (1994).

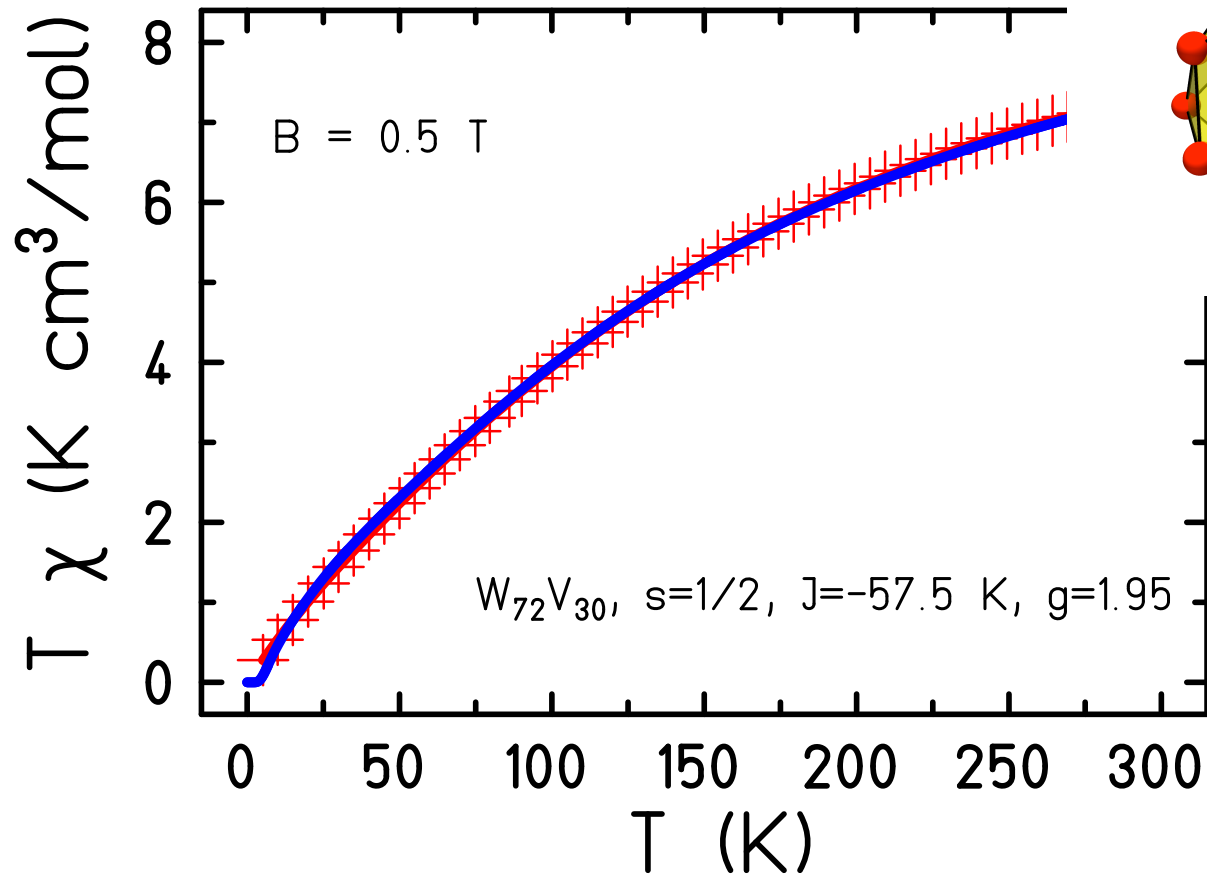
# How good is finite-temperature Lanczos?



- Works very well: compare frustrated cuboctahedron.
- $N = 12, s = 3/2$ : Considered  $< 100,000$  states instead of 16,777,216.

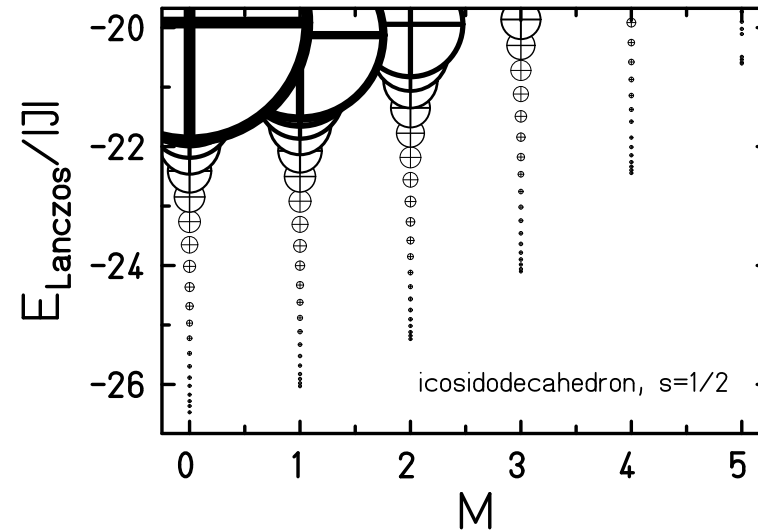
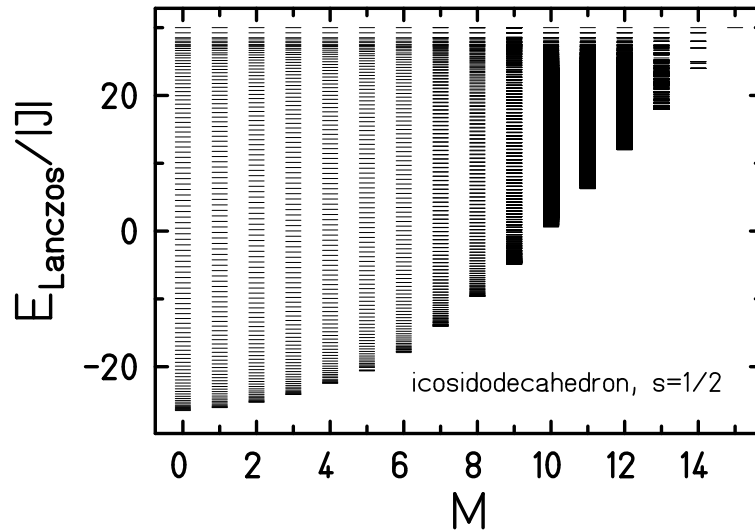
Exact results: R. Schnalle and J. Schnack, *Int. Rev. Phys. Chem.* **29**, 403-452 (2010).  
 FTLM: J. Schnack and O. Wendland, *Eur. Phys. J. B* **78**, 535-541 (2010).

# Icosidodecahedron $s = 1/2$



Exp. data: A. M. Todea, A. Merca, H. Bögge, T. Glaser, L. Engelhardt, R. Prozorov, M. Luban, A. Müller, Chem. Commun., 3351 (2009).

# Icosidodecahedron $s = 1/2$



- The true spectrum will be much denser. This is miraculously compensated for by the weights.

$$Z(T, B) \approx \frac{\dim(\mathcal{H})}{R} \sum_{\nu=1}^R \sum_{n=1}^{N_L} \exp \{ -\beta \epsilon_n \} |\langle n(\nu, \Gamma) | \nu, \Gamma \rangle|^2$$

## Finite-temperature Lanczos Method III

$$\tilde{H} = -2 \sum_{i < j} \vec{\tilde{s}}_i \cdot \mathbf{J}_{ij} \cdot \vec{\tilde{s}}_j + \sum_i \vec{\tilde{s}}_i \cdot \mathbf{D}_i \cdot \vec{\tilde{s}}_i + \mu_B B \sum_i g_i \tilde{s}_i^z$$

- **Problem:** for anisotropic Hamiltonians no symmetry left  
→ accuracy drops (esp. for high  $T$ ).
- Simple traces such as  $\text{Tr} \left( \tilde{S}^z \right) = 0$  tend to be wrong for  $R$  not very big.

O. Hanebaum, J. Schnack, Eur. Phys. J. B **87**, 194 (2014)

## Finite-temperature Lanczos Method IV

Employ very general symmetry (time-reversal invariance)

$$\vec{\mathcal{M}}(T, -\vec{B}) = -\vec{\mathcal{M}}(T, \vec{B})$$

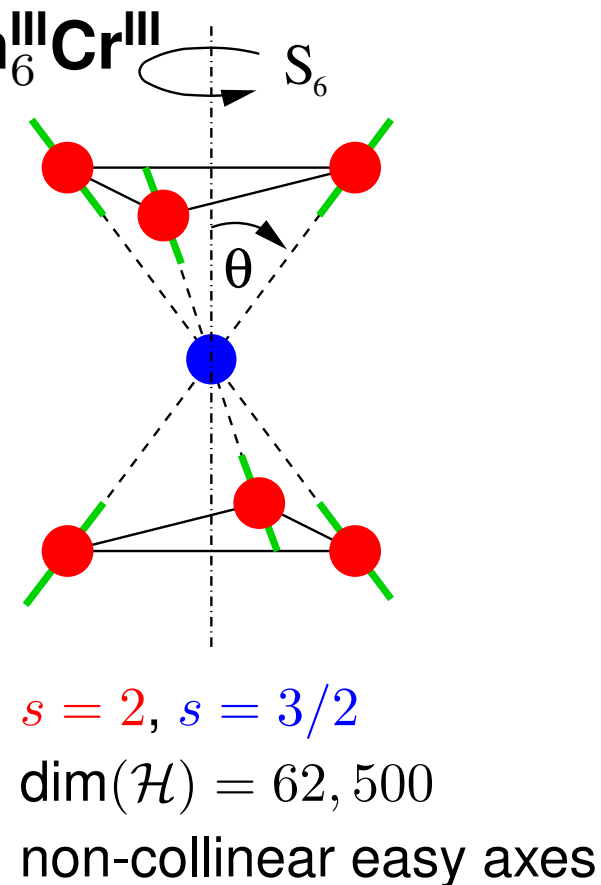
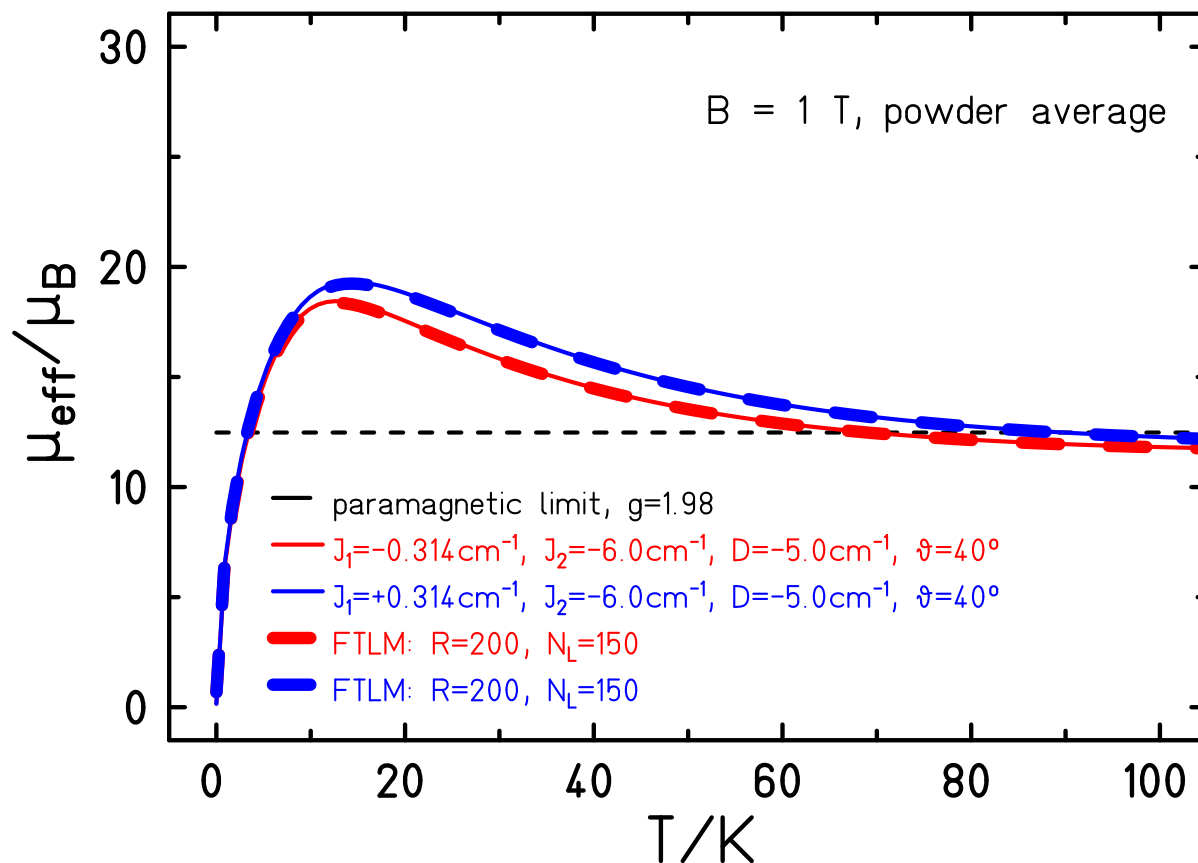
Use Lanczos energy eigenvector  $|n(\nu)\rangle$  and time-reversed counterpart  $|\tilde{n}(\nu)\rangle$

$$|n(\nu)\rangle = \sum_{\vec{m}} c_{\vec{m}} |\vec{m}\rangle, \quad |\tilde{n}(\nu)\rangle = \sum_{\vec{m}} c_{\vec{m}}^* |-\vec{m}\rangle$$

- Restores  $\vec{\mathcal{M}}(T, -\vec{B}) = -\vec{\mathcal{M}}(T, \vec{B})$  and (some) traces.
- More practical: use pairs of time-reversed random vectors; still accurate.

O. Hanebaum, J. Schnack, Eur. Phys. J. B **87**, 194 (2014)

# Glaser-type molecules: $\text{Mn}_6\text{Cr}^{\text{III}}$



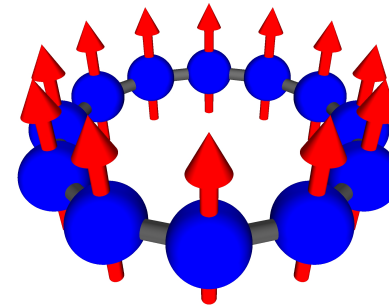
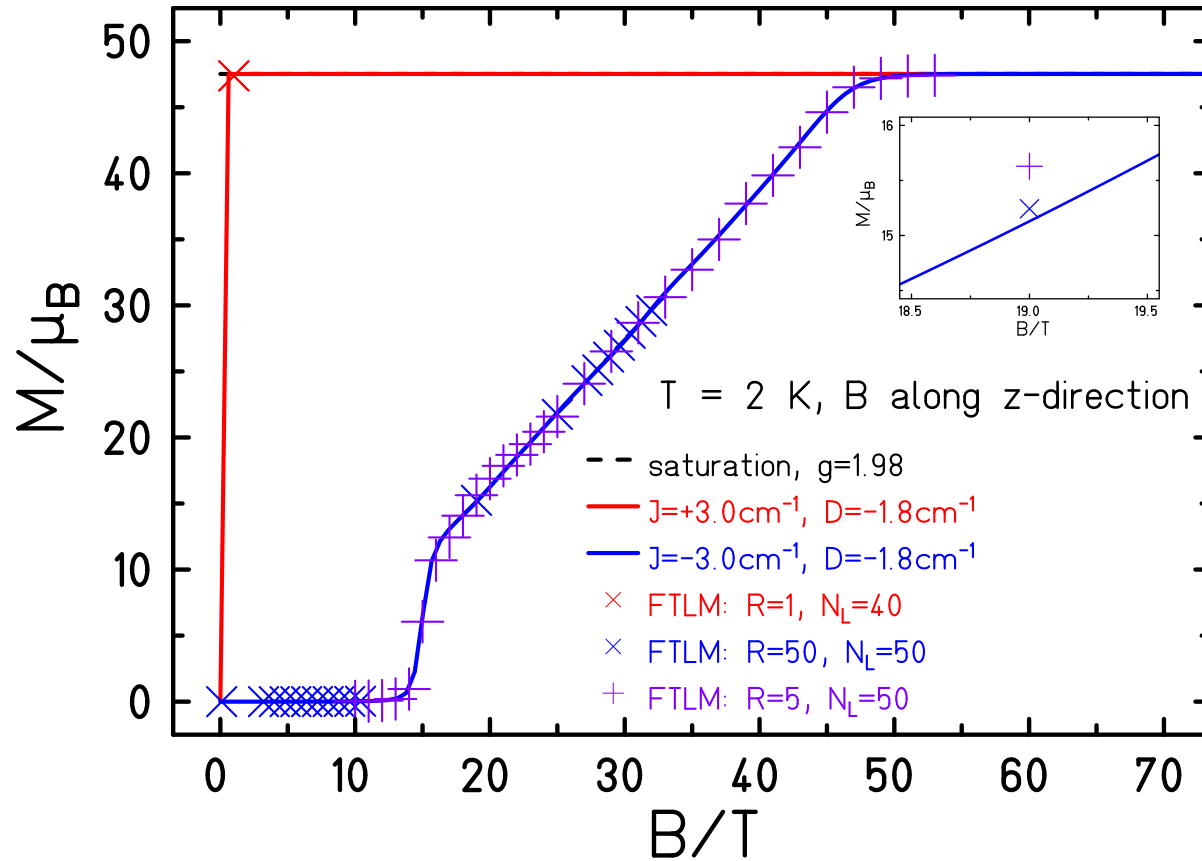
Hours compared to days, notebook compared to supercomputer!

O. Hanebaum, J. Schnack, Eur. Phys. J. B **87**, 194 (2014)

T. Glaser, Chem. Commun. **47**, 116-130 (2011)



# A fictitious $\text{Mn}_{12}^{\text{III}}$ – $M_z$ vs $B_z$



$s = 2$

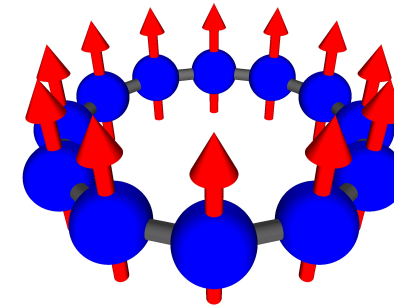
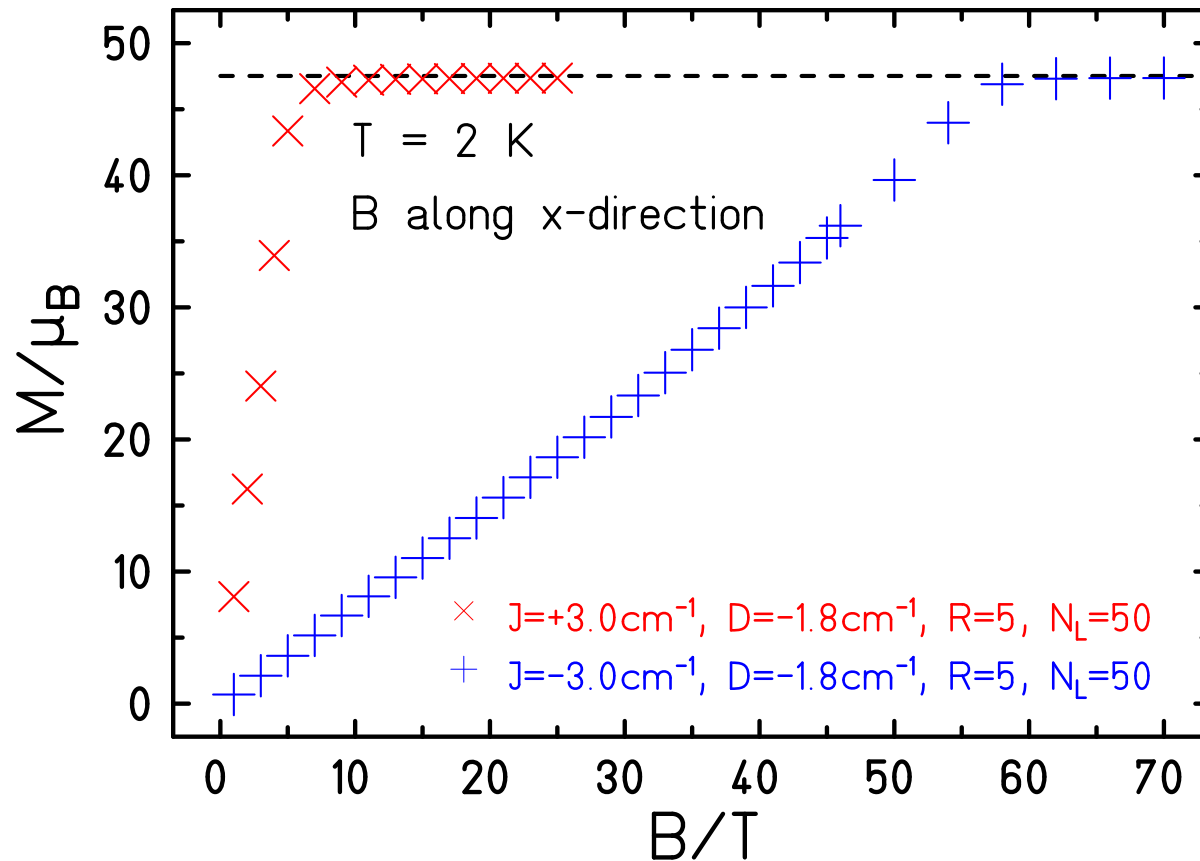
$\dim(\mathcal{H}) = 244, 140, 625$

collinear easy axes

A few days compared to *impossible!*

O. Hanebaum, J. Schnack, Eur. Phys. J. B **87**, 194 (2014)

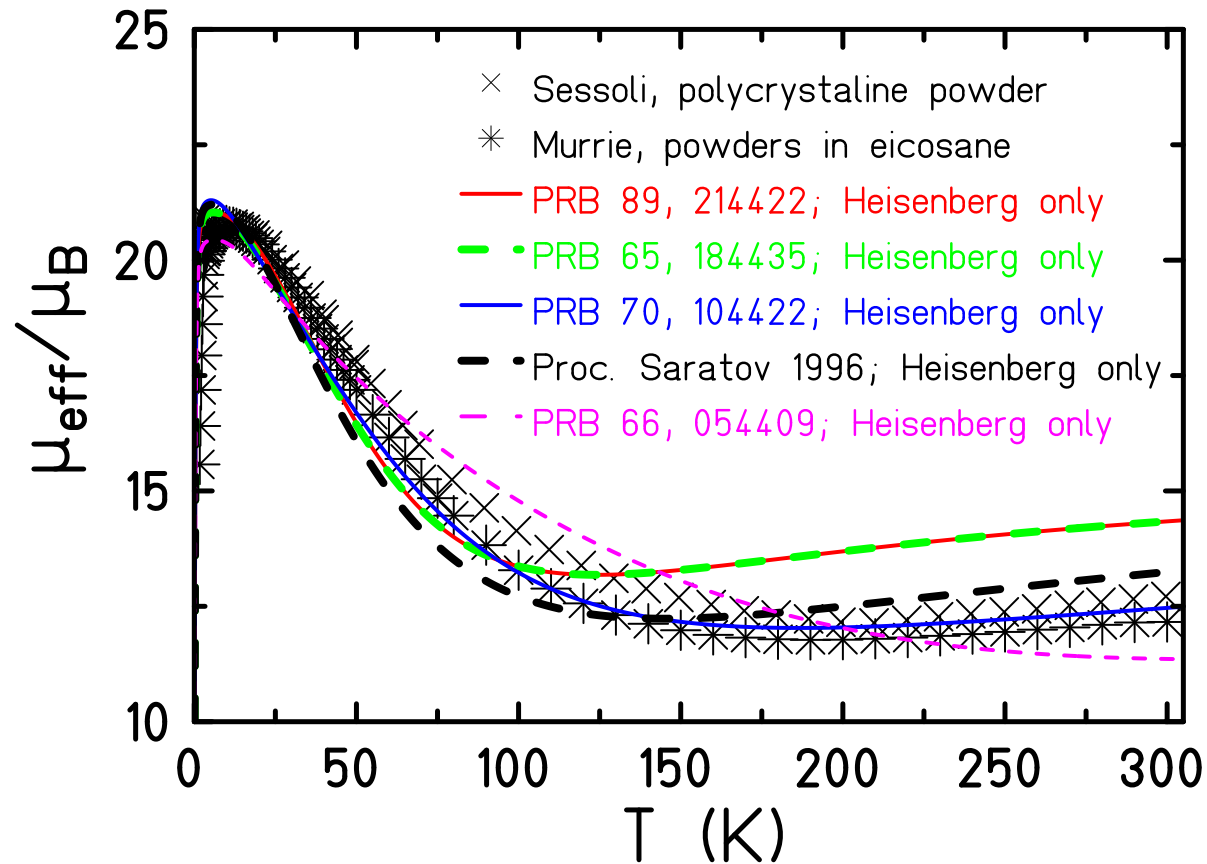
# A fictitious $\text{Mn}_{12}^{\text{III}}$ – $M_x$ vs $B_x$



No other method can deliver these curves!

O. Hanebaum, J. Schnack, Eur. Phys. J. B **87**, 194 (2014)

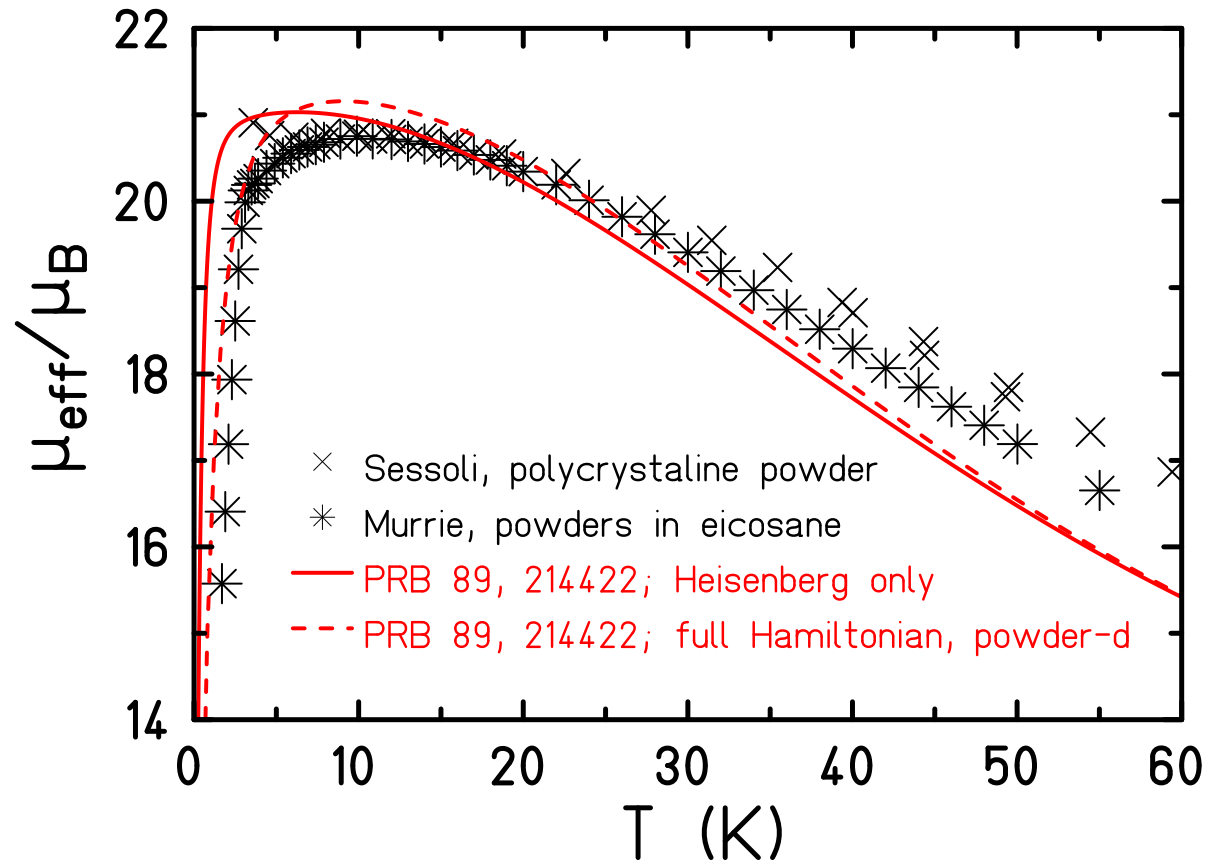
# Effective magnetic moment of Mn<sub>12</sub>-acetate



We can check DFT parameter predictions for large molecules!

O. Hanebaum, J. Schnack, work in progress

# Effective magnetic moment of Mn<sub>12</sub>-acetate



We can check DFT parameter predictions for large molecules!

O. Hanebaum, J. Schnack, work in progress

# The magnetocaloric effect

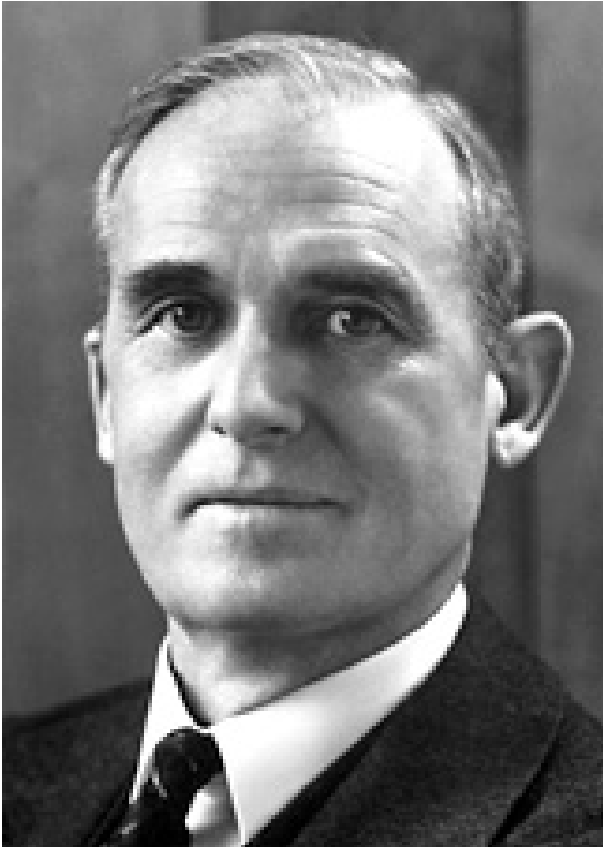
# Magnetocaloric effect – Basics



- Heating or cooling in a varying magnetic field. Predicted, discussed, discovered by Thomson, Warburg, Weiss, and Piccard (1).
- Typical rates: 0.5 . . . 2 K/T.
- Giant magnetocaloric effect: 3 . . . 4 K/T e.g. in  $\text{Gd}_5(\text{Si}_x\text{Ge}_{1-x})_4$  alloys ( $x \leq 0.5$ ).
- **Scientific goal I: room temperature applications.**
- **Scientific goal II: sub-Kelvin cooling.**

(1) A. Smith, Eur. Phys. J. H **38**, 507 (2013).

## Sub-Kelvin cooling: Nobel prize 1949



The Nobel Prize in Chemistry 1949 was awarded to William F. Giaouque *for his contributions in the field of chemical thermodynamics, particularly concerning the behaviour of substances at extremely low temperatures.*

# Sub-Kelvin cooling: Nobel prize 1949

768

LETTERS TO THE EDITOR

## Attainment of Temperatures Below 1° Absolute by Demagnetization of $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$

We have recently carried out some preliminary experiments on the adiabatic demagnetization of  $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$  at the temperatures of liquid helium. As previously predicted by one of us, a large fractional lowering of the absolute temperature was obtained.

An iron-free solenoid producing a field of about 8000 gauss was used for all the measurements. The amount of  $\text{Gd}_2(\text{SO}_4)_3 \cdot 8\text{H}_2\text{O}$  was 61 g. The observations were checked by many repetitions of the cooling. The temperatures were measured by means of the inductance of a coil surrounding the gadolinium sulfate. The coil was immersed in liquid helium and isolated from the gadolinium by means of an evacuated space. The thermometer was in excellent agreement with the temperature of liquid helium as indicated by its vapor pressure down to 1.5°K.

On March 19, starting at a temperature of about 3.4°K, the material cooled to 0.53°K. On April 8, starting at about 2°, a temperature of 0.34°K was reached. On April 9, starting at about 1.5°, a temperature of 0.25°K was attained.

It is apparent that it will be possible to obtain much lower temperatures, especially when successive demagnetizations are utilized.

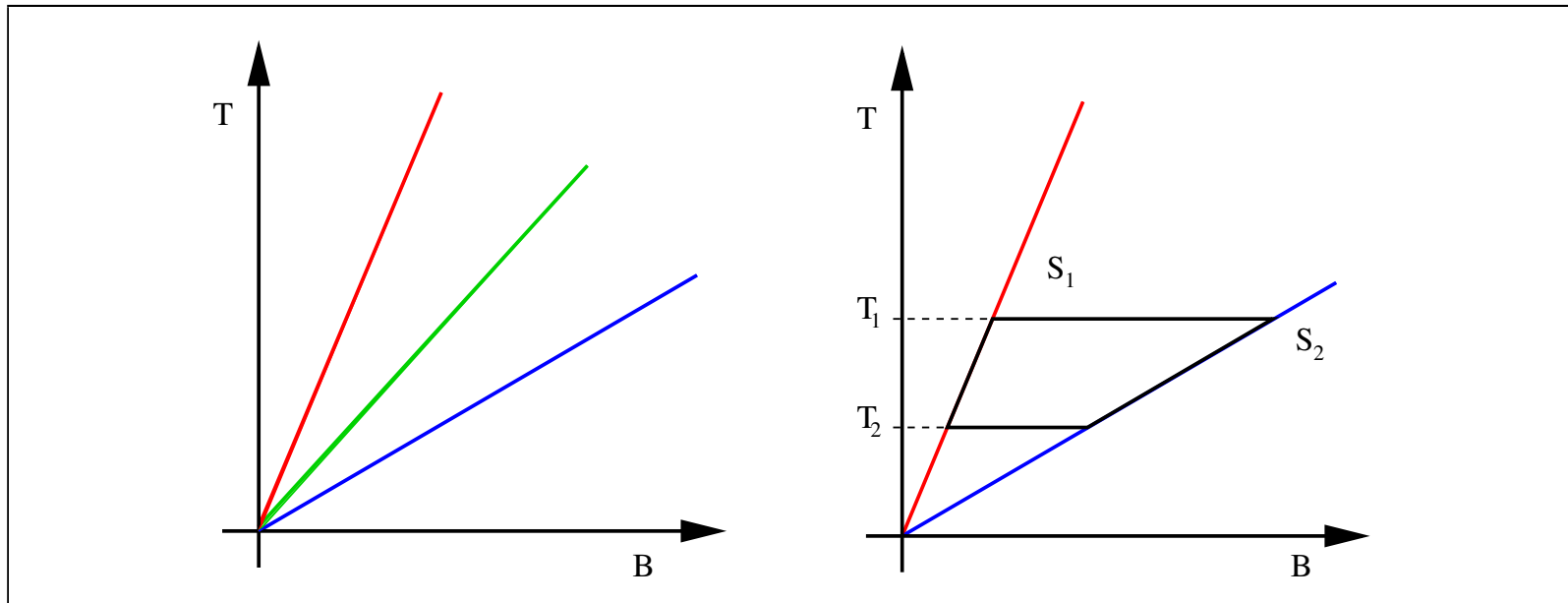
W. F. GIAUQUE  
D. P. MACDOUGALL

Department of Chemistry,  
University of California,  
Berkeley, California,  
April 12, 1933.

W. F. Giauque and D. MacDougall, *Phys. Rev.* **43**, 768 (1933).

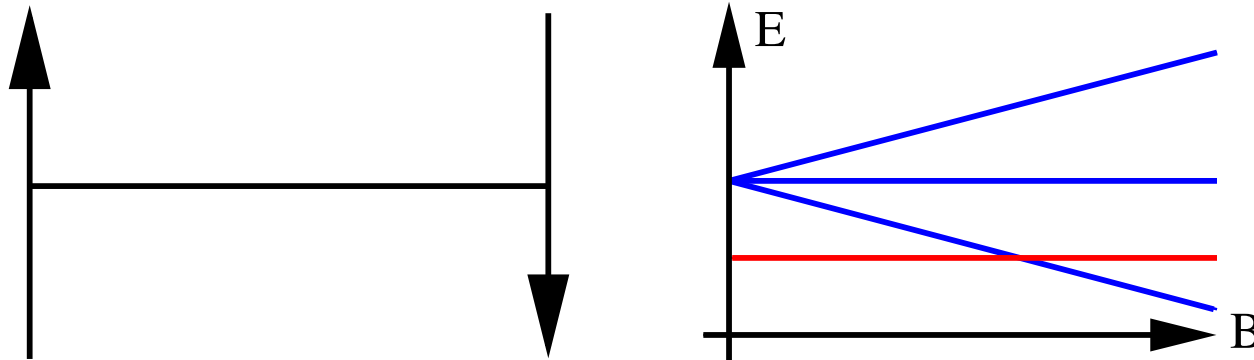


# Magnetocaloric effect – Paramagnets



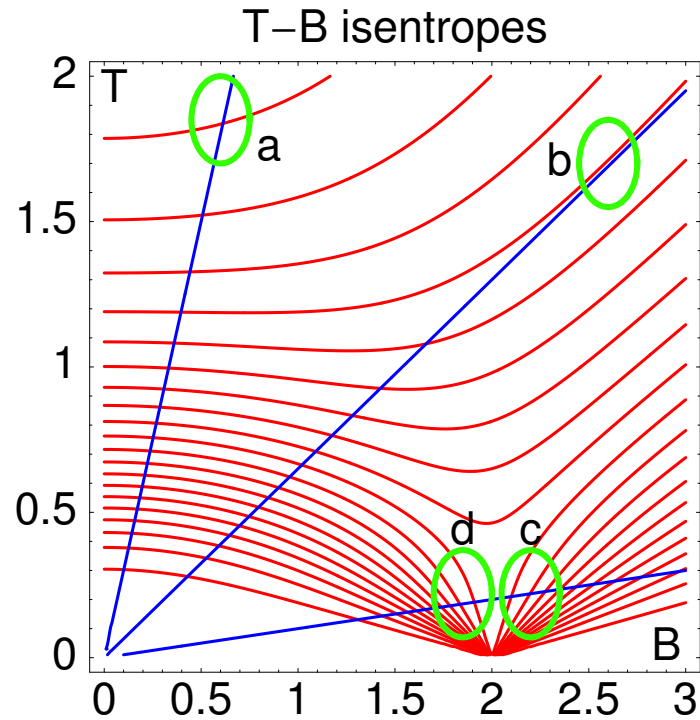
- **Ideal paramagnet:  $S(T, B) = f(B/T)$ , i.e.  $S = const \Rightarrow T \propto B$ .**
- At low  $T$  pronounced effects of dipolar interaction prevent further effective cooling.

## Magnetocaloric effect – af $s = 1/2$ dimer



- Singlet-triplet level crossing causes a peak of  $S$  at  $T \approx 0$  as function of  $B$ .
- $M(T = 0, B)$  and  $S(T = 0, B)$  not analytic as function of  $B$ .
- $M(T = 0, B)$  jumps at  $B_c$ ;  $S(T = 0, B_c) = k_B \ln 2$ , otherwise zero.

# Magnetocaloric effect – af $s = 1/2$ dimer



blue lines: ideal paramagnet, red curves: af dimer

Magnetocaloric effect:

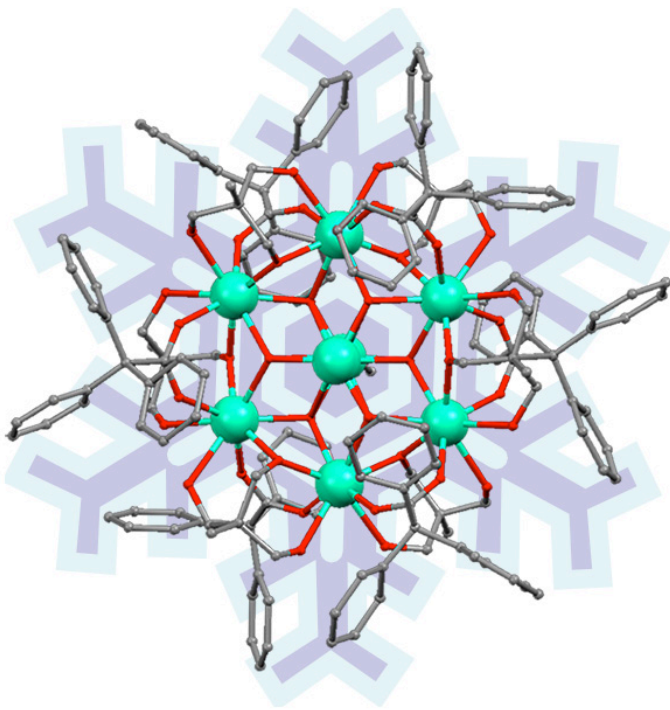
- (a) reduced,
- (b) the same,
- (c) enhanced,
- (d) opposite

when compared to an ideal paramagnet.

Case (d) does not occur for a paramagnet.

Nice ideas,  
but can one measure it?  
With magnetic molecules?

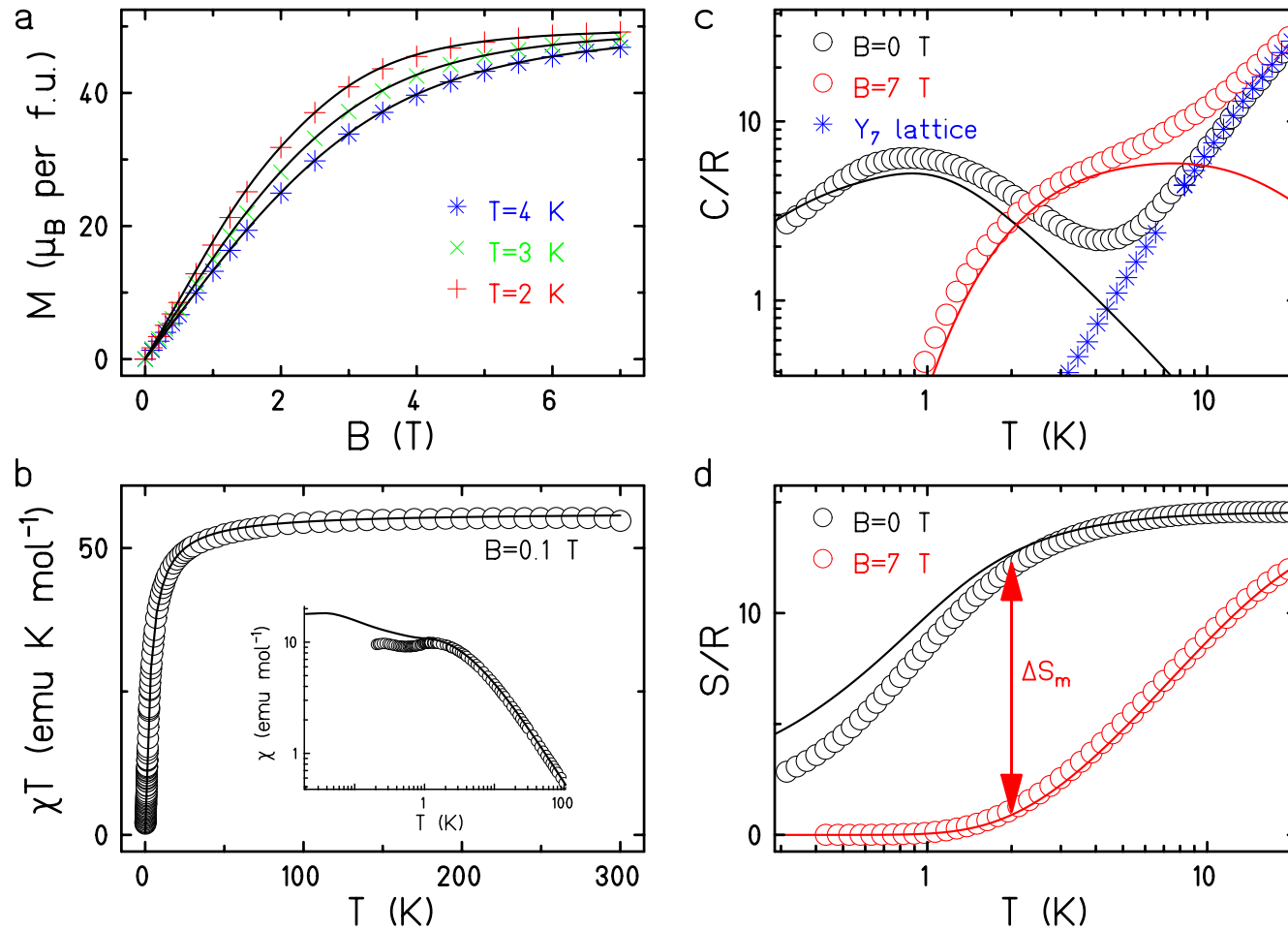
## Gd<sub>7</sub> – Basics



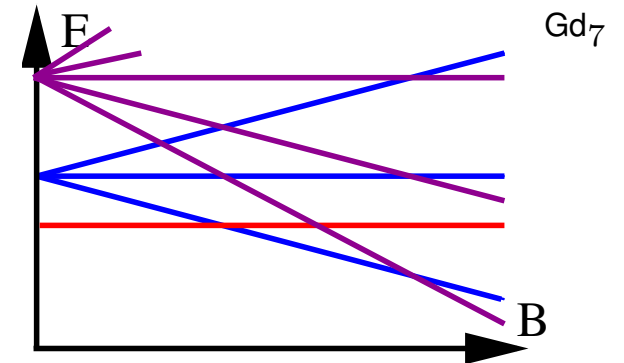
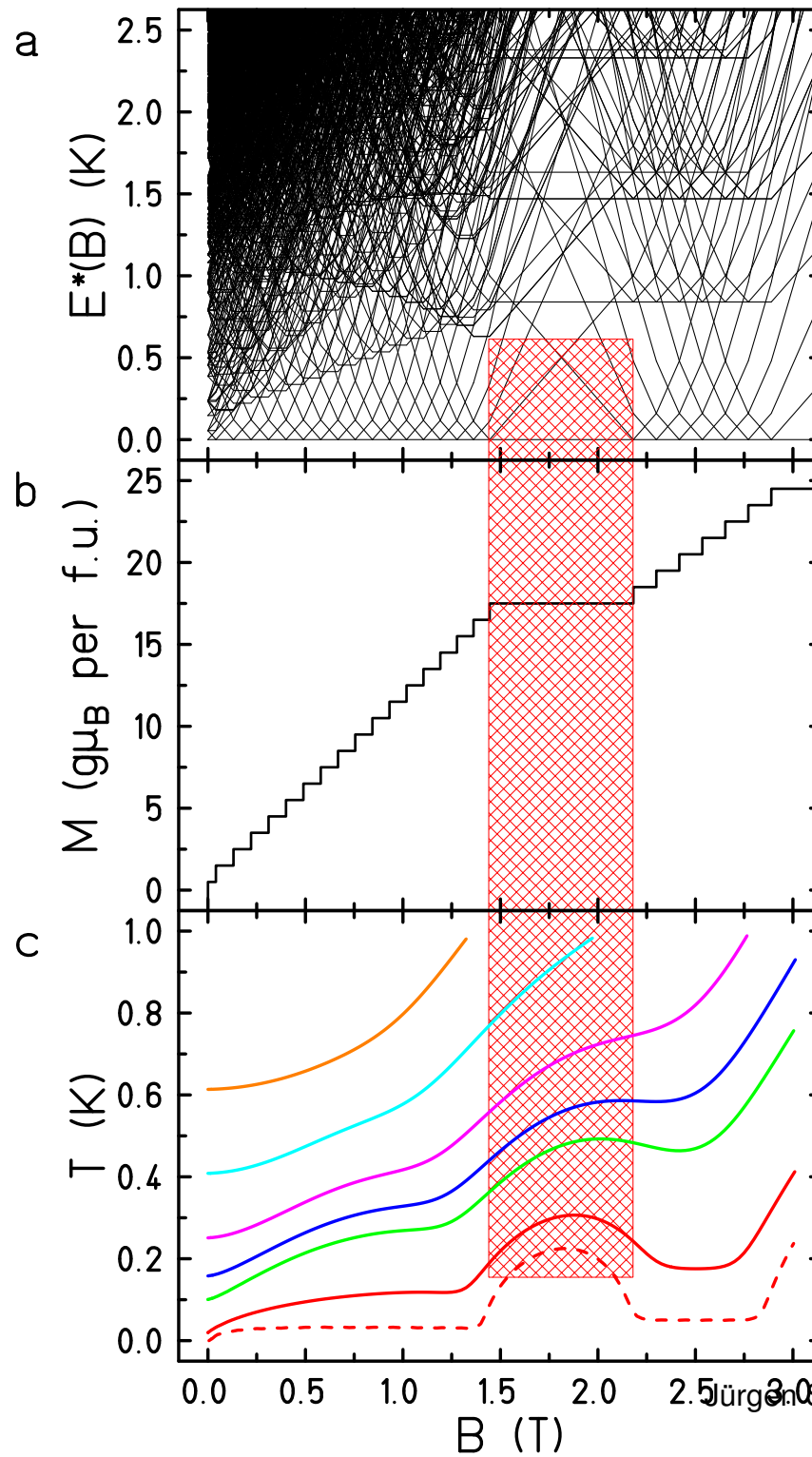
- Often magnetocaloric observables not directly measured, but inferred from Maxwell's relations.
- First real cooling experiment with a molecule.
- $$\underline{H} = -2 \sum_{i < j} J_{ij} \vec{\tilde{s}}_i \cdot \vec{\tilde{s}}_j + g \mu_B B \sum_i^N \tilde{s}_i^z$$
  
 $J_1 = -0.090(5) \text{ K}, J_2 = -0.080(5) \text{ K}$   
 and  $g = 2.02$ .
- **Very good agreement down to the lowest temperatures.**

J. W. Sharples, D. Collison, E. J. L. McInnes, J. Schnack, E. Palacios, M. Evangelisti, Nat. Commun. **5**, 5321 (2014).

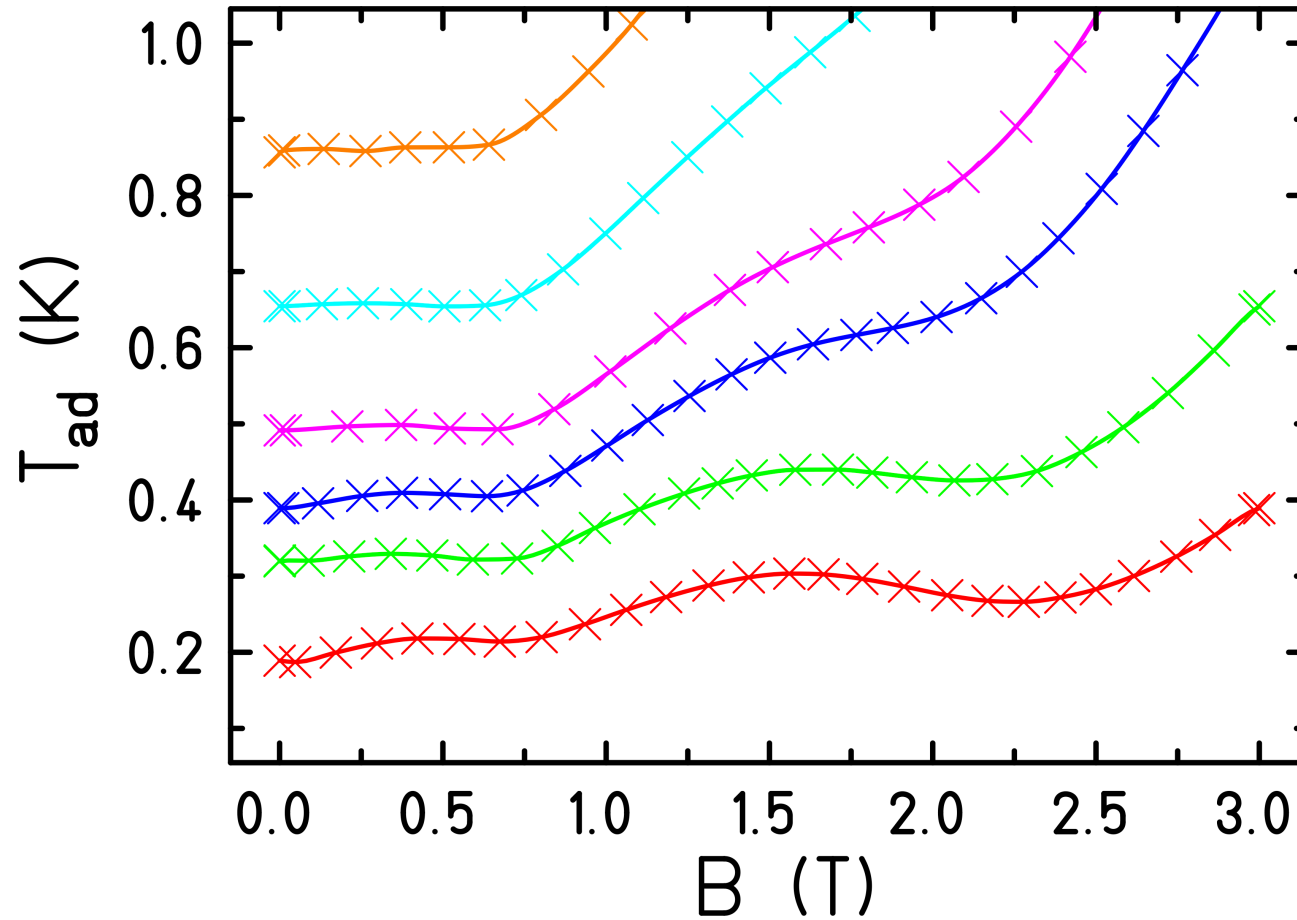
# Gd<sub>7</sub> – experiment & theory



J. W. Sharples, D. Collison, E. J. L. McInnes, J. Schnack, E. Palacios, M. Evangelisti, Nat. Commun. **5**, 5321 (2014).



## Gd<sub>7</sub> – Experimental cooling



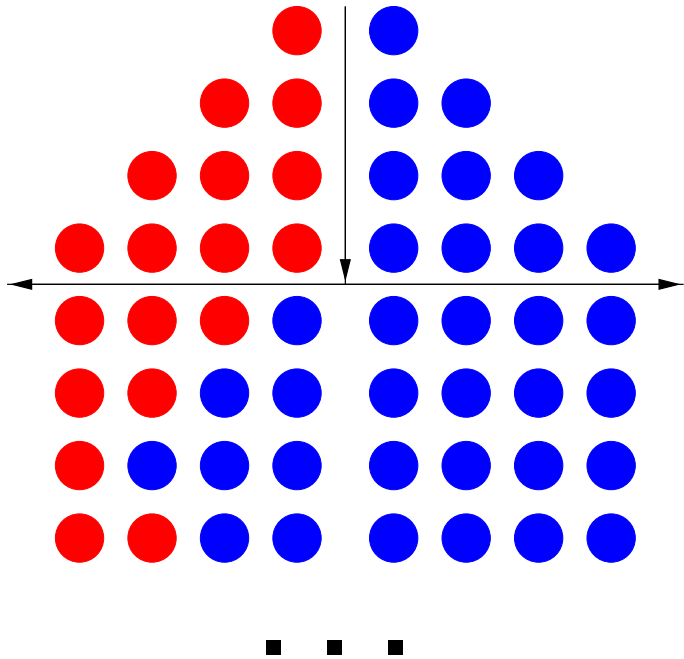
J. W. Sharples, D. Collison, E. J. L. McInnes, J. Schnack, E. Palacios, M. Evangelisti, Nat. Commun. **5**, 5321 (2014).



# Density Matrix Renormalization Group

(Best for one-dimensional systems, even for huge sizes.)

# Density Matrix Renormalization Group

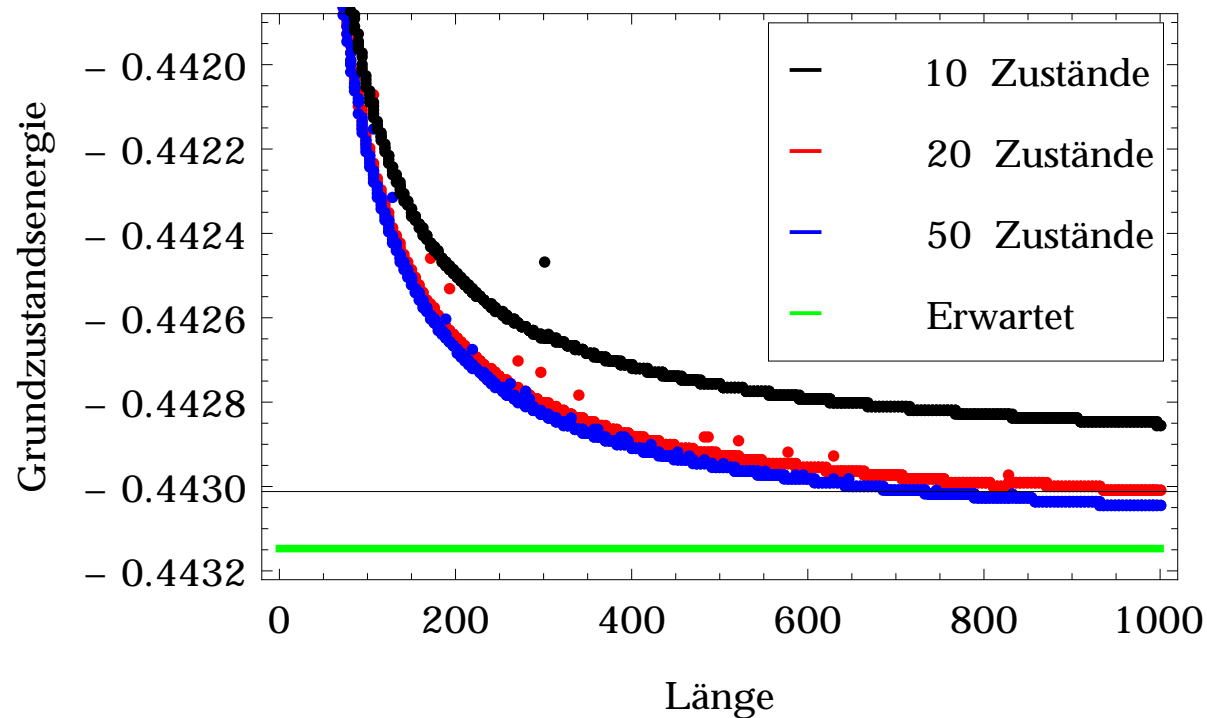


**Again: build your appropriate reduced basis set**

- Naive idea: start with small system, diagonalize  $\tilde{H}$ , keep only  $m$  lowest states, enlarge system, diagonalize  $\tilde{H}$ , keep only  $m$  lowest states, ...
- **Better: similar idea, use low-lying eigenstates of density matrix of part of system (1,2,3).**
- Technical procedure: growth of system & sweeps.

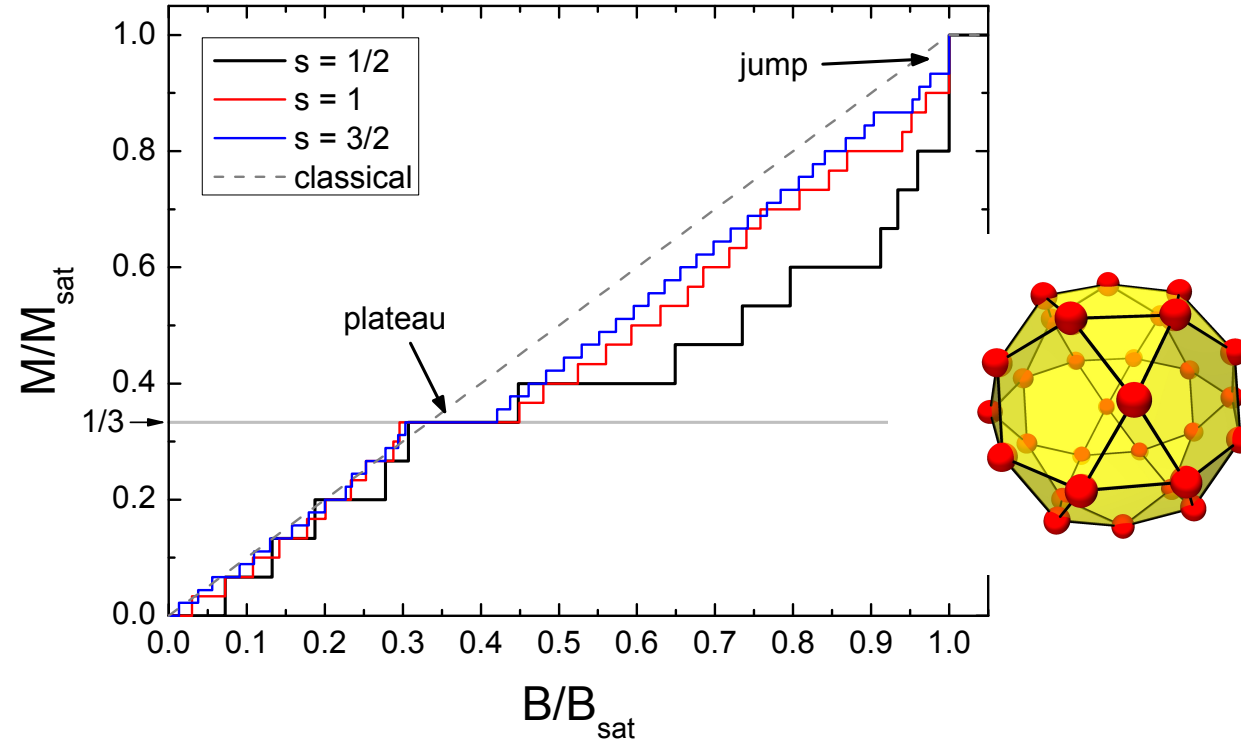
(1) S. R. White, Phys. Rev. Lett. **69**, 2863 (1992).  
 (2) S. R. White, Phys. Rev. B **48**, 10345 (1993).  
 (3) U. Schollwöck, Rev. Mod. Phys. **77**, 259 (2005).

# DMRG spin chain $s = 1/2$



- Simple example: 1000 spins with  $s = 1/2$ ; Hilbert space dimension  $2^{1000} \approx 10^{301}$ .
- Approaches result known from Bethe ansatz with matrices as small as  $50 \times 50$ !

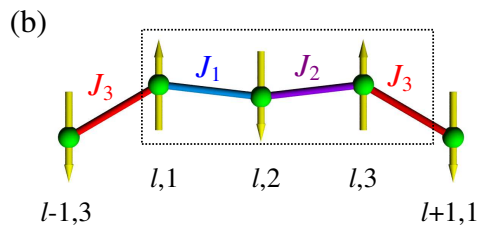
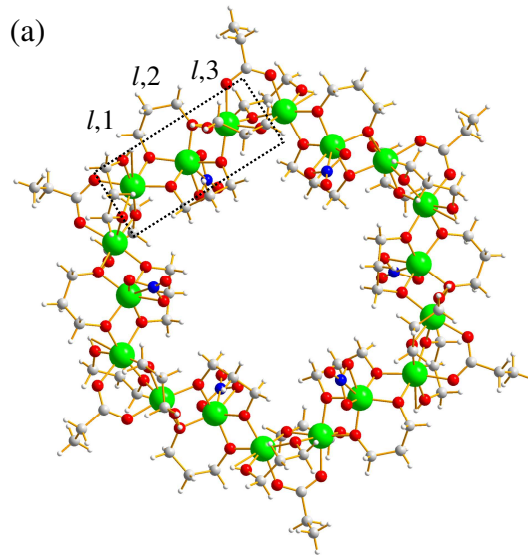
# Density Matrix Renormalization Group



- DMRG yields ground states + very few low-lying states in orthogonal subspaces.
- Magnetization curve for  $T = 0$ , resonance energies for spectroscopy.

(1) J. Ummethum, J. Schnack, and A. Laeuchli, J. Magn. Magn. Mater. **327** (2013) 103

# Dynamical Density Matrix Renormalization Group

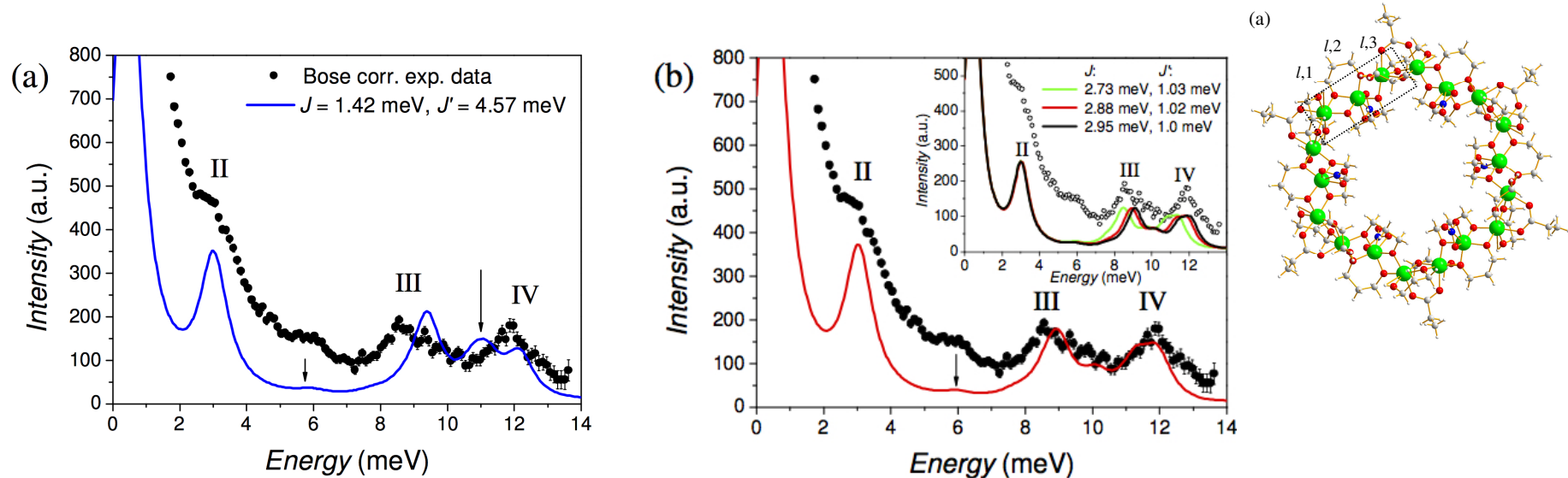


## Evaluation of correlation functions, e.g. for INS:

- $S_{jj'}^{zz}(\omega) \equiv \sum_n \langle 0 | \mathcal{S}_j^z | n \rangle \langle n | \mathcal{S}_{j'}^z | 0 \rangle \delta(\hbar\omega - E_n + E_0)$ ; transitions from the ground state;
- $S_{jj'}^{zz}(\omega) \approx \frac{1}{\pi} \langle 0 | \mathcal{S}_j^z \frac{\eta}{(E_0 + \hbar\omega - \tilde{H})^2 + \eta^2} \mathcal{S}_{j'}^z | 0 \rangle$ ;
- Use DMRG ground state and DMRG representation of  $\tilde{H}$  (1,2);  $\eta$  – finite broadening.

(1) T. D. Kühner and S. R. White, Phys. Rev. B **60**, 335 (1999).  
 (2) E. Jeckelmann, Phys. Rev. B **66**, 045114 (2002).  
 (3) P. King, T. C. Stamatatos, K. A. Abboud, and G. Christou, Angew. Chem. Int. Ed. **45**, 7379 (2006).  
 (4) O. Waldmann *et al.*, Phys. Rev. Lett. **102**, 157202 (2009).

# Dynamical Density Matrix Renormalization Group



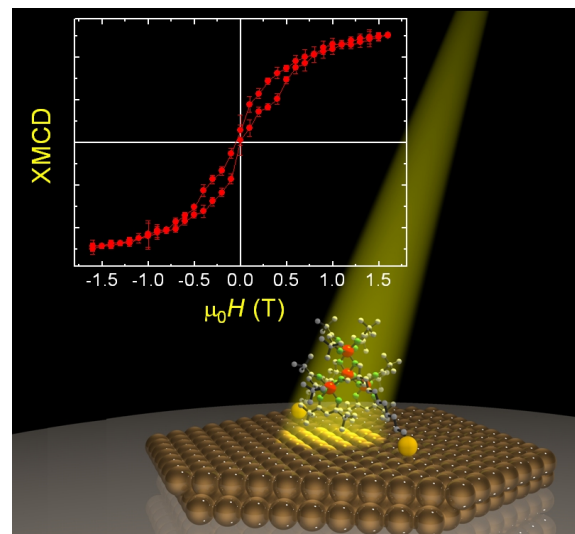
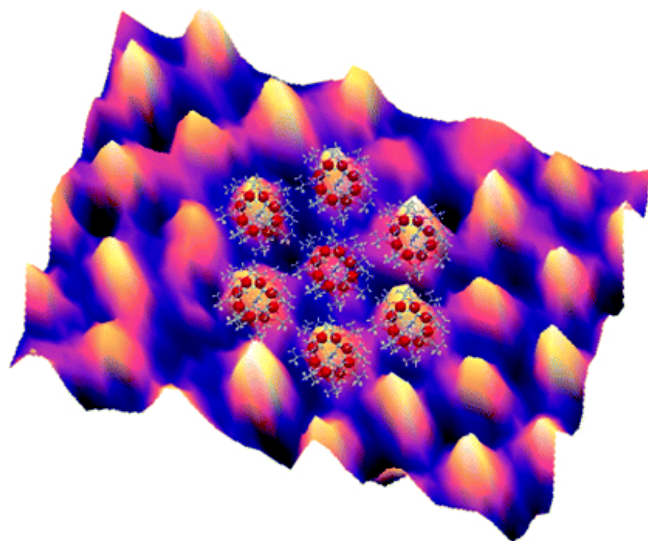
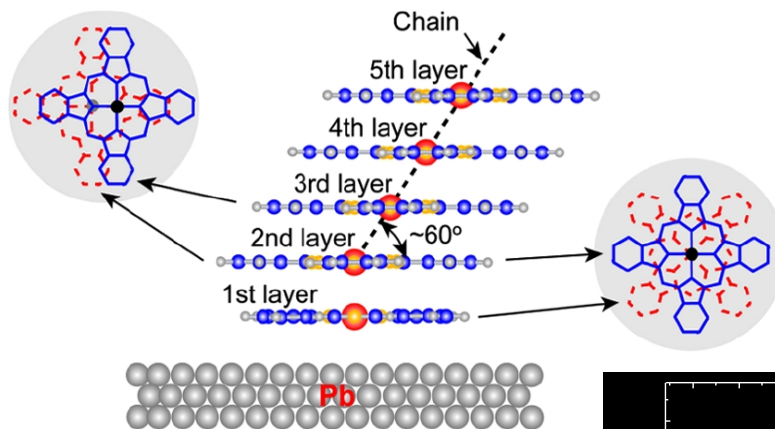
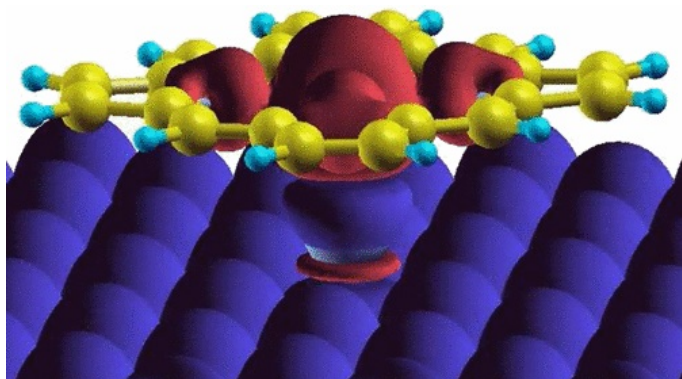
- Accurate description of low-lying excitations for the giant ferric wheel  $\text{Fe}_{18}$ . Hilbert space dimension  $10^{14}$ .
- Determination of model parameters.

(1) J. Ummethum, J. Nehr Korn, S. Mukherjee, N. B. Ivanov, S. Stuiber, Th. Strässle, P. L. W. Tregenna-Piggott, H. Mutka, G. Christou, O. Waldmann, J. Schnack, Phys. Rev. B **86**, 104403 (2012).

# Numerical Renormalization Group calculations

(Good for deposited molecules.)

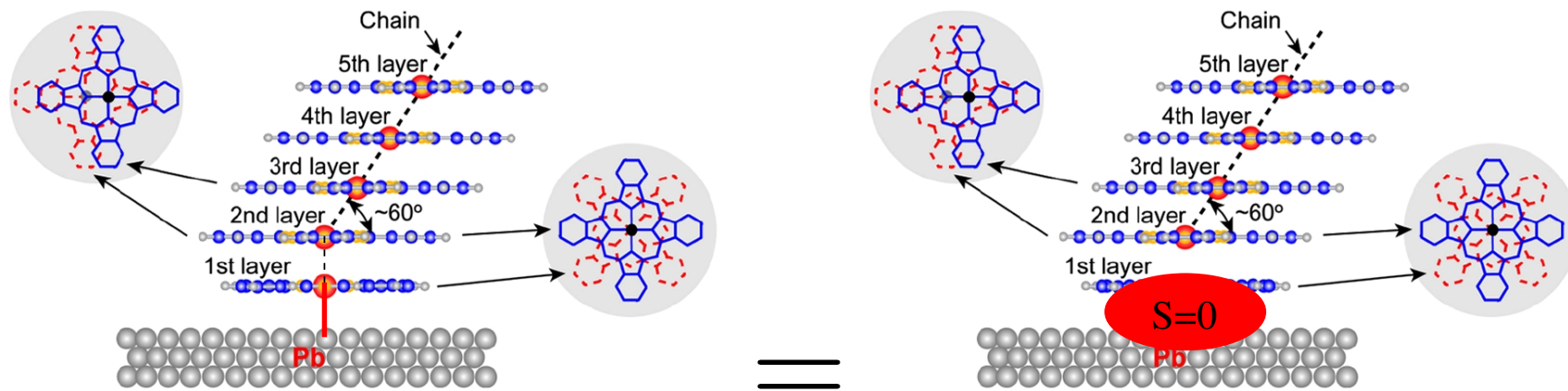
# You want to deposit a molecule



M. Bernien *et al.*, Phys. Rev. Lett. **102**, 047202 (2009); A. Ghirriet *et al.*, ACS Nano, **5**, 7090-7099 (2011); X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008); M. Mannini *et al.*, Nature Materials **8**, 194 - 197 (2009).



# Physical example (ICMM 2010)

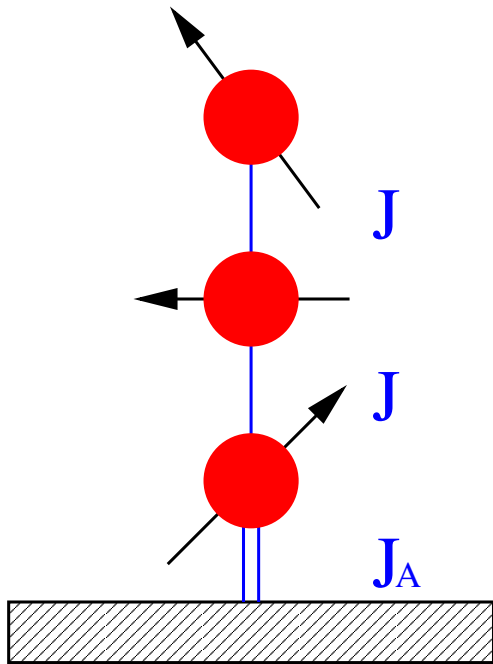


Stack of deposited Cobalt phthalocyanine (CoPc) molecules;  
 $\text{Co}^{2+}$  with spin  $s = 1/2$ .

Under which circumstances is the picture of total screening correct?

X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008).

# NRG – minimal model (**already an approximation!**)



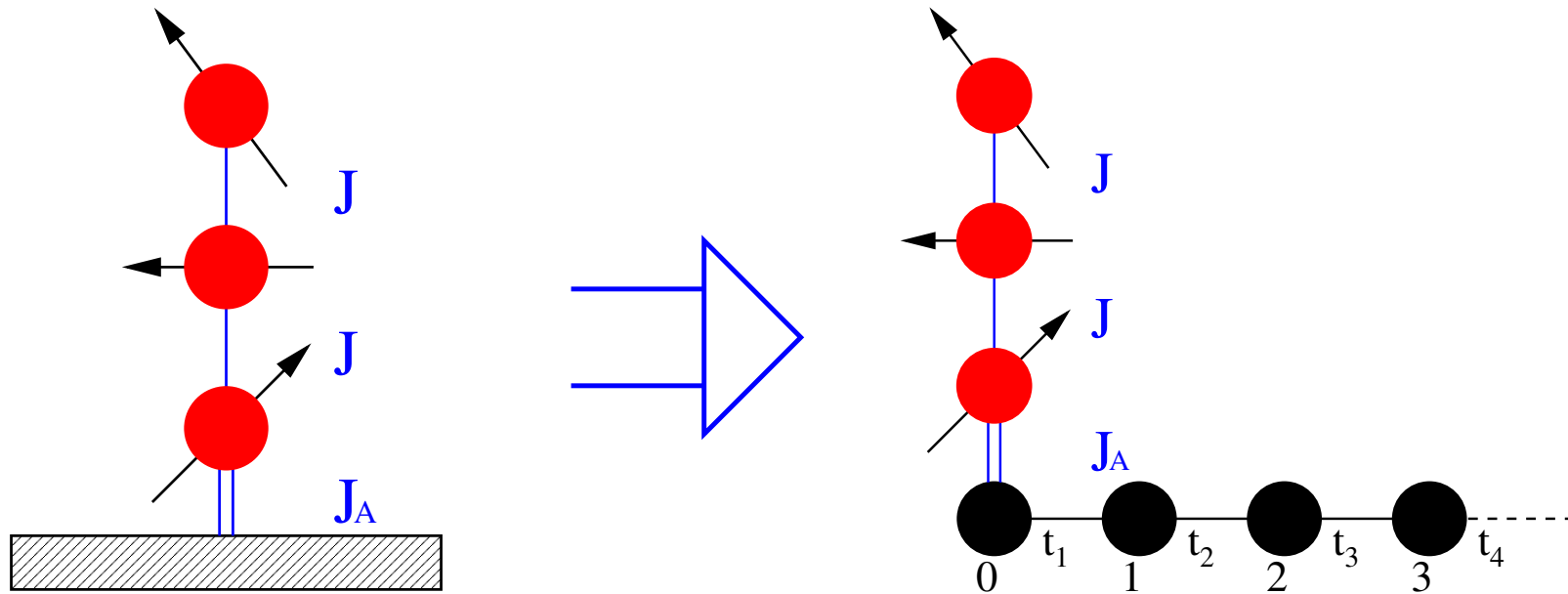
- $\tilde{H} = \tilde{H}_{\text{electrons}} + \tilde{H}_{\text{coupling}} + \tilde{H}_{\text{impurity}}$   

$$\tilde{H}_{\text{electrons}} = \sum_{i \neq j, \sigma} t_{ij} d_{i\sigma}^\dagger d_{j\sigma} + g_e \mu_B B \mathcal{S}^z$$

$$\tilde{H}_{\text{coupling}} = -2J_A \tilde{\mathcal{S}} \cdot \underline{s}_0 \quad , \quad \underline{s}_0 - \text{spin density at contact}$$
- $\tilde{H}_{\text{impurity}} = \text{Hamiltonian of your molecule!}$
- **NRG  $\equiv$  construction of a small (!) effective model in order to evaluate properties of the deposited cluster, the impurity (3).**

(1) K. G. Wilson, Rev. Mod. Phys. **47**, 773 (1975)  
 (2) M. Höck, J. Schnack, Phys. Rev. B **87**, 184408 (2013)  
 (3) *Impurity* is a technical term in this context and not an insult to chemists.

# NRG in a cartoon



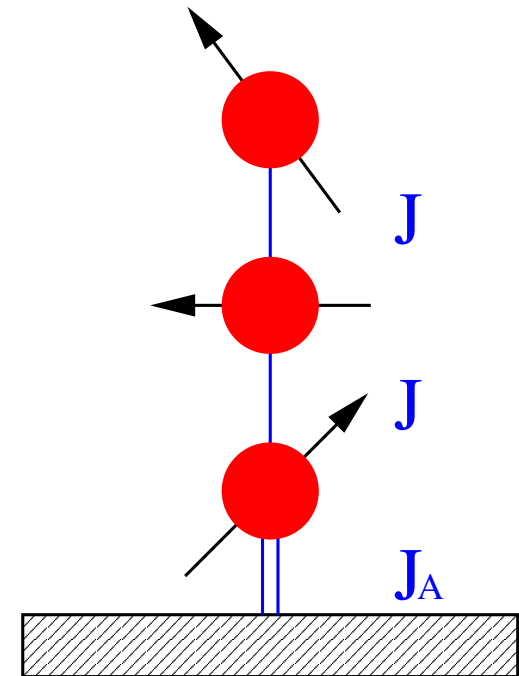
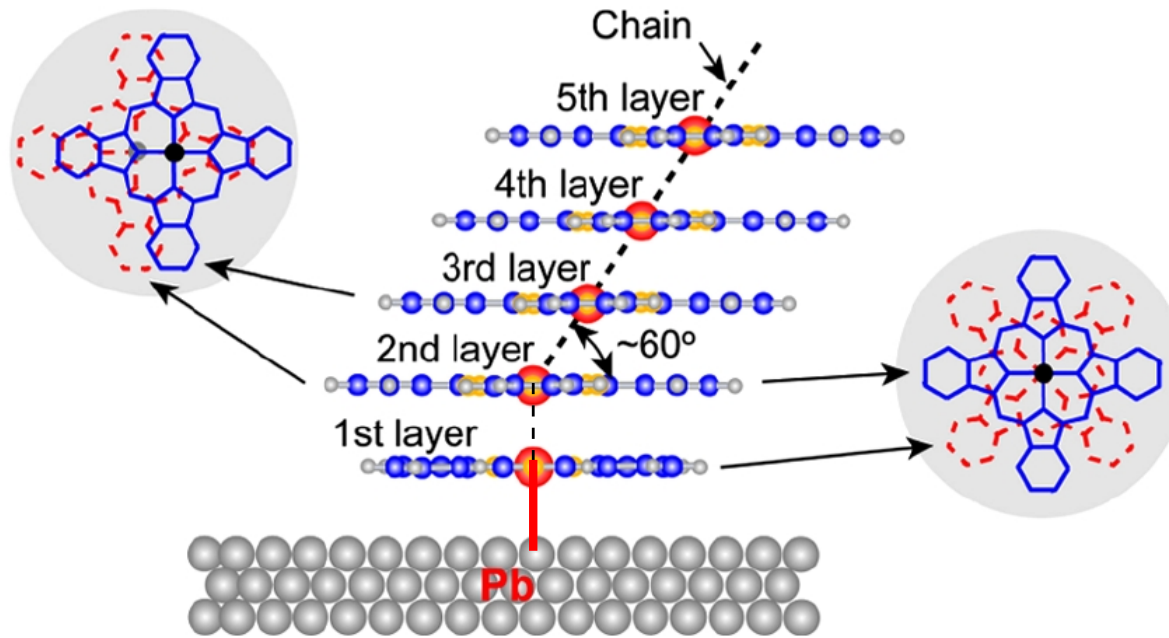
Metallic surface is replaced by semi-infinite Hubbard chain;

Parameters of the chain: hopping matrix elements and on-site energies;

Stepwise enlargement of the chain ( $t_1 > t_2 > t_3 \dots$ );

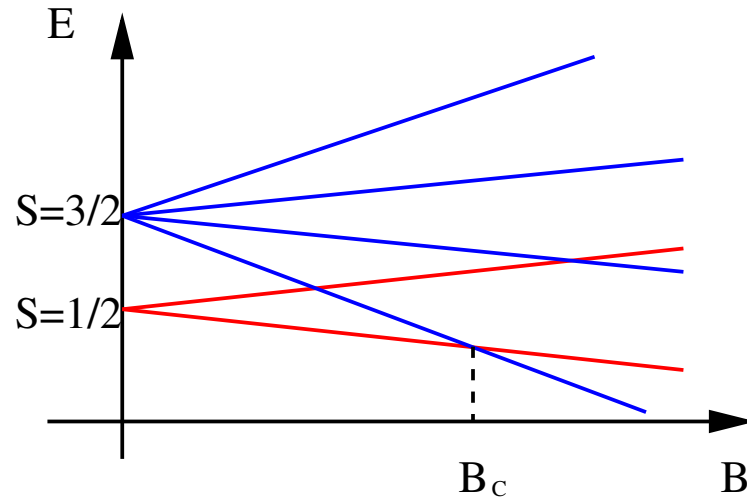
Truncation of basis set when matrices grow too big.

# Once more: deposited chain

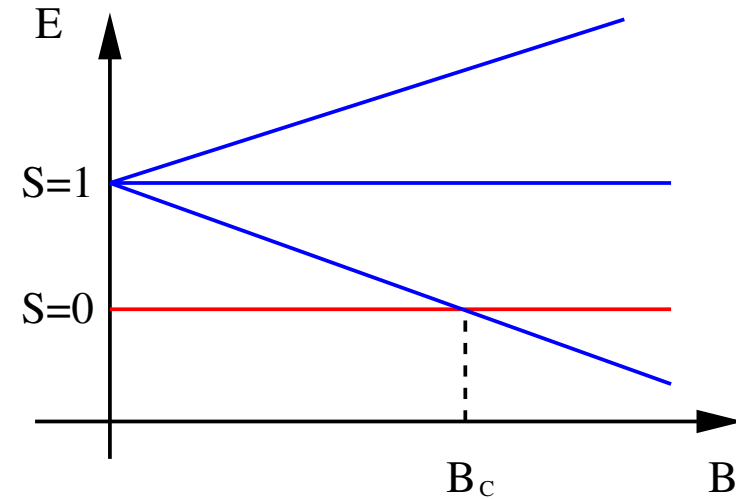


X. Chen *et al.*, Phys. Rev. Lett. **101**, 197208 (2008).

# Energy levels of limiting cases for deposited trimer



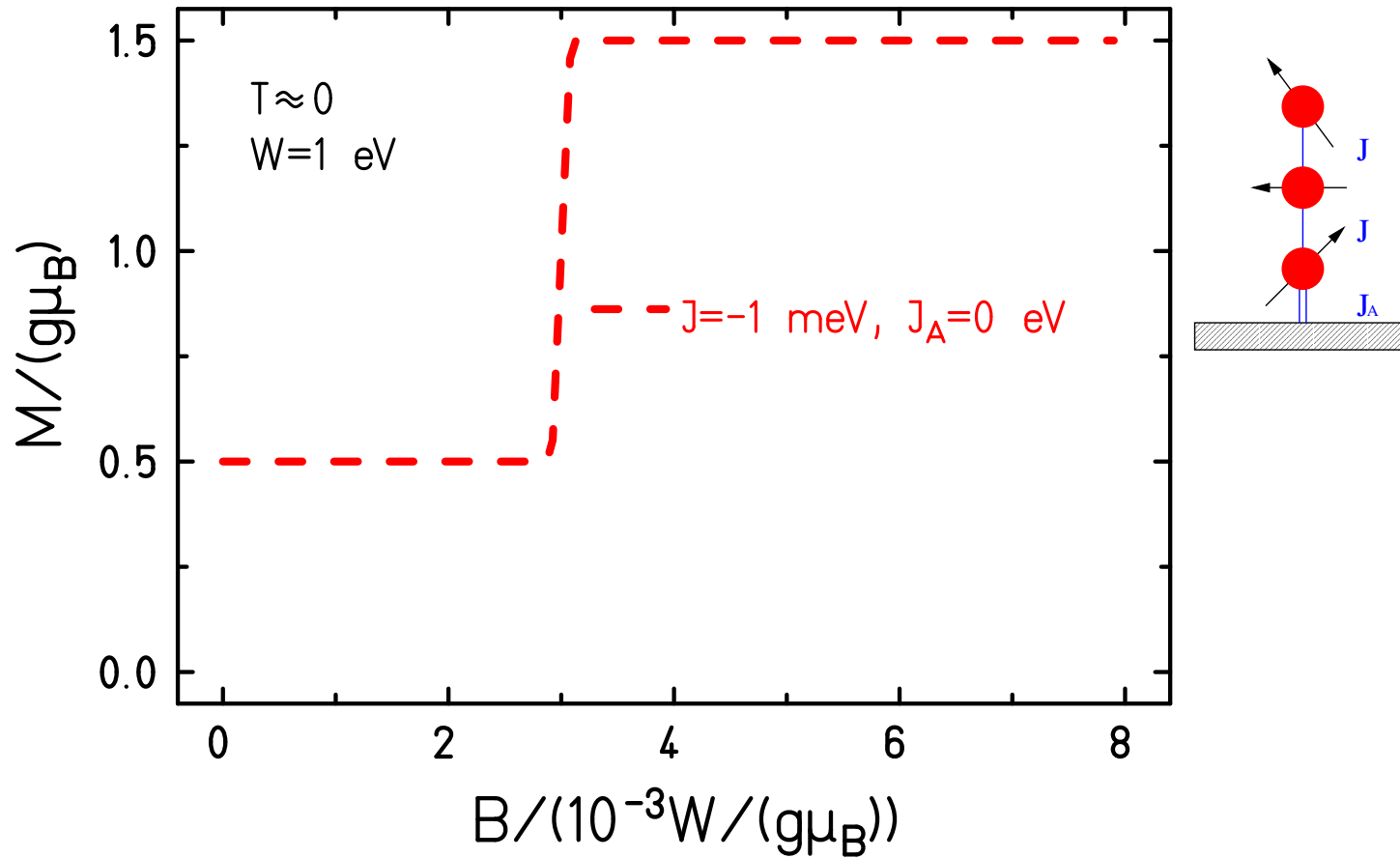
- energy levels of a trimer



- energy levels of a dimer

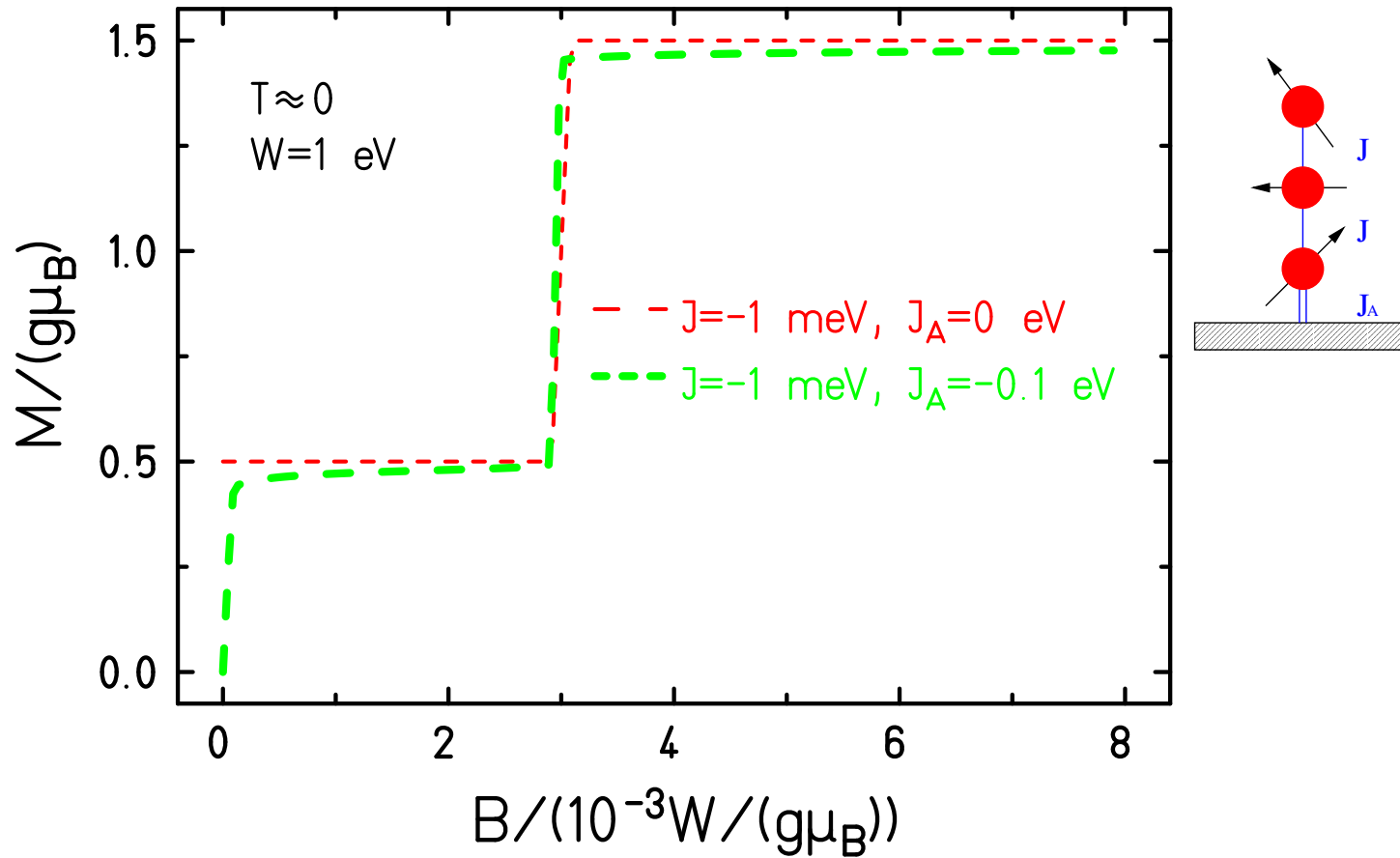
Magnetization curves different; could be seen in XMCD.  
 NRG calculates observables also between limiting cases  
 and can thus tell under which circumstances a limiting case applies.

# Increasing coupling to the substrate



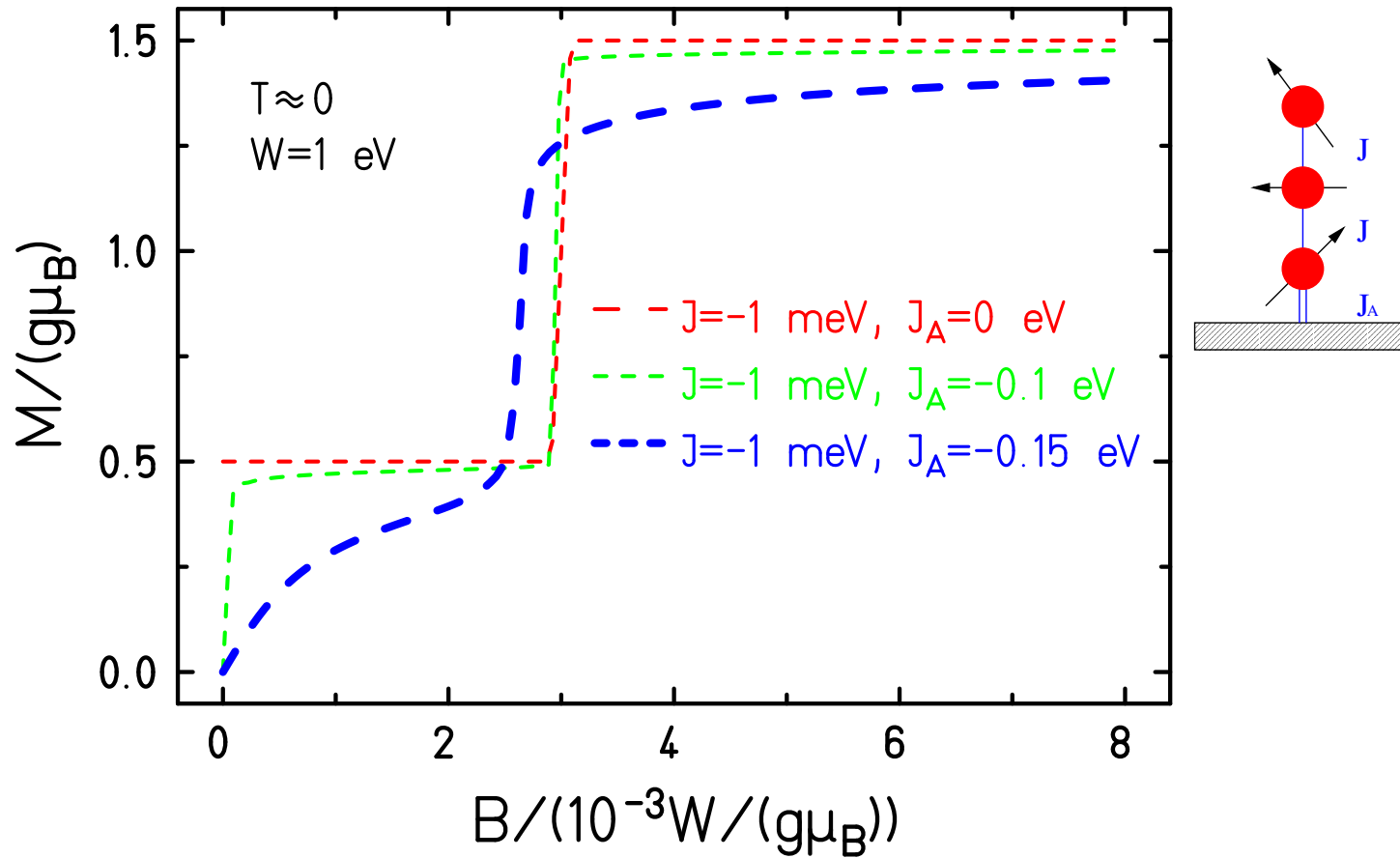
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

# Increasing coupling to the substrate



H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

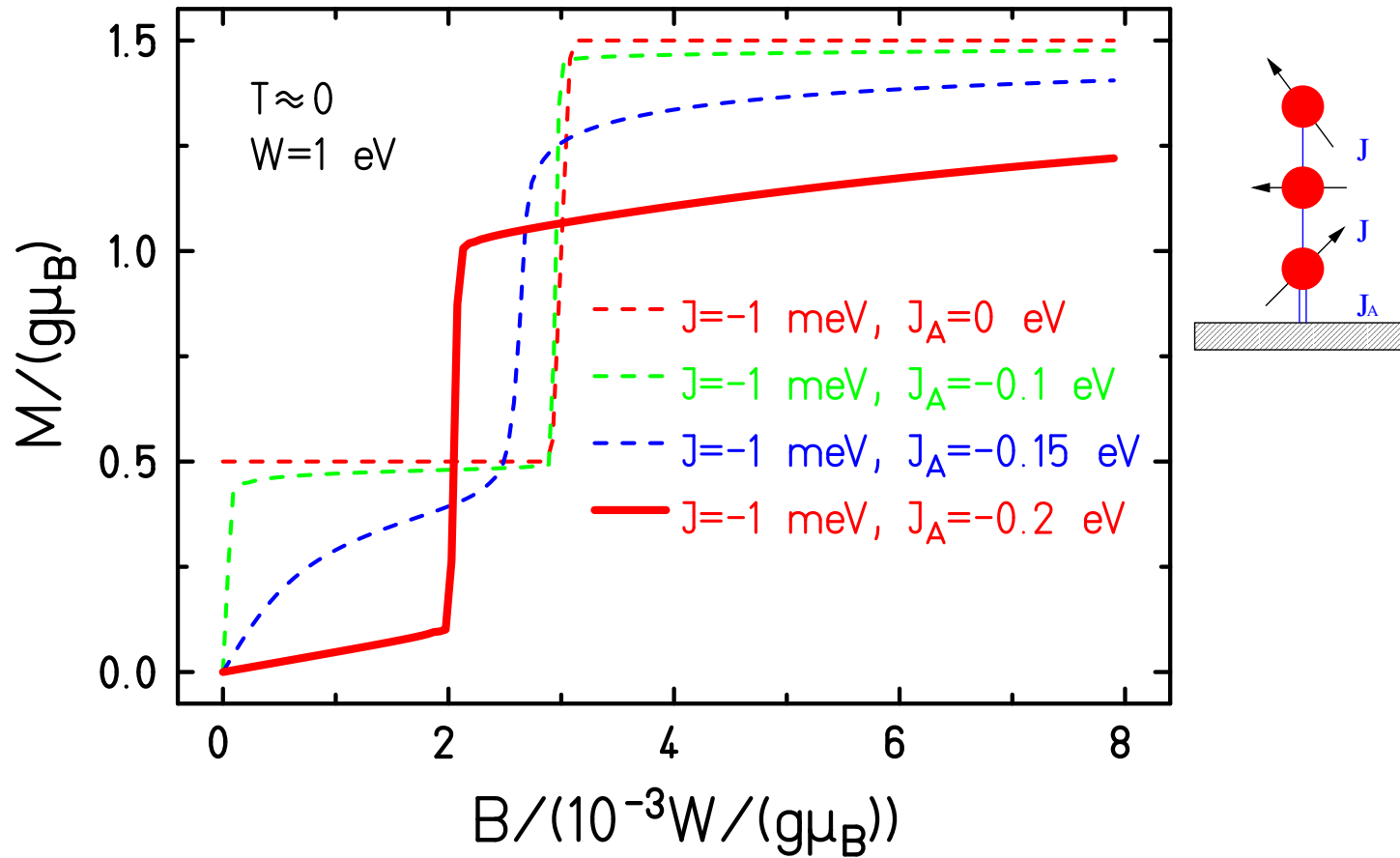
# Increasing coupling to the substrate



H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

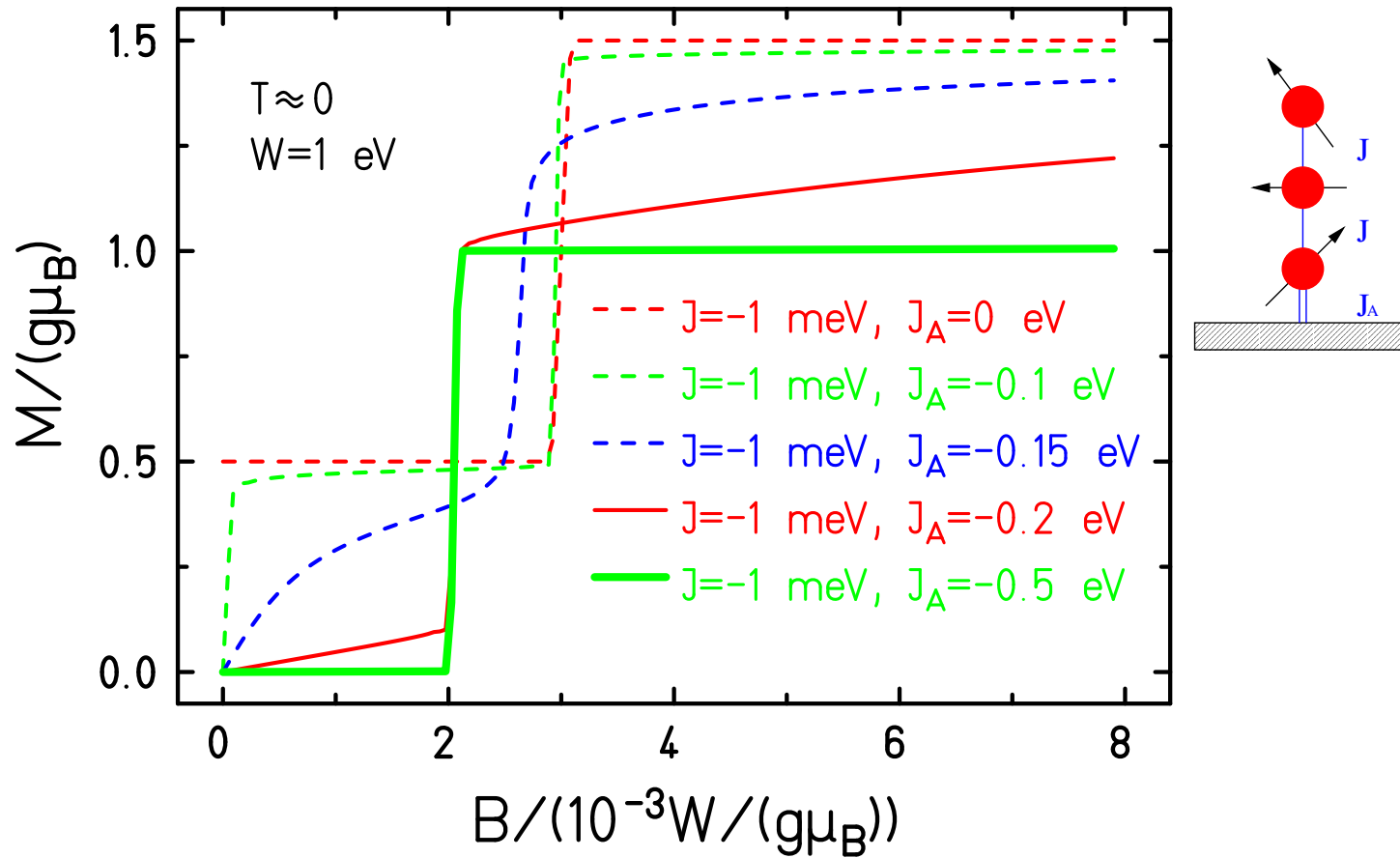


# Increasing coupling to the substrate



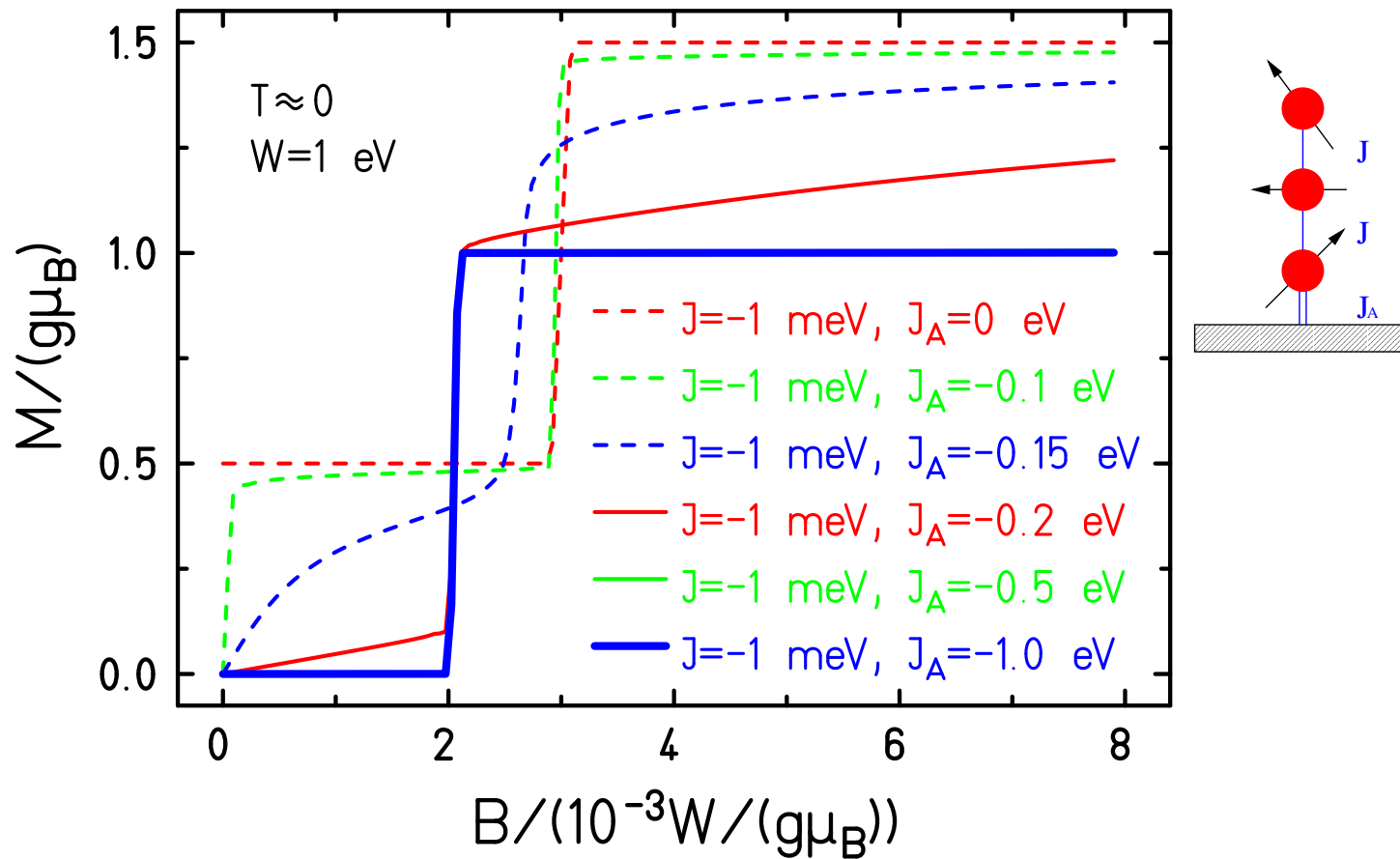
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

# Increasing coupling to the substrate



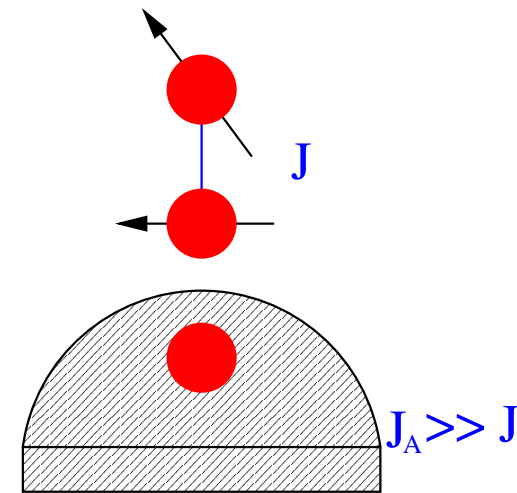
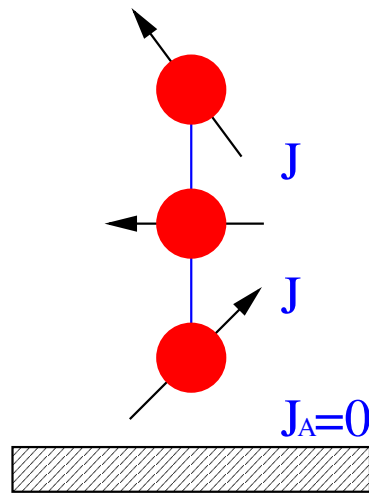
H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

# Increasing coupling to the substrate



H.-T. Langwald and J. Schnack, submitted; arXiv:1312.0864.

# Weak vs. strong coupling



- weak coupling limit:  
unperturbed molecule (trimer)

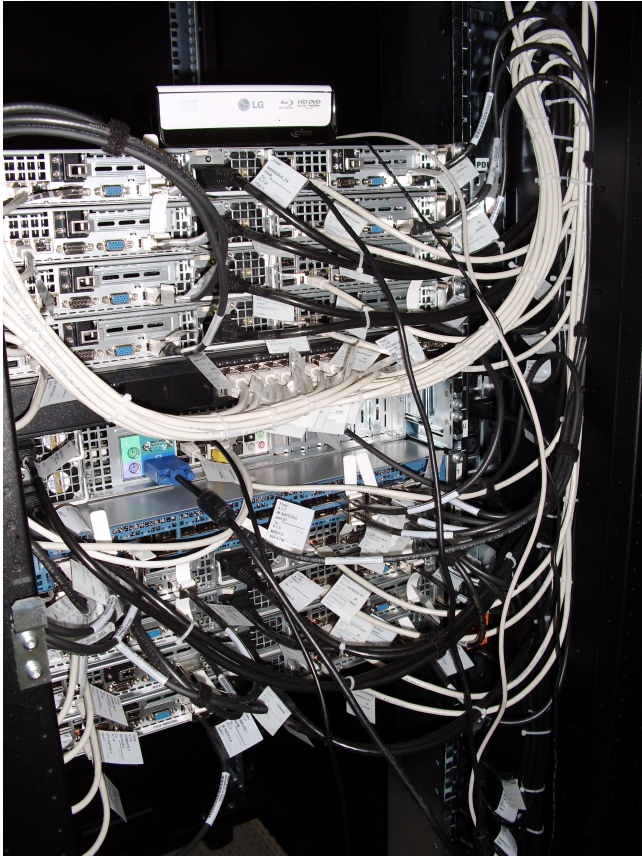
- $|J_A| \lesssim 0.1W$

- strong coupling limit:  
effective remainder (dimer)

- $|J_A| \gtrsim 0.5W$

Inbetween: no simple characterization + further sequential screening possible

# Summary



- Exact diagonalization is great but limited.
- Finite-Temperature Lanczos is a good approximate method for Hilbert space dimensions smaller than  $10^{10}$ . The accuracy is amazing!
- FTLM works for anisotropic spin systems.
- Magnetic molecules for storage, q-bits, MCE, and since they are nice.

## Many thanks to my collaborators worldwide

- M. Czopnik, T. Glaser, O. Hanebaum, Chr. Heesing, M. Höck, N.B. Ivanov, F. Kaiser, H.-T. Langwald, S. Leiding, A. Müller, R. Schnalle, Chr. Schröder, J. Ummethum (Bielefeld)
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Thank you very much for your  
attention.

The end.

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