

# Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions

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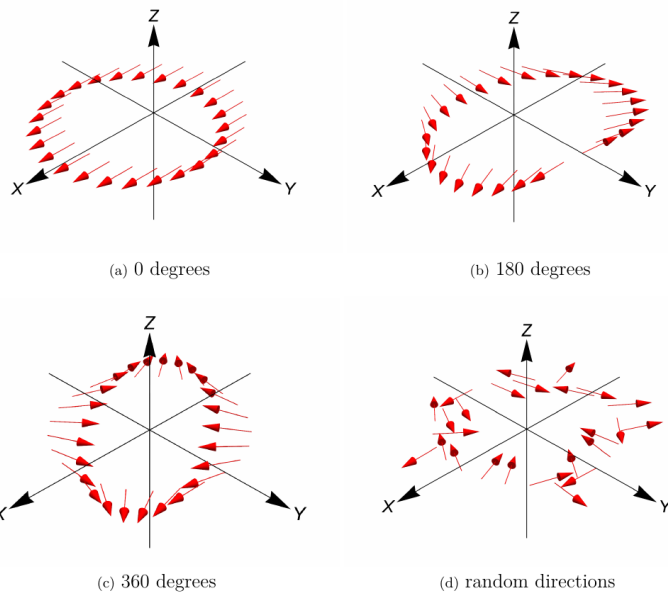
# Context

Investigation of relaxation, equilibration, and possibly thermalization during unitary time-evolution, i.e. of a closed system.

# Movie 1

# Synchronization I – Setting

- System of  $N$  spins (mostly  $s = 1/2$ );
- Unitary time evolution with Hamiltonian  $\tilde{H}$ ;
- Zeeman term included, field along  $z$ -direction;
- Initial state, e.g. product state, with single spin expectation values in  $x$ - $y$ -plane;
- Let go!
- What do you expect?



P. Vorndamme, H.-J. Schmidt, Chr. Schröder, J. Schnack, *Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions*, New J. Phys. **23**, 083038 (2021)

# Synchronization II – Heisenberg case

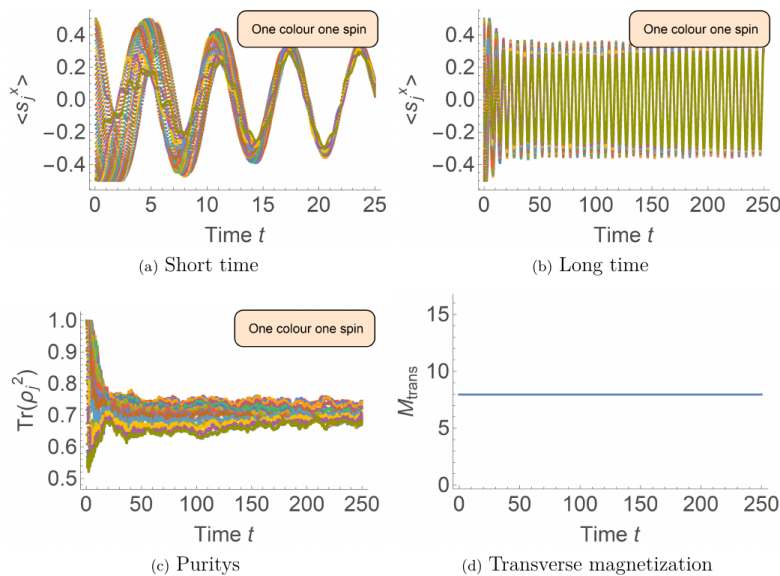
- $$\underline{H} = - \sum_{j=1}^N J_j \underline{\tilde{s}}_j \cdot \underline{\tilde{s}}_{j+1} - \sum_{j=1}^N h_j s_j^z \quad (1);$$

- $\forall j : h_j = h$  : total spin and transverse magnetization conserved;

$$M_{\text{trans}} := \sqrt{\langle \underline{S}^x \rangle^2 + \langle \underline{S}^y \rangle^2};$$

- Not entangled: purity  $\text{Tr}(\rho_j^2) = 1$ ,  
maximally entangled: purity  $\text{Tr}(\rho_j^2) = 0.5$ ;

- Let go with random  $J_j$ !
- What do you expect?



Time evolution of initial state  $|\psi_B\rangle$  w.r.t. Hamiltonian (1) with isotropic Heisenberg interactions and  $J_j \in [1.6, 2.4]$ ,  $h_j = -1 \forall j$ ,  $N = 25$ .

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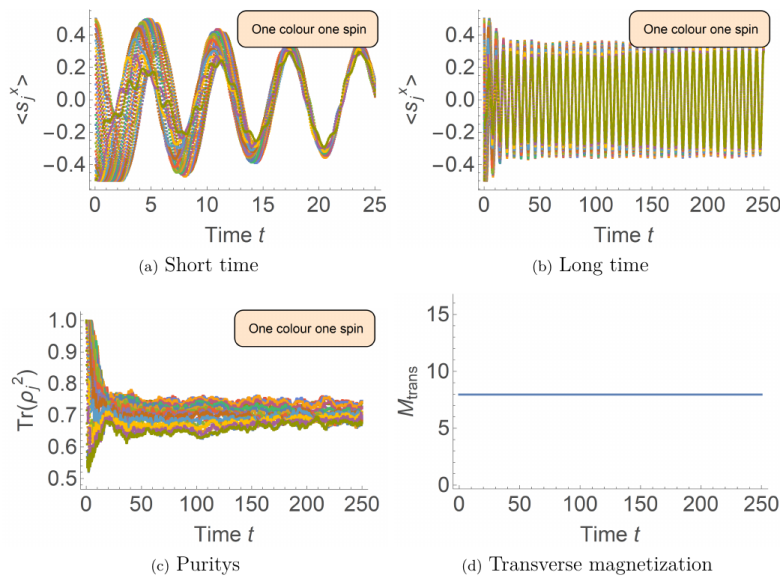
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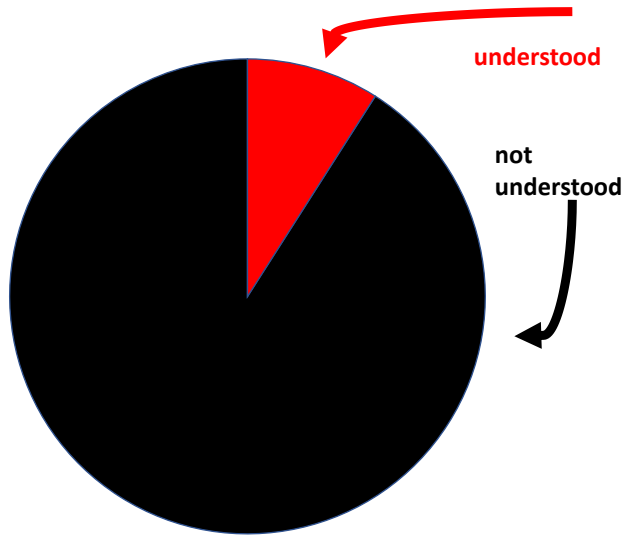
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- What do you expect?  
Want to see the movie again?



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# Synchronization III – our understanding



- We understand the case where all  $J_j = J$  and all  $h_j = h$ , i.e. all spins equivalent!
- Total spin and transverse magnetization are conserved.
- $\Rightarrow$  If one assumes local equilibration to a state compatible with the conserved quantities, then all spins need to have the same vector expectation value  $\langle \vec{S} \rangle / N$ .
- BUT: Synchronization is observed for the vast majority of all initial states and Heisenberg Hamiltonians that we investigated so far.

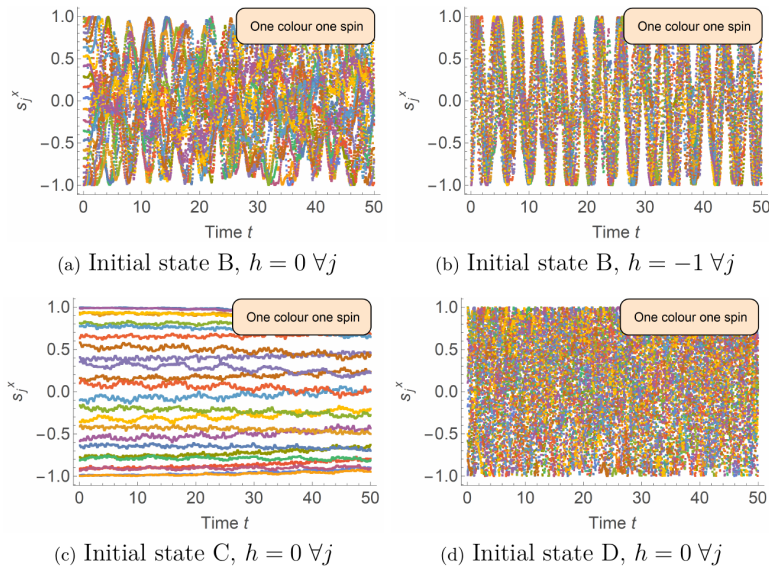
# What about similar systems of classical spins?



# Movie 2

(Christian Schröder)

# Synchronization IV – classical Heisenberg case



Time evolution of initial states A, dots, D w.r.t. classical Hamiltonian (1) with isotropic Heisenberg interactions and  $J_j \in [1.6, 2.4]$ ,  $h_j = -1 \forall j$ ,  $N = 24$ .

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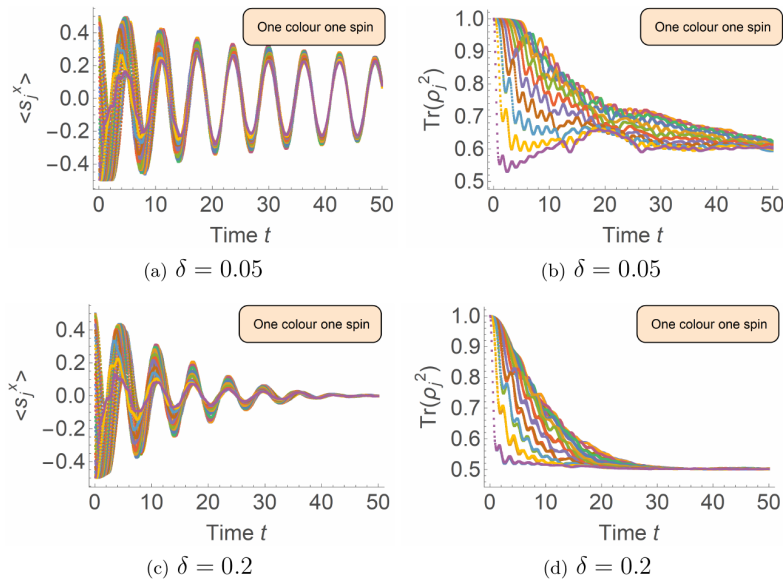
- $H = - \sum_{j=1}^N J_j \vec{s}_j \cdot \vec{s}_{j+1} - \sum_{j=1}^N h_j s_j^z \quad (1);$
- **Classical spins do not synchronize in a closed system. Never!**
- Classical spins have  $N$  additional conserved quantities, the length of the classical spins. The synchronized state is not on the energy shell.
- **Classical spins cannot entangle.**

What about other systems  
in the zoo of spin Hamiltonians?

# Movie 3

(Guess what happens to the purity!)

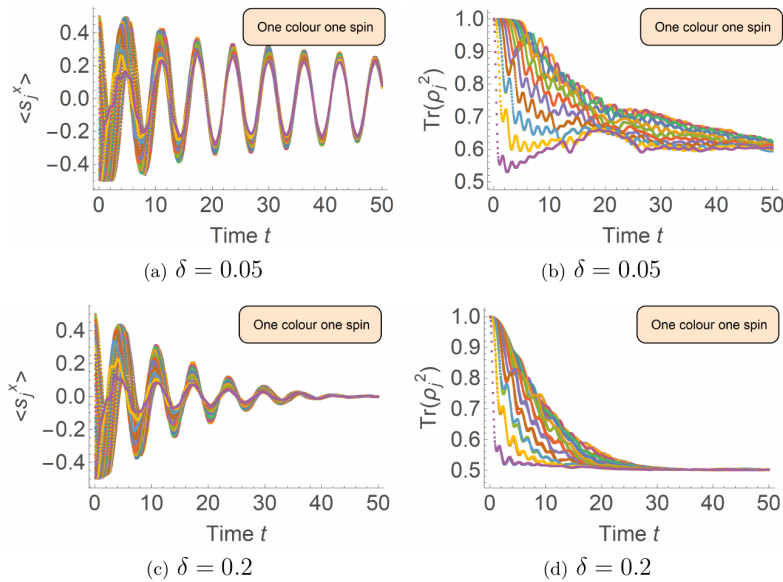
# Synchronization V – loss of symmetries



Time evolution of initial state  $|\psi_B\rangle$  w.r.t. Hamiltonian (2) with for two values of  $\delta$ , and  $N = 24$ ,  $J = 2$ ,  $h = -1$ .

- $$\begin{aligned}
 \tilde{H}_{XYZ} = & -J \sum_{j=1}^N \tilde{s}_j^x \tilde{s}_{j+1}^x \\
 & - (J - \delta) \sum_{j=1}^N \tilde{s}_j^y \tilde{s}_{j+1}^y \\
 & - (J - 2\delta) \sum_{j=1}^N \tilde{s}_j^z \tilde{s}_{j+1}^z - h \sum_{j=1}^N \tilde{s}_j^z \quad (2);
 \end{aligned}$$
- Hamiltonians with less symmetries down to none;
- What do you expect?

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- Hamiltonians with less symmetries down to none;
- What do you expect?  
Transient synchronization and decay to zero!

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# Summary

Heisenberg systems ( $SU(2)$  symmetry) appear to exhibit robust synchronization of single-spin expectation values under unitary time evolution.

Classical Heisenberg as well as quantum spin systems without  $SU(2)$  symmetry do not synchronize or in a transient way at most.

# Thank you very much for your attention.



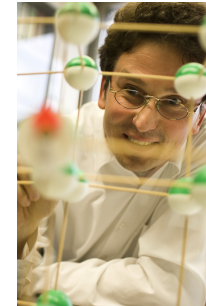
Patrick Vorndamme



Christian Schröder



Heinz-Jürgen Schmidt



Jürgen Schnack

The end.



# Magnon crystallization in the kagome lattice antiferromagnet



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