Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions

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Fun Seminar

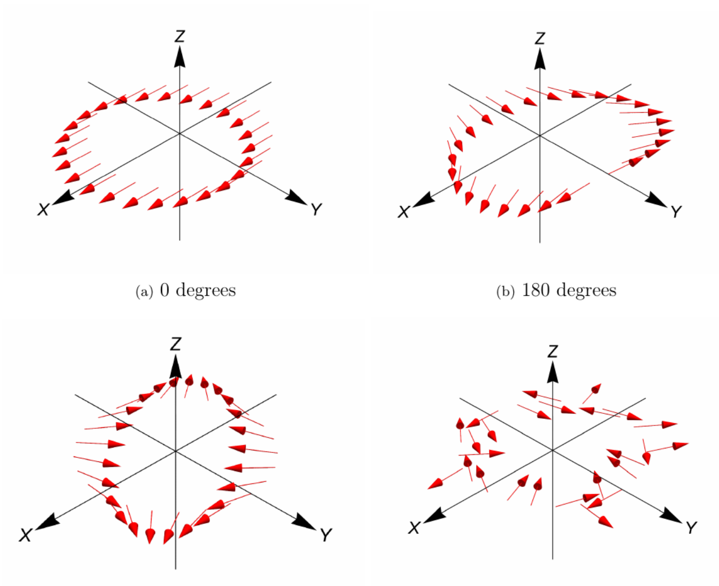
University of Oxford, Clarendon Laboratory, 13 May 2022

UNIVERSITÄT BIELEFELD
Fakultät für Physik

Movie 1

(Don’t relax! You will be asked what you saw!)

Synchronization I – Setting



(a) 0 degrees (b) 180 degrees

(c) 360 degrees (d) random directions

- System of N spins (mostly $s = 1/2$);
- Unitary time evolution with Hamitonian $\underset{\sim}{H}$;
- Zeeman term included, field along z–direction;
- Initial state, e.g. product state, with single spin expectation values in x–y–plane;
- Let go!
- What do you expect?

P. Vorndamme, H.-J. Schmidt, Chr. Schröder, J. Schnack, *Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions,* New J. Phys. **23**, 083038 (2021)

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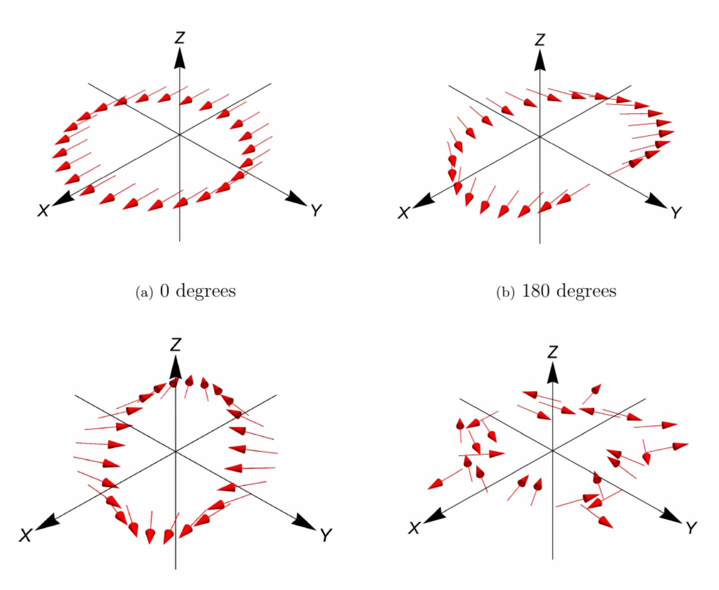

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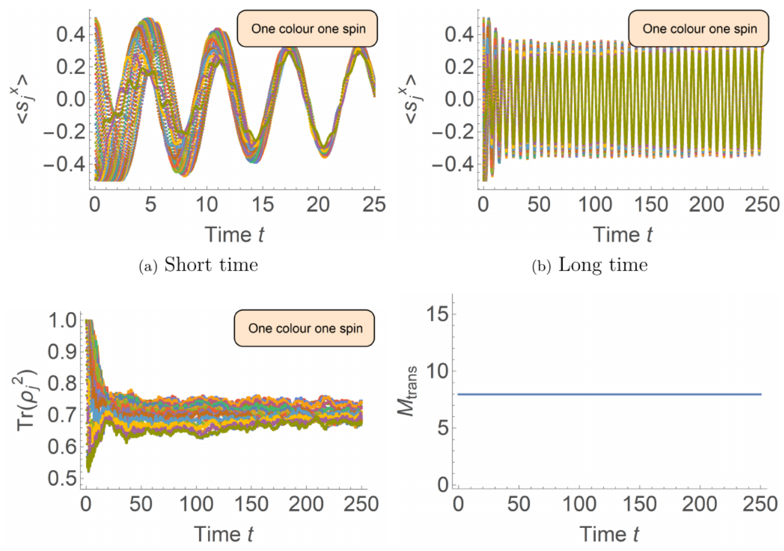
Synchronization II – Heisenberg case



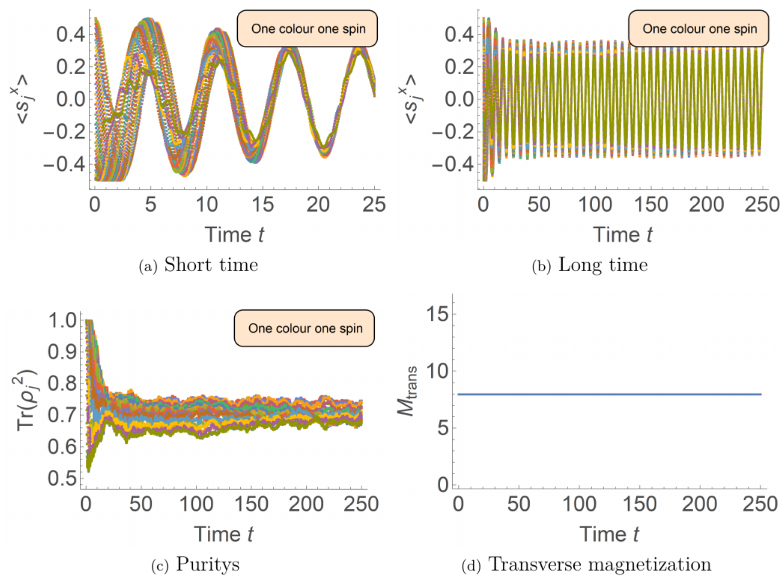
(a) Short time (b) Long time

(c) Puritys (d) Transverse magnetization

Time evolution of initial state $\mid \psi_B \,\rangle$ w.r.t. Hamiltonian (1) with isotropic Heisenberg interactions and $J_j \in [1.6, 2.4]$, $h_j = -1 \;\forall j$, $N = 25$.

- $\underset{\sim}{H} = -\sum_{j=1}^{N} J_j \underset{\sim}{\vec{s}}_j \cdot \underset{\sim}{\vec{s}}_{j+1} - \sum_{j=1}^{N} h_j \underset{\sim}{s}_j^z$ (1);
- $\forall j : h_j = h$: total spin and transverse magnetization conserved;
  $M_{\text{trans}} := \sqrt{\langle\, \underset{\sim}{S}^x \,\rangle^2 + \langle\, \underset{\sim}{S}^y \,\rangle^2}$;
- Not entangled: purity $Tr\left(\underset{\sim}{\rho}_j^2\right) = 1$,
  maximally entangled: purity $Tr\left(\underset{\sim}{\rho}_j^2\right) = 0.5$;
- Let go with random J_j!
- What do you expect?

Synchronization II – Heisenberg case



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- What do you expect?
  Want to see the movie again?

Synchronization III – our understanding

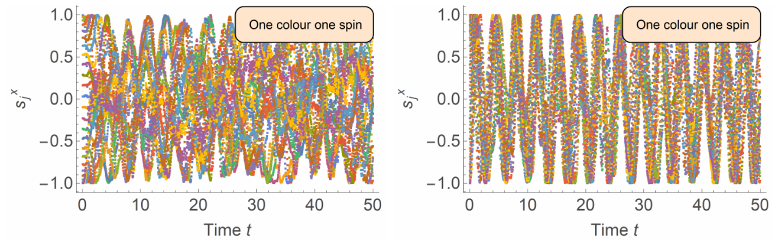
- We understand the case where all $J_j = J$ and all $h_j = h$.
- Total spin and transverse magnetization conserved;
- AND: all spins equivalent!
- If one assumes local equilibration to a state compatible with the conserved quantities, then all spins need to have the same expectation value.
- Synchronization is observed for the vast majority of all initial states and Heisenberg Hamiltonians that we investigated so far.

What about similar systems of classical spins?

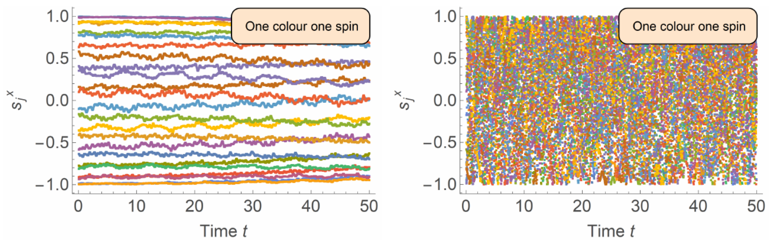
Movie 2

(Schröder Bros. Inc. – Bielefeld, Melle, Hollywood)

Synchronization IV – classical Heisenberg case



(a) Initial state B, $h = 0\ \forall j$ (b) Initial state B, $h = -1\ \forall j$



(c) Initial state C, $h = 0\ \forall j$ (d) Initial state D, $h = 0\ \forall j$

Time evolution of initial states A, dots, D w.r.t. classical Hamiltonian (1) with isotropic Heisenberg interactions and $J_j \in [1.6, 2.4]$, $h_j = -1\ \forall j$, $N = 24$.

- $H = -\sum_{j=1}^{N} J_j \vec{s}_j \cdot \vec{s}_{j+1} - \sum_{j=1}^{N} h_j s_j^z$ (1);
- Classical spins do not synchronize in a closed system. Never!
- Classical spins have N additional conserved quantities, the length of the classical spins.
- Classical spins cannot entangle.

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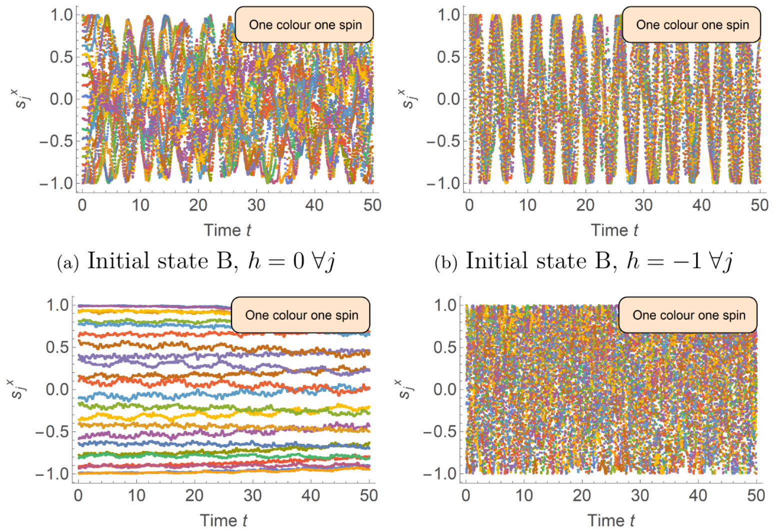
Synchronization IV – classical Heisenberg case



(a) Initial state B, $h = 0 \; \forall j$
(b) Initial state B, $h = -1 \; \forall j$
(c) Initial state C, $h = 0 \; \forall j$
(d) Initial state D, $h = 0 \; \forall j$

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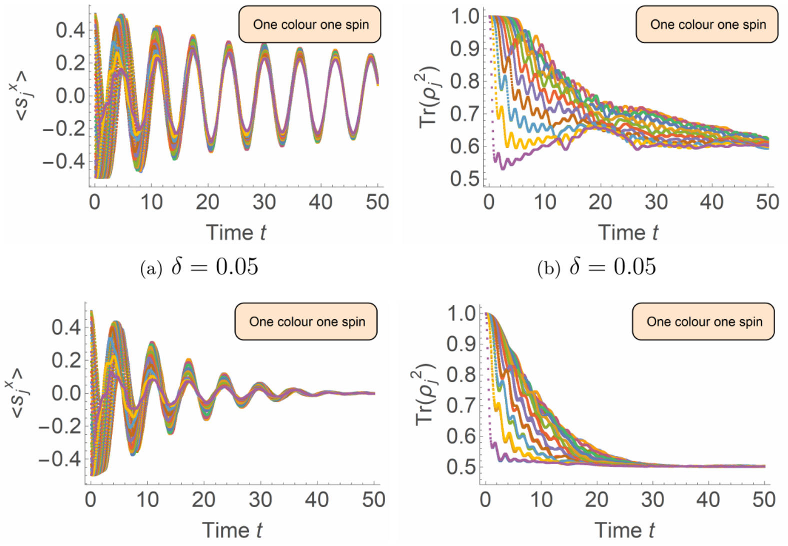
What about other systems

in the zoo of spin Hamiltonians?

Movie 3

(Guess what happens to the purity!)

Synchronization V – loss of symmetries



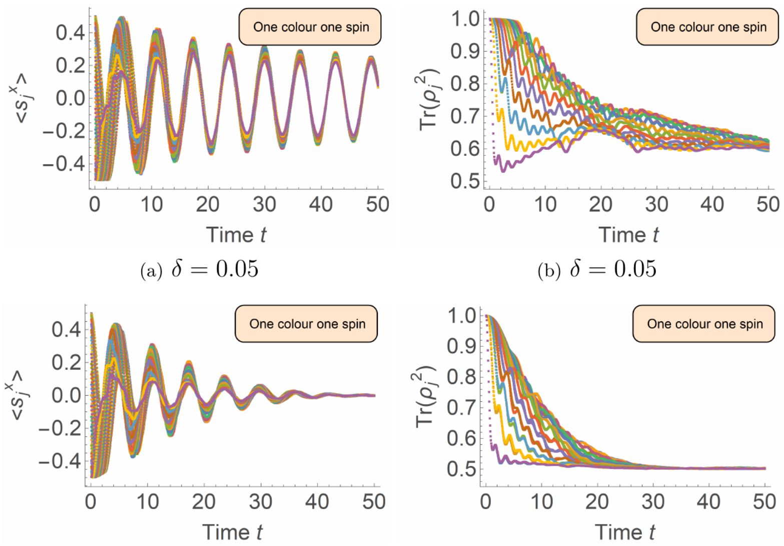
(a) $\delta = 0.05$ (b) $\delta = 0.05$

(c) $\delta = 0.2$ (d) $\delta = 0.2$

Time evolution of initial state $\mid \psi_B \rangle$ w.r.t. Hamiltonian (2) with for two values of δ, and $N = 24$, $J = 2$, $h = -1$.

- $\underset{\sim}{H}_{XYZ} = -J \sum_{j=1}^{N} \underset{\sim}{s}_j^x \underset{\sim}{s}_{j+1}^x - (J-\delta) \sum_{j=1}^{N} \underset{\sim}{s}_j^y \underset{\sim}{s}_{j+1}^y - (J-2\delta) \sum_{j=1}^{N} \underset{\sim}{s}_j^z \underset{\sim}{s}_{j+1}^z - h \sum_{j=1}^{N} \underset{\sim}{s}_j^z$ (2);
- Hamiltonians with less symmetries down to none;
- What do you expect?

Synchronization V – loss of symmetries



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- Hamiltonians with less symmetries down to none;

- What do you expect?
  Transient synchronization and decay to zero!
  Want to see the movie again?

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Thank you very much for your attention.


Patrick Vorndamme


Christian Schröder


Heinz-Jürgen Schmidt


Jürgen Schnack

The end.

Magnon crystallization in the kagome lattice antiferromagnet

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