

FOR2692 meets SFB/TRR288: typicality and exact quantum dynamics for equilibrium and non-equilibrium properties

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Online, 23 April 2021

Simplified remarks on typicality

Typicality bill of rights – FOR 2692

“All” non-equilibrium states relax equal

- C. Bartsch, J. Gemmer, Dynamical typicality of quantum expectation values, Phys. Rev. Lett., 102, 110403 (2009)
- P. Reimann, J. Gemmer, Why are macroscopic experiments reproducible? Imitating the behavior of an ensemble by single pure states, Physica A 552, 121840 (2020)
- T. Heitmann, J. Richter, D. Schubert, R. Steinigeweg, Selected applications of typicality to real-time dynamics of quantum many-body systems, Zeitschrift für Naturforschung A 75, 421 (2020)

“All” similar Hamiltonians produce equal time-evolution

- P. Reimann, Typical fast thermalization processes in closed many-body systems, Nat. Commun. 7, 10821 (2016)
- L. Dabelow, P. Reimann, Relaxation Theory for Perturbed Many-Body Quantum Systems versus Numerics and Experiment, Phys. Rev. Lett. 124, 120602 (2020)

“All” random states are equal(ly hot)

- A. Hams, H. De Raedt, Fast algorithm for finding the eigenvalue distribution of very large matrices, Phys. Rev. E 62, 4365 (2000)
- J. Schnack, J. Richter, R. Steinigeweg, Accuracy of the finite-temperature Lanczos method compared to simple typicality-based estimates, Phys. Rev. Research 2, 013186 (2020)

S. Lloyd@arXiv:1307.0378: Pure state quantum statistical mechanics and black holes, submitted to PRB in 1988 but rejected by one sentence referee report: “There is no physics.”

Deutsch, Srednicki, Goldstein, Lebowitz, Tumulka, Zanghi, Popescu, Short, Winter, Sugiura, Shimizu, . . .

Yes, we can!



$$\begin{pmatrix} 3 & 42 & 4711 \\ 42 & 0 & 3.14 \\ 4711 & 3.14 & 8 \\ -17 & 007 & 13 \\ 1.8 & 15 & 081 \end{pmatrix}$$

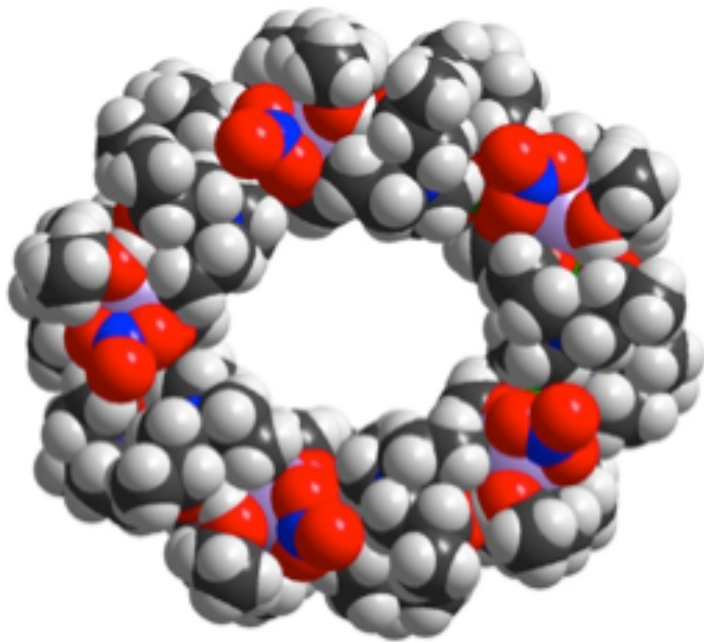
1. A flash on magnetic molecules
2. **Typicality approach to equilibrium**
3. Stability of clock transitions
4. **Spin-phonon issues**

We are the sledgehammer team of matrix diagonalization.
Please send inquiries to jschnack@uni-bielefeld.de!

We investigate magnetic molecules

J. Schnack, Contemporary Physics **60**, 127-144 (2019)

You have got a molecule!

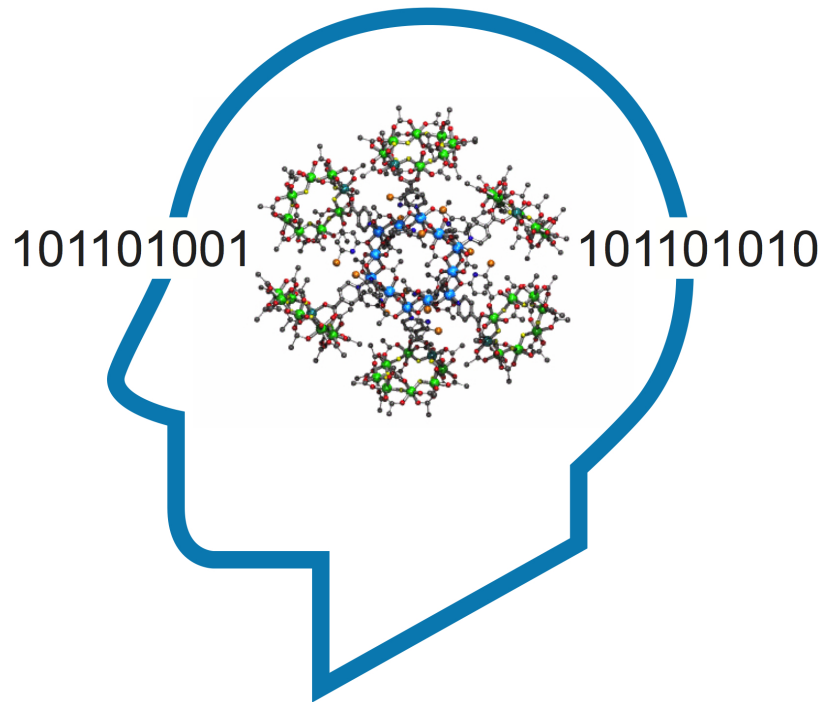


$$S = 60!$$

Congratulations!

Powell group: npj Quantum Materials **3**, 10 (2018)

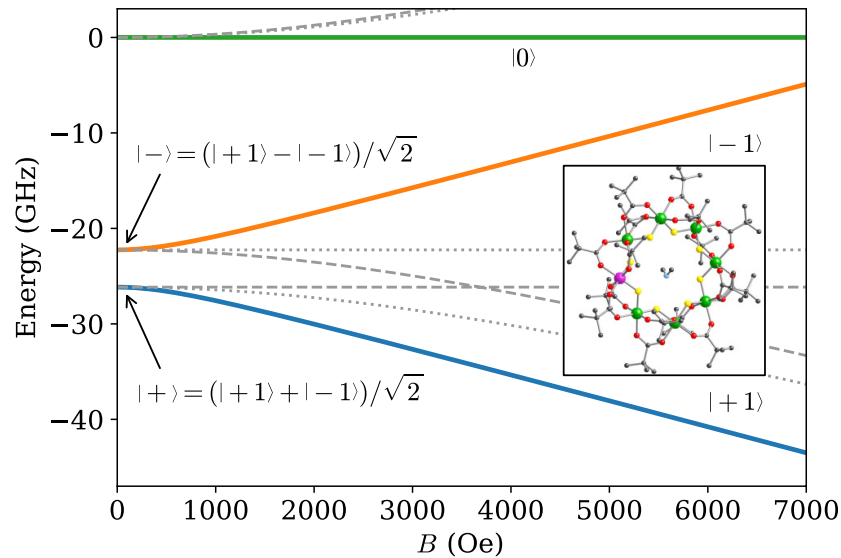
You want to build a quantum computer!



Very smart!

Wernsdorfer group: Phys. Rev. Lett. **119**, 187702 (2017)

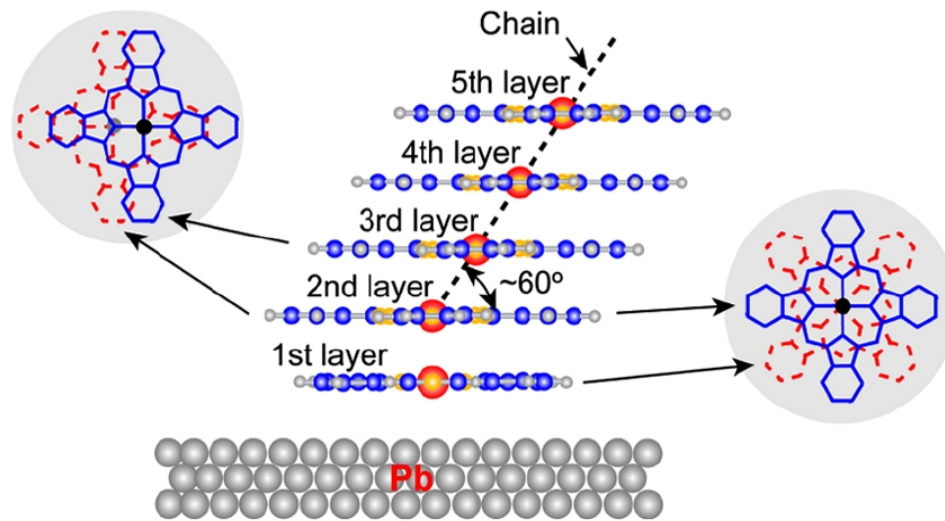
You want to achieve quantum coherence!



Desperately needed!

Friedman group: Phys. Rev. Research **2**, 032037(R) (2020)

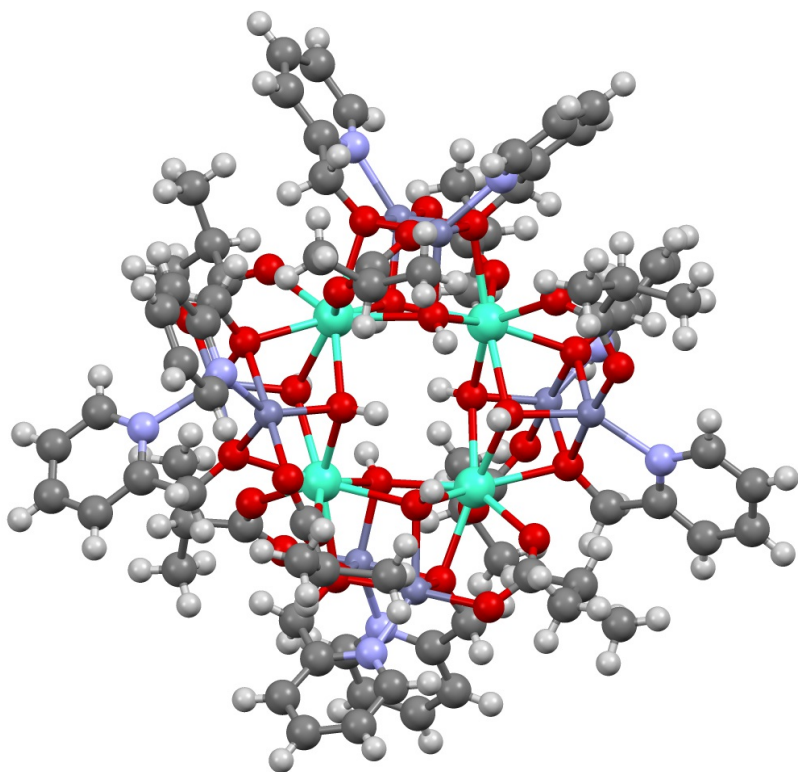
You want to deposit your molecule!



Next generation magnetic storage!

Xue group: Phys. Rev. Lett. **101**, 197208 (2008)

You want molecular magnetocalorics!



Cool!

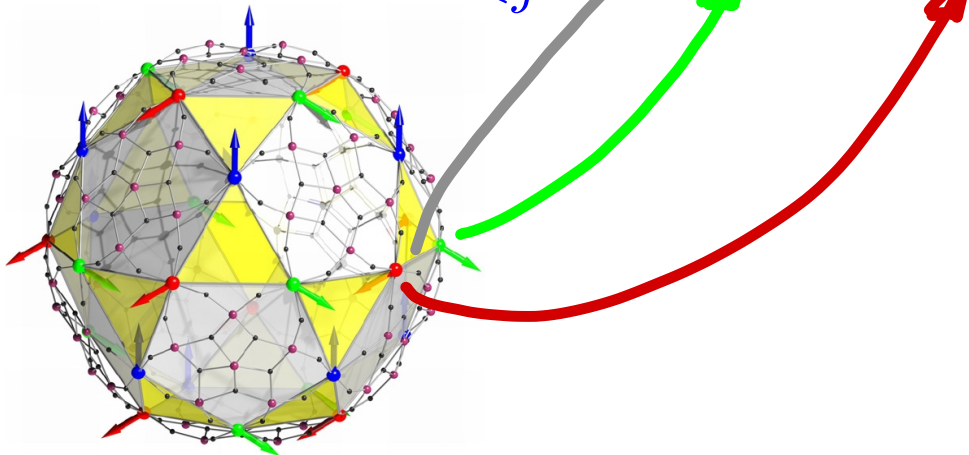
Brechin group: *Angew. Chem. Int. Ed.* **51**, 4633 (2012)

You have got an idea about the modeling!

Heisenberg

Zeeman

$$\underline{H} = -2 \sum_{i < j} J_{ij} \underline{\vec{s}}(i) \cdot \underline{\vec{s}}(j) + g \mu_B B \sum_i^N s_z(i)$$



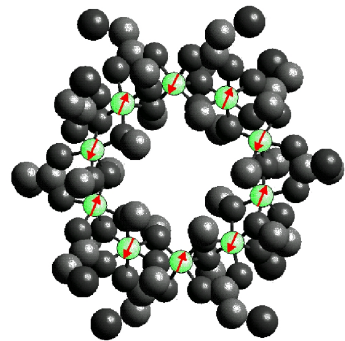
You have to solve the Schrödinger equation!

$$\underline{H} |\phi_n\rangle = E_n |\phi_n\rangle$$

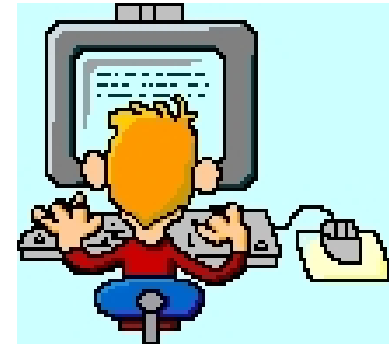
Eigenvalues E_n and eigenvectors $|\phi_n\rangle$

- needed for spectroscopy (EPR, INS, NMR);
- needed for thermodynamic functions (magnetization, susceptibility, heat capacity);
- needed for time evolution (pulsed EPR, simulate quantum computing, thermalization).

In the end it's always a big matrix!



$$\Rightarrow \begin{pmatrix} -27.8 & 3.46 & 0.18 & \cdots \\ 3.46 & -2.35 & -1.7 & \cdots \\ 0.18 & -1.7 & 5.64 & \cdots \\ \vdots & \vdots & \vdots & \cdots \end{pmatrix} \Rightarrow$$



$$\text{Fe}_{10}^{\text{III}}: N = 10, s = 5/2, \dim(\mathcal{H}) = (2s + 1)^N$$

Dimension=**60,466,176**. Maybe too big?

Can we evaluate the partition function

$$Z(T, B) = \text{tr} \left(\exp \left[-\beta \underline{H} \right] \right)$$

without diagonalizing the Hamiltonian?

Yes, with magic!

Typicality approach to molecular magnetism

Solution I: trace estimators

$$\text{tr}(\tilde{Q}) \approx \langle r | \tilde{Q} | r \rangle = \sum_{\nu} \langle \nu | \tilde{Q} | \nu \rangle + \sum_{\nu \neq \mu} r_{\nu} r_{\mu} \langle \nu | \tilde{Q} | \mu \rangle$$

$$|r\rangle = \sum_{\nu} r_{\nu} |\nu\rangle, \quad r_{\nu} = \pm 1$$

- $|\nu\rangle$ some orthonormal basis of your choice; not the eigenbasis of \tilde{Q} , since we don't know it.
- $r_{\nu} = \pm 1$ random, equally distributed. Rademacher vectors.
- **Amazingly accurate, bigger (Hilbert space dimension) is better.**

M. Hutchinson, Communications in Statistics - Simulation and Computation **18**, 1059 (1989).

Solution II: Krylov space representation

$$\exp \left[-\beta \underline{H} \right] \approx \underline{1} - \beta \underline{H} + \frac{\beta^2}{2!} \underline{H}^2 - \dots - \frac{\beta^{N_L-1}}{(N_L-1)!} \underline{H}^{N_L-1}$$

applied to a state $|r\rangle$ yields a superposition of

$$\underline{1} |r\rangle, \quad \underline{H} |r\rangle, \quad \underline{H}^2 |r\rangle, \quad \dots \underline{H}^{N_L-1} |r\rangle.$$

These (linearly independent) vectors span a small space of dimension N_L ; it is called Krylov space.

Let's diagonalize \underline{H} in this space!

Partition function I: simple approximation

$$Z(T, B) \approx \langle r | e^{-\beta \tilde{H}} | r \rangle \approx \sum_{n=1}^{N_L} e^{-\beta \epsilon_n^{(r)}} |\langle n(r) | r \rangle|^2$$

$$O^r(T, B) \approx \frac{\langle r | \tilde{Q} e^{-\beta \tilde{H}} | r \rangle}{\langle r | e^{-\beta \tilde{H}} | r \rangle} = \frac{\langle r | e^{-\beta \tilde{H}/2} \tilde{Q} e^{-\beta \tilde{H}/2} | r \rangle}{\langle r | e^{-\beta \tilde{H}/2} e^{-\beta \tilde{H}/2} | r \rangle}$$

- Wow!!!
- One can replace a trace involving an intractable operator by an expectation value with respect to just ONE random vector evaluated by means of a Krylov space representation???
- Typicality = any random vector will do: $|r\rangle \equiv (T = \infty)$

J. Jaklic and P. Prelovsek, Phys. Rev. B **49**, 5065 (1994).

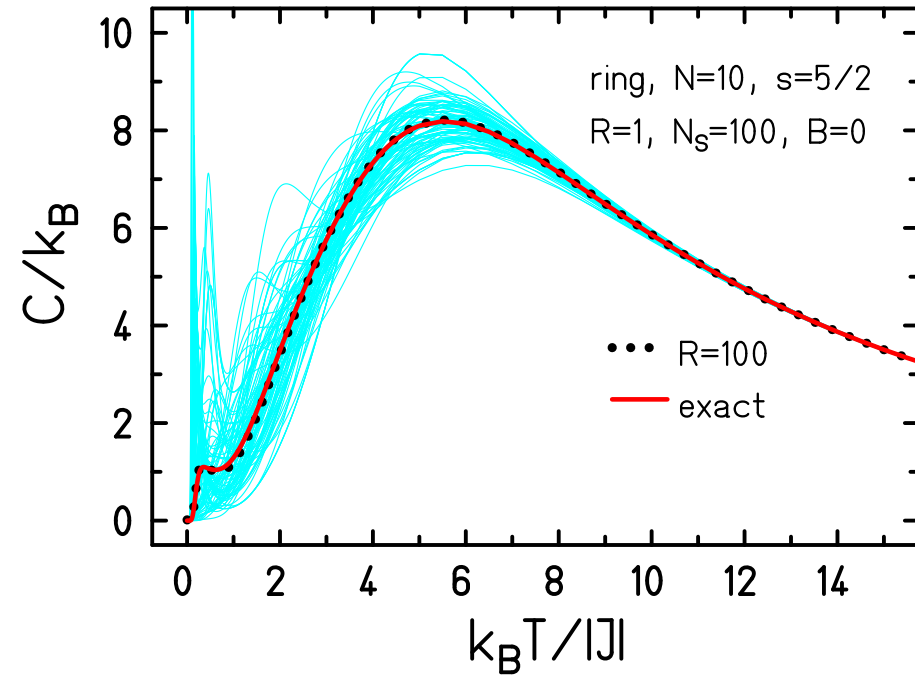
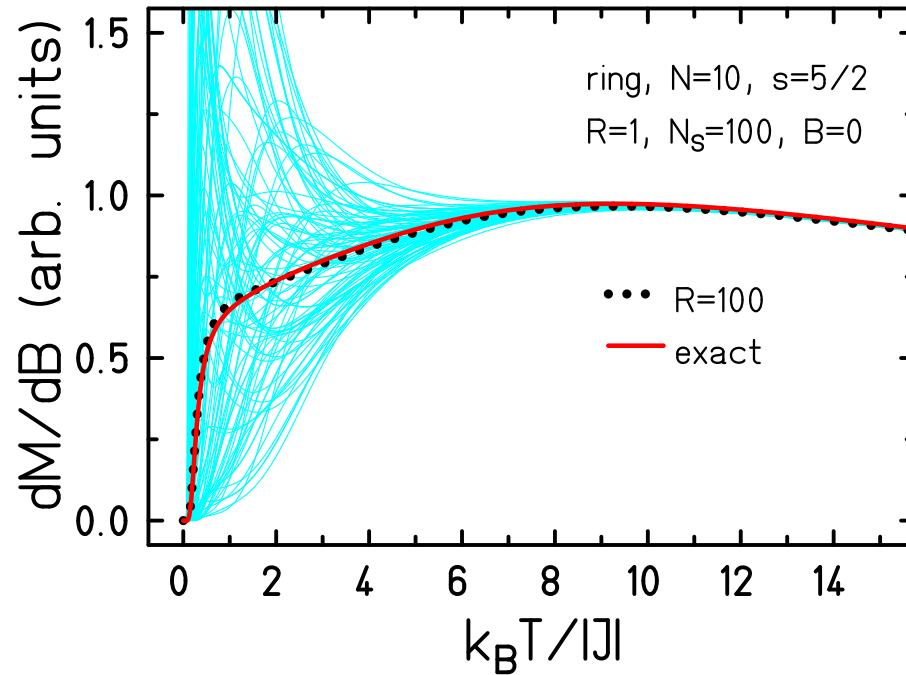
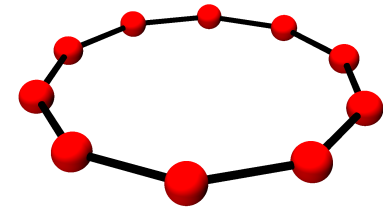
Partition function II: Finite-temperature Lanczos Method

$$Z^{\text{FTLM}}(T, B) \approx \frac{1}{R} \sum_{r=1}^R \sum_{n=1}^{N_L} e^{-\beta \epsilon_n^{(r)}} |\langle n(r) | r \rangle|^2$$

- Averaging over R random vectors is better.
- $|n(r)\rangle$ n -th Lanczos eigenvector starting from $|r\rangle$ (Rademacher vectors).
- **Partition function replaced by a small sum: $R = 1 \dots 100, N_L \approx 100$.**
- Use symmetries!

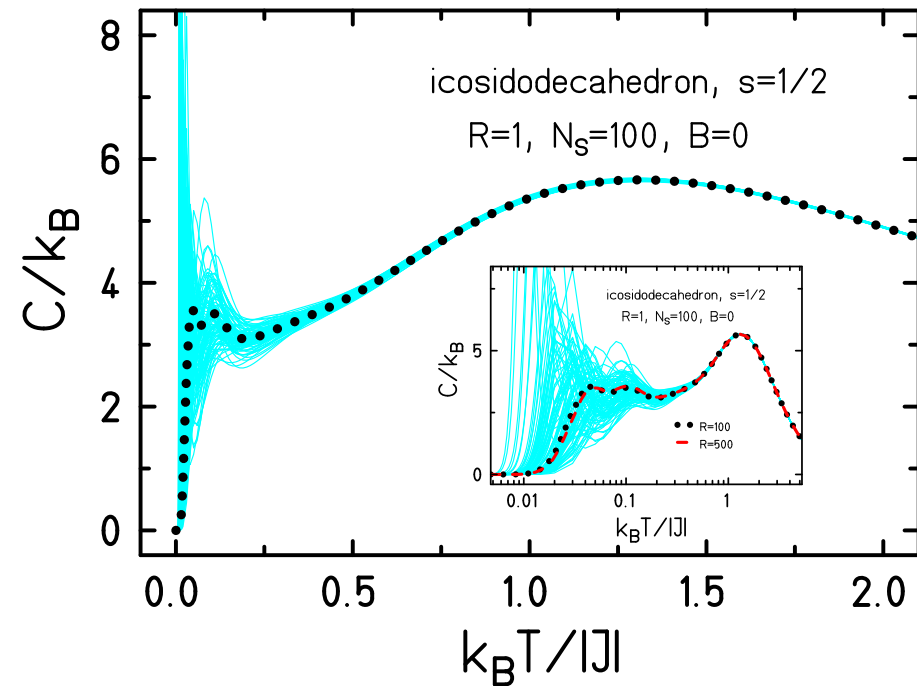
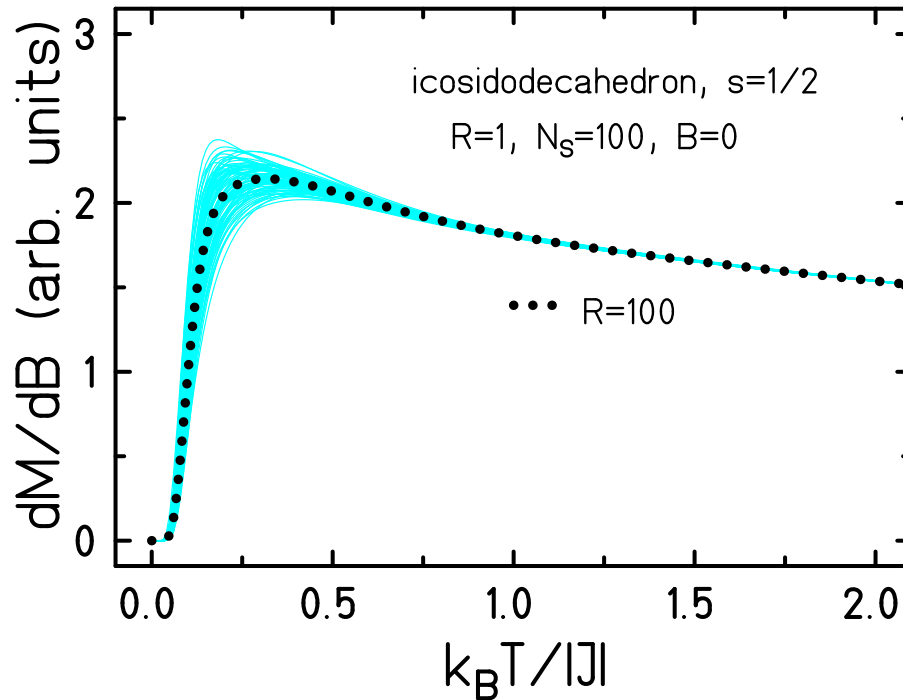
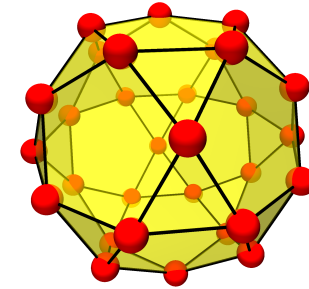
J. Jaklic and P. Prelovsek, Phys. Rev. B **49**, 5065 (1994).

FTLM 1: ferric wheel



- (1) J. Schnack, J. Richter, R. Steinigeweg, Phys. Rev. Research **2**, 013186 (2020).
- (2) SU(2) & D₂: R. Schnalle and J. Schnack, Int. Rev. Phys. Chem. **29**, 403 (2010).
- (3) SU(2) & C_N: T. Heitmann, J. Schnack, Phys. Rev. B **99**, 134405 (2019)

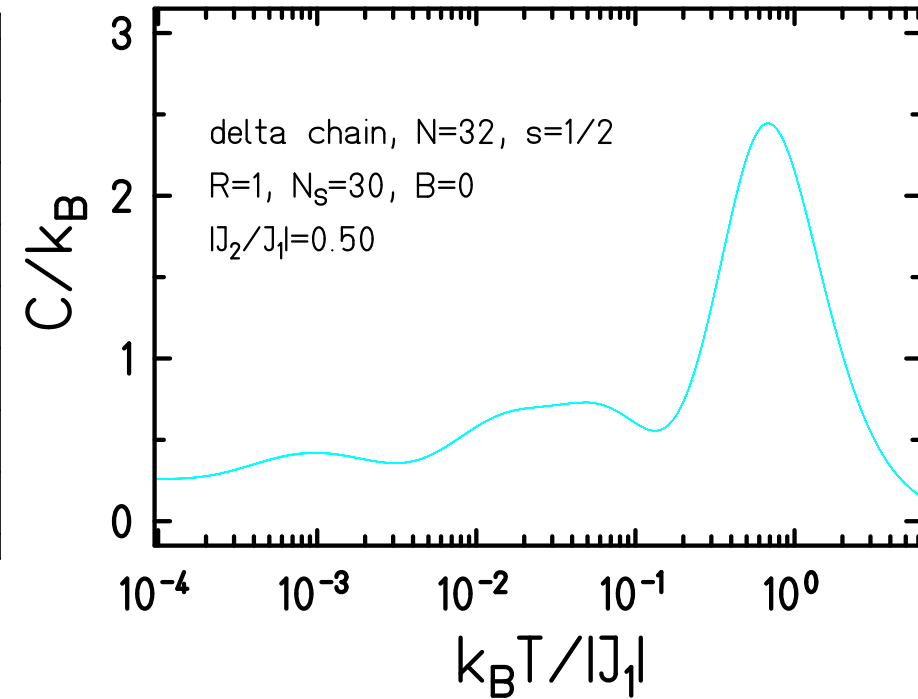
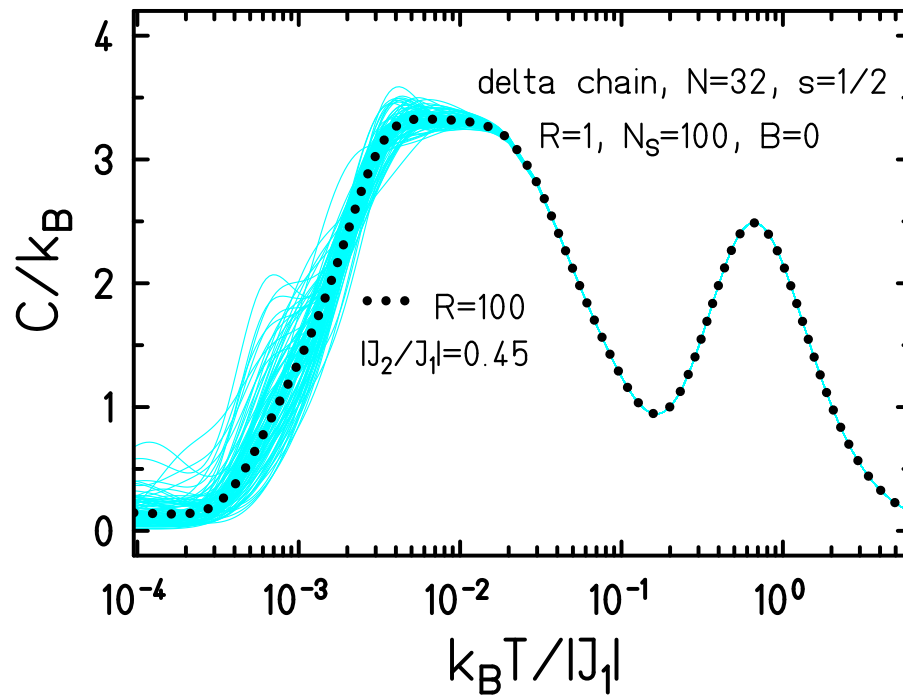
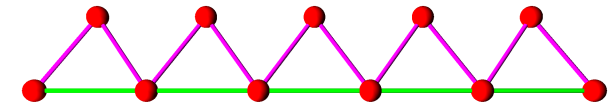
FTLM 2: icosidodecahedron



(1) J. Schnack, J. Richter, R. Steinigeweg, Phys. Rev. Research **2**, 013186 (2020).

(2) J. Schnack and O. Wendland, Eur. Phys. J. B **78**, 535 (2010).

FTLM 3: sawtooth chain



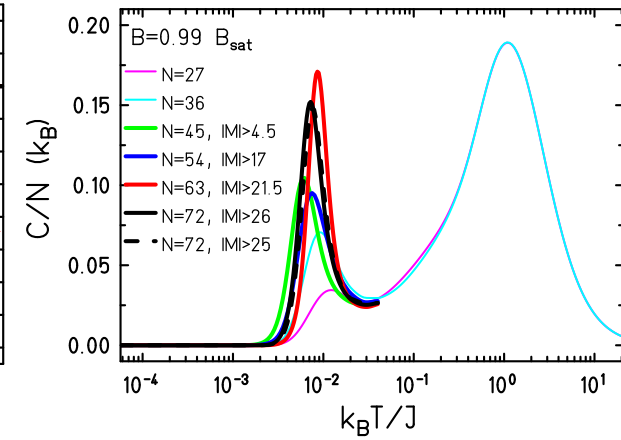
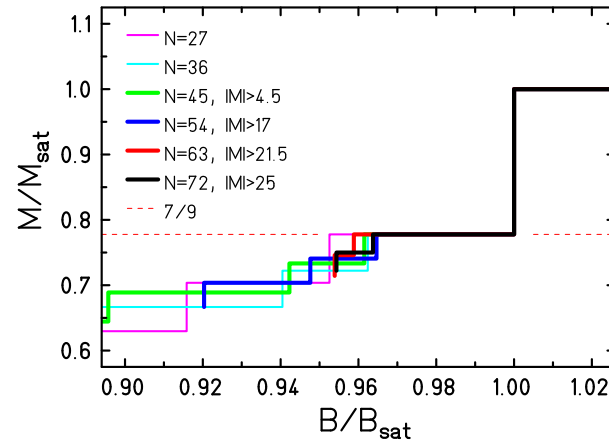
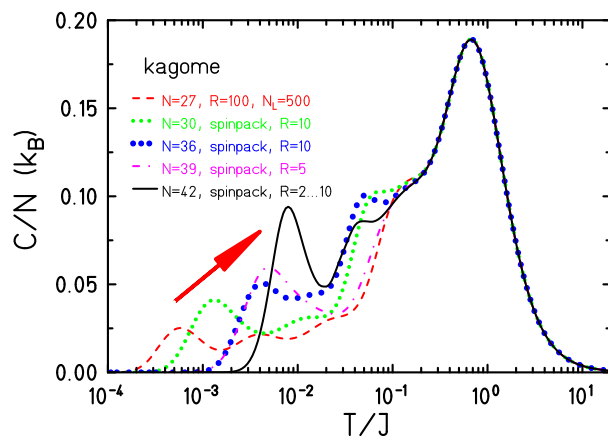
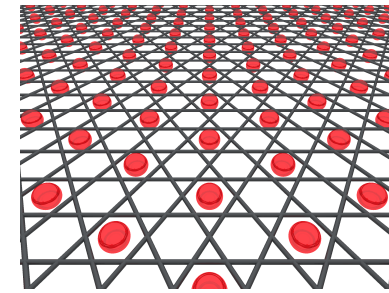
$|J_2/J_1| = 0.45$ – near critical, $|J_2/J_1| = 0.50$ – critical.

Frustration, technically speaking, works in your favour.

(1) J. Schnack, J. Richter, R. Steinigeweg, Phys. Rev. Research **2**, 013186 (2020)

(2) J. Schnack, J. Richter, T. Heitmann, J. Richter, R. Steinigeweg, Z. Naturforsch. A **75**, 465 (2020)

FTLM 4: kagome



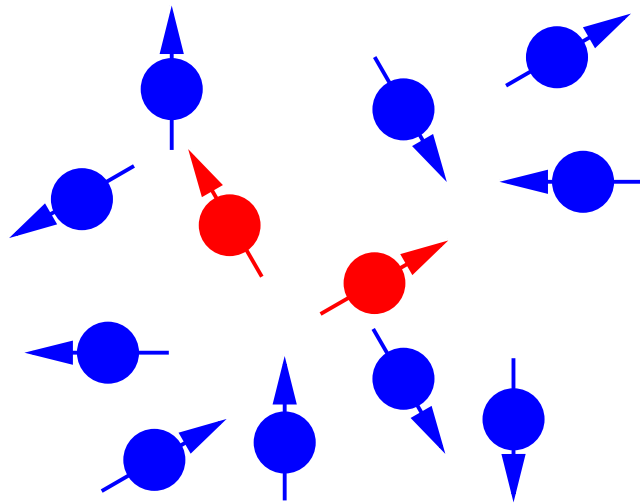
Specific heat of kagome with $N = 42$ – role of low-lying singlets, and magnon crystalization at high field.

(1) J. Schnack, J. Schulenburg, J. Richter, Phys. Rev. B **98**, 094423 (2018)

(2) J. Schnack, J. Schulenburg, A. Honecker, J. Richter, Phys. Rev. Lett. **125**, 117207 (2020)

Stability of clock transitions

Context



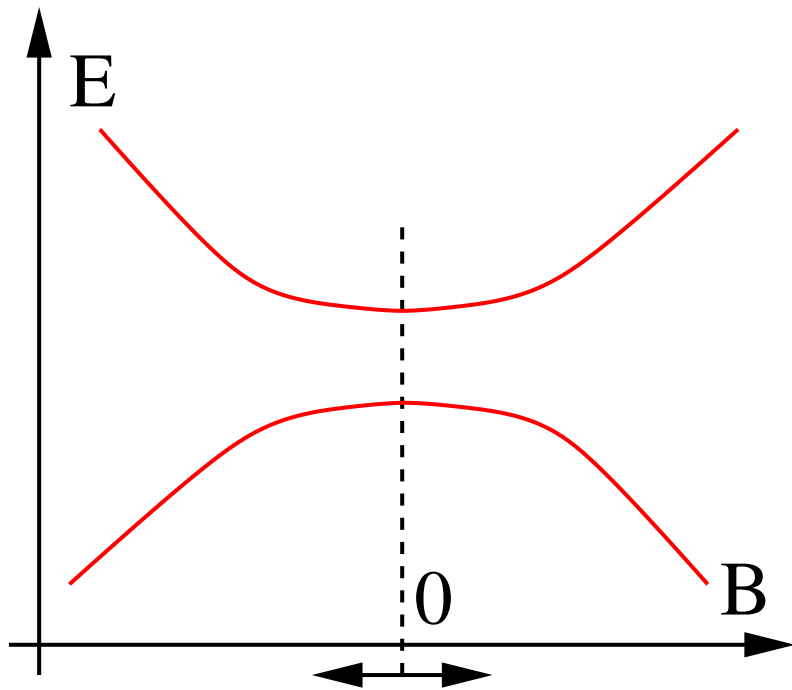
Investigation of **decoherence of a subsystem** if the combined system (including bath) is evolved via the time-dependent Schrödinger equation.

Employed measure of decoherence: reduced density matrix

$$\rho_{\text{system}} = \text{Tr}_{\text{bath}} \left(\rho \right)$$

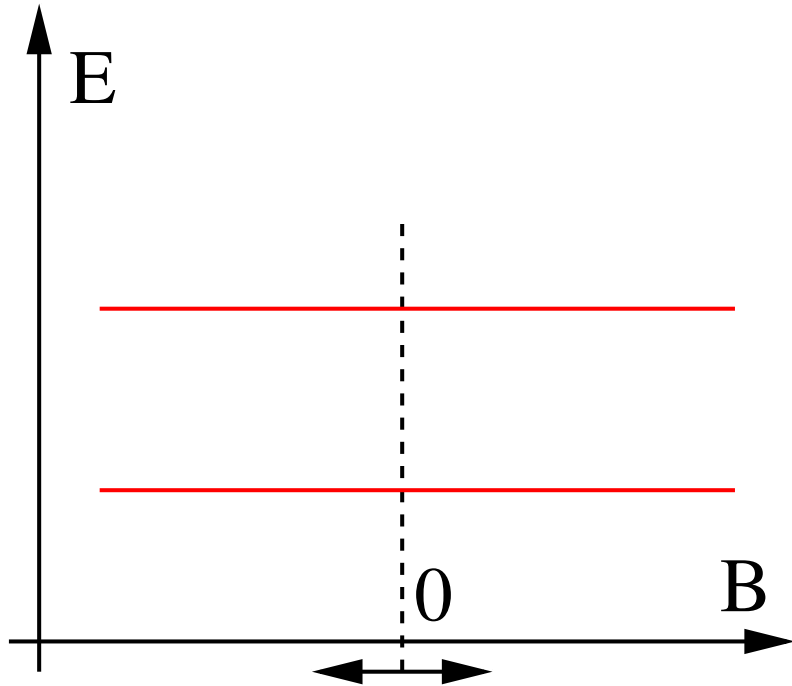
Typicality: unitary-time evolution of pure state approximates dynamics in environment.

Clock transitions

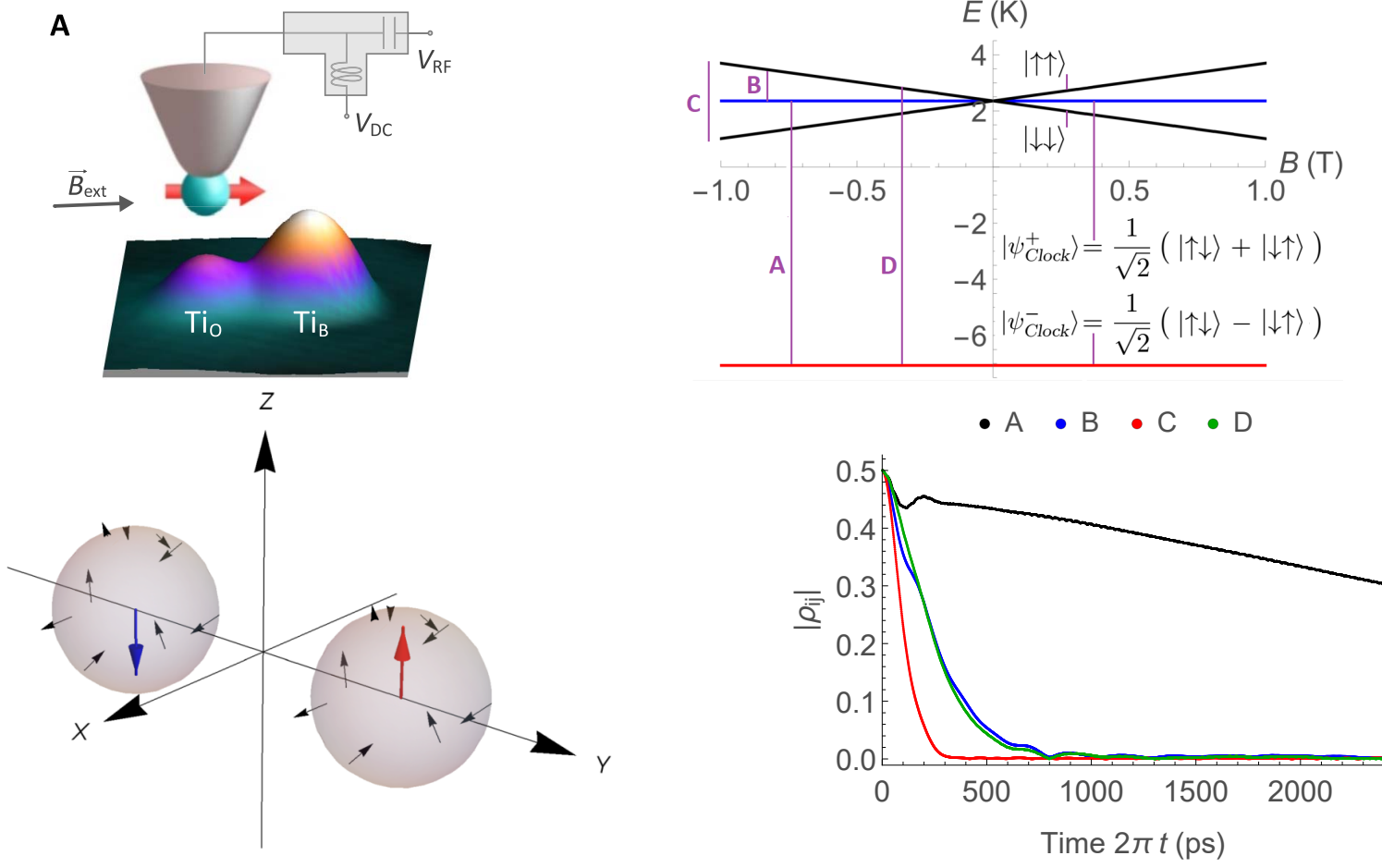


Fluctuations produce little effect on dynamics of superposition since ΔE of clock transition is independent of field at $B = 0$, at least to some order of a Taylor expansion.

Perfect clock transitions



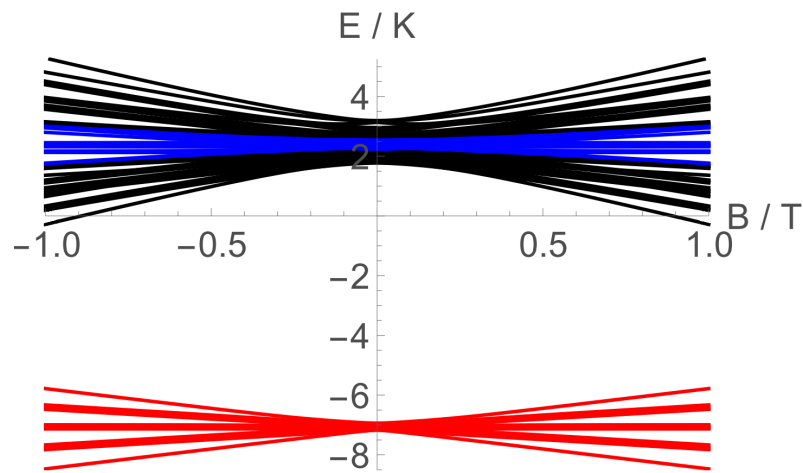
Fluctuations produce very small effect on superposition since ΔE of transition is *totally* independent of field.



P. Vorndamme, J. Schnack, Phys. Rev. B 101, 075101 (2020)

Y. Bae, K. Yang, P. Willke, T. Choi, A. J. Heinrich, and C. P. Lutz, Sci. Adv. 4, eaau4159 (2018)

Decoherence of clock transitions III



Single-particle/mean-field picture only valid for small couplings to a few bath spins.

Initial product state entangles in the course of time. Eigenstates of the full Hamiltonian lose clock property.

P. Vorndamme, J. Schnack, Phys. Rev. B 101, 075101 (2020)

Spin-phonon interaction

Model Hamiltonian (effective, spin-only, bilinear)

$$\underline{H} = \sum_{i,j} \underline{\vec{s}}(i) \cdot \mathbf{J}_{ij} \cdot \underline{\vec{s}}(j) + \mu_B \vec{B} \cdot \sum_i^N \mathbf{g}_i \cdot \underline{\vec{s}}(i)$$

Exchange/Anisotropy Zeeman

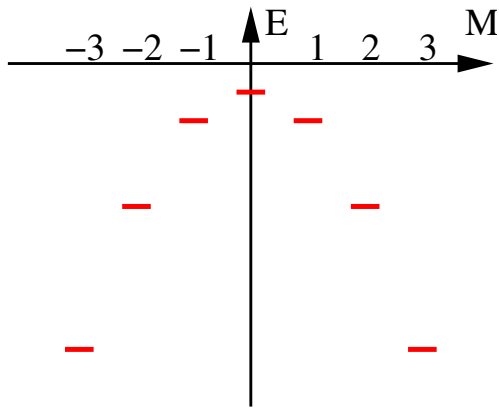
\mathbf{J}_{ij} : Heisenberg exchange, anisotropic exchange, and single-ion anisotropy.

Isotropic Heisenberg Hamiltonian

$$\underline{H} = -2 \sum_{i<j} J_{ij} \underline{\vec{s}}(i) \cdot \underline{\vec{s}}(j) + g \mu_B B \sum_i^N s_z(i)$$

Heisenberg Zeeman

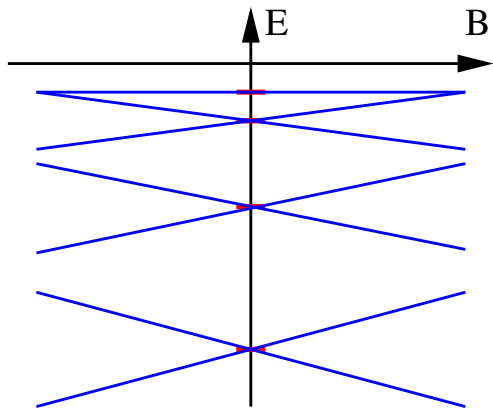
Single-ion anisotropy – single spin I



$$\tilde{H} = D(\tilde{s}^z)^2 + g\mu_B B \tilde{s}^z$$

$D < 0$ easy axis, $D > 0$ hard axis;

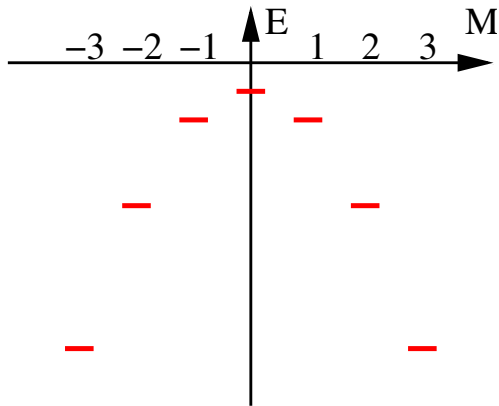
eigenvectors: $|s, m\rangle$



eigenvalues: $E_m = Dm^2 + g\mu_B Bm$, $m = -s, \dots, s$

IMPORTANT: $[\tilde{H}, \tilde{s}^z] = 0 \Rightarrow$ level crossings at $B = 0$

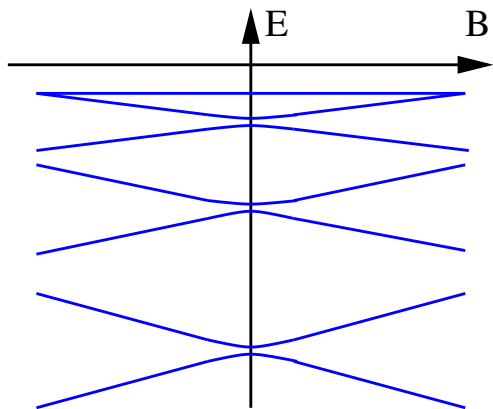
Single-ion anisotropy – single spin II



$$\tilde{H} = D(\tilde{s}^z)^2 + E \left\{ (\tilde{s}^x)^2 - (\tilde{s}^y)^2 \right\} + g\mu_B B \tilde{s}^z$$

$|E| < |D|$ – major axes of the anisotropy tensor;

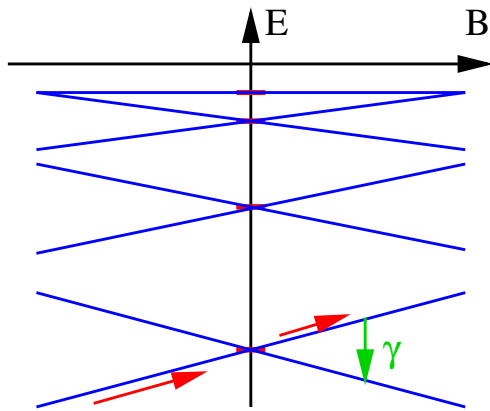
NO LONGER eigenvectors: $|s, m\rangle$



eigenvalues are more complicated functions of $\vec{B} = B\vec{e}_z$: $E_\mu(B)$

IMPORTANT: $[\tilde{H}, \tilde{s}^z] \neq 0 \Rightarrow$ avoided level crossings at $B = 0$ for integer spins (otherwise Kramers degeneracy)

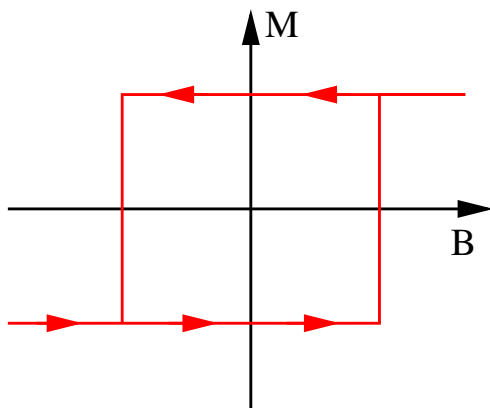
Bistability – uniaxial system – \tilde{S}^z -symmetry



Goal: single-molecule magnets (SMM)

$$\tilde{H} = \sum_i D_i (\tilde{S}_i^z)^2 + \mu_B B \sum_i g_i \tilde{S}_i^z + \tilde{H}_{\text{ferro int}}$$

IMPORTANT: $[\tilde{H}, \tilde{S}^z] = 0 \Rightarrow$ level crossings at $B = 0$



\Rightarrow low-temperature **TIME-DEPENDENT** hysteresis

Side remark: For macroscopic systems in the ferromagnetic phase the relaxation time is HUGE, that's why we don't experience it.

Bistability – general system – NO \tilde{S}^z -symmetry

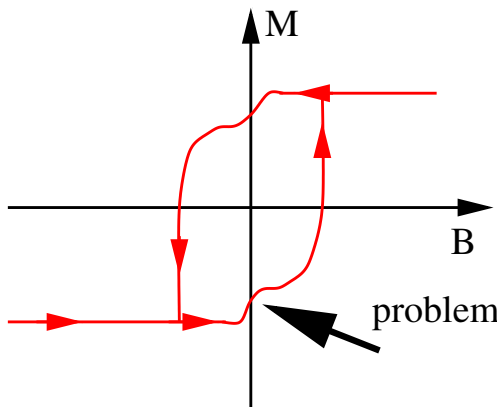
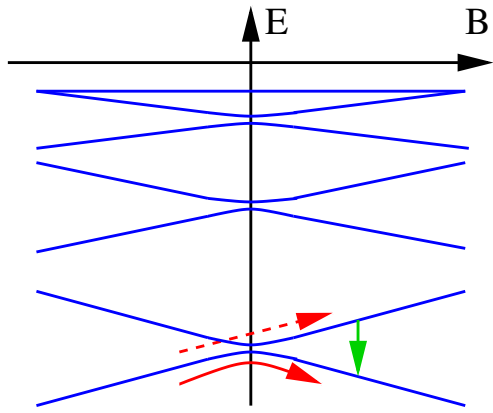
$$\underline{H} = \sum_i \vec{\tilde{S}}_i \cdot \mathbf{D}_i \cdot \vec{\tilde{S}}_i + \mu_B B \sum_i g_i \tilde{S}_i^z + \underline{H}_{\text{ferro int}}$$

\mathbf{D}_i individual anisotropy tensors

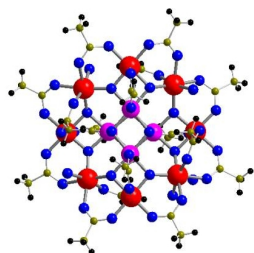
⇒ low-temperature TIME-DEPENDENT hysteresis closes at $B = 0$ – not bistable & bad for storage

REASON: branching at avoided level crossings; strong dependence on tunneling gap and \dot{B} ;

slow change of $B \Rightarrow$ system follows ground state, compare Landau-Zener-Stückelberg or slow/fast train at switch



Bistability – state of the art



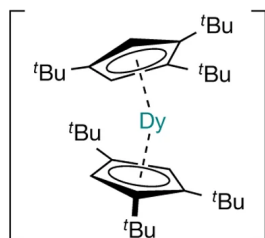
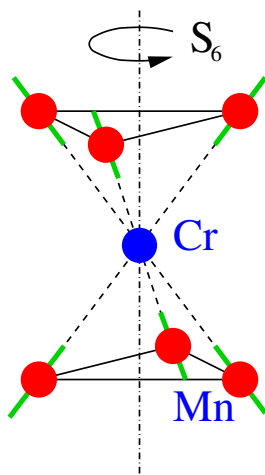
Today's major goals:

ferromagnetic spin-spin interaction

uniaxial anisotropy tensors

symmetry that does not permit E -terms

PERSISTENT PROBLEM: phonons



Nick Chilton, Thorsten Glaser, Jeff Long, Alessandro Lunghi, Mark Murrie, Frank Neese, Stefano Sanvito, Roberta Sessoli, Richard Winpenny, Yan-Zhen Zheng, ...

Spin-phonon interaction – DFT view of the problem

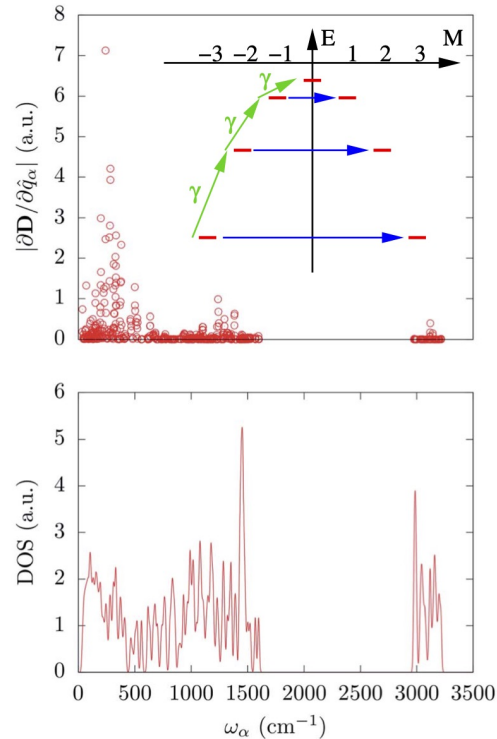


Fig. 2 Top panel: calculated spin-phonon coupling coefficients projected onto the normal modes basis set and displayed as a function of the modes frequency. Bottom panel: DFT calculated density of states for the Γ -point normal modes of vibration.

Calculate structure by means of DFT (1)

Calculate phonon density of states by means of DFT + molecular dynamics (1,2,3)

Calculate coupling coefficients from DFT (2,3,4)

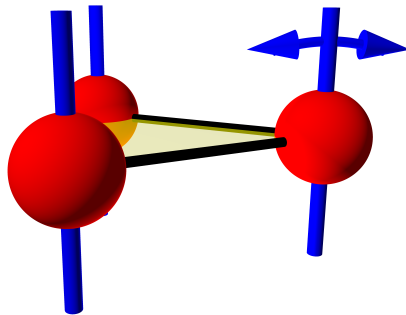
Perturbation picture: set up rate equations for phonon transitions between eigenstates of unperturbed spin Hamiltonian (2,3)

ADVANTAGE: many realistic phonons

- (1) A. V. Postnikov, J. Kortus, and M. R. Pederson, *physica status solidi (b)* **243**, 2533 (2006).
- (2) A. Lunghi and S. Sanvito, *Science Advances* **5**, eaax7163 (2019).
- (3) A. Albino, S. Benci, L. Tesi, M. Atzori, R. Torre, S. Sanvito, R. Sessoli, and A. Lunghi, *Inorg. Chem.* **58**, 10260 (2019);
 \Rightarrow figure.
- (4) D.A.S. Kaib, S. Biswas, K. Riedl, S.M. Winter, R. Valentí, *Phys. Rev. B* **103**, L140402 (2021).

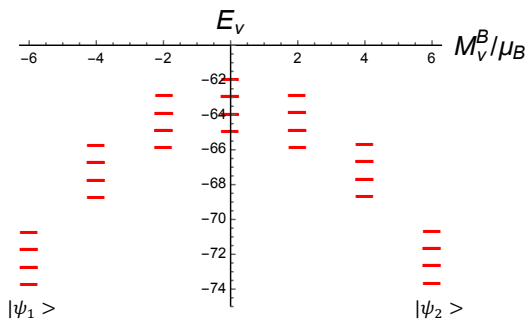
Spin-phonon interaction – our question

Can phonons induce a tunnel splitting?



Know that non-collinear easy axes produce tunnel splitting

Set up special phonon modes that tilt easy axes in plane with C_3 axis out of uniaxial alignment



ADVANTAGE: quantum many-body solution for spins and phonons

⇒ correlated spin-phonon states:

$$\Psi_\nu = \sum c_{m_1, m_2, m_3, n_1, n_2, n_3}^\nu |m_1, m_2, m_3, n_1, n_2, n_3\rangle$$

(1) K. Irländer and J. Schnack, Phys. Rev. B **102**, 054407 (2020).

Spin-phonon interaction – Hamiltonian

$$\begin{aligned}
 \tilde{H} = & -2J \left(\vec{s}_{\tilde{1}} \cdot \vec{s}_{\tilde{2}} + \vec{s}_{\tilde{2}} \cdot \vec{s}_{\tilde{3}} + \vec{s}_{\tilde{3}} \cdot \vec{s}_{\tilde{1}} \right) \\
 & + \vec{s}_{\tilde{1}} \cdot \mathbf{D}_1(\theta_{\tilde{1}}) \cdot \vec{s}_{\tilde{1}} + \vec{s}_{\tilde{2}} \cdot \mathbf{D}_2(\theta_{\tilde{2}}) \cdot \vec{s}_{\tilde{2}} + \vec{s}_{\tilde{3}} \cdot \mathbf{D}_3(\theta_{\tilde{3}}) \cdot \vec{s}_{\tilde{3}} \\
 & + \omega_1 \left(a_{\tilde{1}}^\dagger a_{\tilde{1}} + \frac{1}{2} \right) + \omega_2 \left(a_{\tilde{2}}^\dagger a_{\tilde{2}} + \frac{1}{2} \right) + \omega_3 \left(a_{\tilde{3}}^\dagger a_{\tilde{3}} + \frac{1}{2} \right) \\
 & + g\mu_B \cdot \vec{B} \cdot \left(\vec{s}_{\tilde{1}} + \vec{s}_{\tilde{2}} + \vec{s}_{\tilde{3}} \right)
 \end{aligned}$$

$$\mathbf{D}_i(\theta_i) = D \vec{e}_i(\theta_i, \phi_i) \otimes \vec{e}_i(\theta_i, \phi_i)$$

$$\theta_i = \theta_{i,0} + \alpha \left(a_{\tilde{i}}^\dagger + a_{\tilde{i}} \right), \quad \theta_{i,0} = 0, \quad \text{i.e., zero mean tilt}$$

Spin-phonon interaction – our result (applies to integer spins)

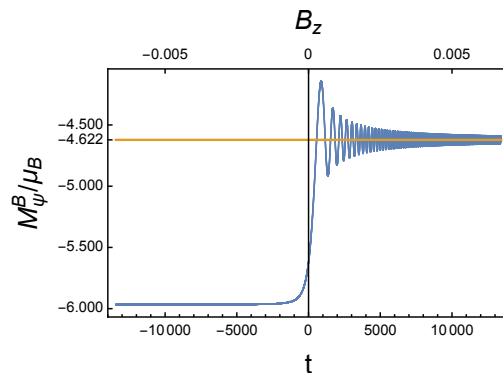
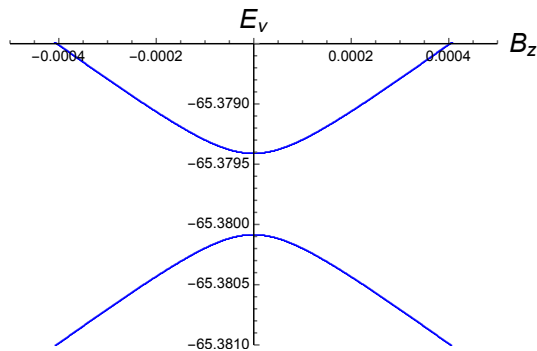
Can phonons induce a tunnel splitting?

⇒ Yes, they can!

Ground state, practically, does not contain any phonons, nevertheless tunneling occurs. Coupling to zero-point motion suffices (2).

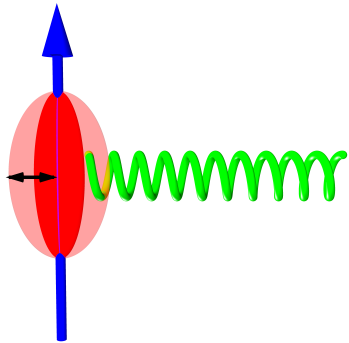
BAD NEWS: It is not enough to cool quantum devices, you have to prevent the coupling to disturbing sources at all.

Side remark: result probably already known in field of vibronic coupling (Atanasov, Shrivastava, Tsukerblat, Coronado).



- (1) K. Irländer and J. Schnack, Phys. Rev. B **102**, 054407 (2020).
 (2) F. Ortu *et al.*, Dalton Trans. **48**, 8541 (2019).

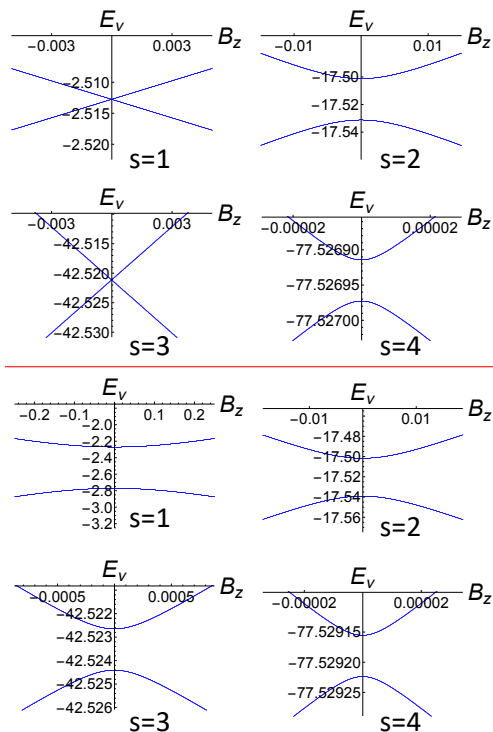
SUSY spin-phonon interaction (applies to integer spins)



$$\underline{H} = D(\underline{s}^z)^2 + E \left\{ (\underline{s}^x)^2 - (\underline{s}^y)^2 \right\} + g\mu_B B \underline{s}^z + \underline{H}_{\text{HO}}$$

Special phonons that modify only:

$$L: E = \alpha \left(\underline{a}^\dagger + \underline{a} \right) \quad \text{or} \quad Q: E = \alpha \left(\underline{a}^\dagger + \underline{a} \right)^2$$



L: tunneling gap for even s , no gap for odd s .

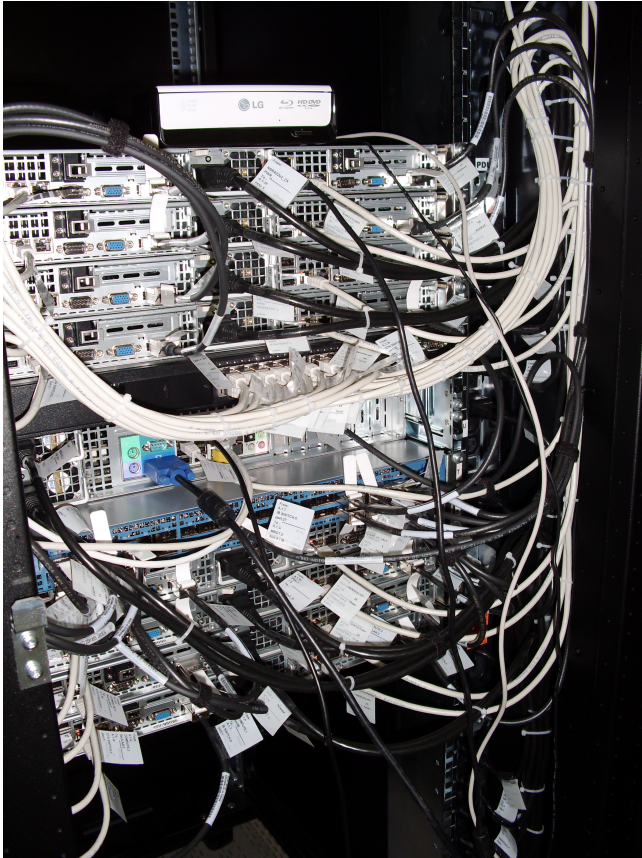
This is not Kramers, but related to another symmetry.

Q: tunneling gap for all s .

RESULT: very interesting behavior; there are some phonons that do not produce a tunneling gap thanks to the way they couple. SUSY at work.

(1) K. Irländer, H.-J. Schmidt, J. Schnack, Eur. Phys. J. B **94**, 68 (2021)

Summary



- Magnetic molecules for storage, q-bits, MCE, and since they are nice.
- SMM challenges: quantum tunneling and phonons
- Magnetism is much richer and more complicated than shown here. Talk focused on 3d ions with weak spin-orbit interaction.
- Typicality is a powerful approach.
- ED, HTE, CMC, QMC, FTLM, DMRG, DDMRG, thDMRG, DFT for magnetic molecules.

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Thank you very much for your
attention.

The end.

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