Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions

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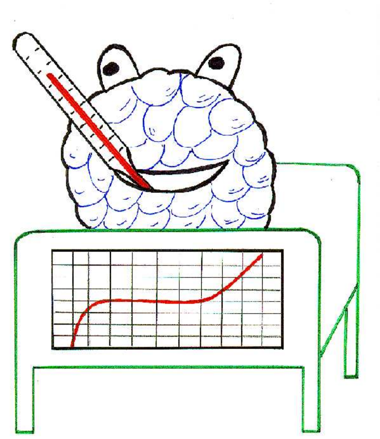
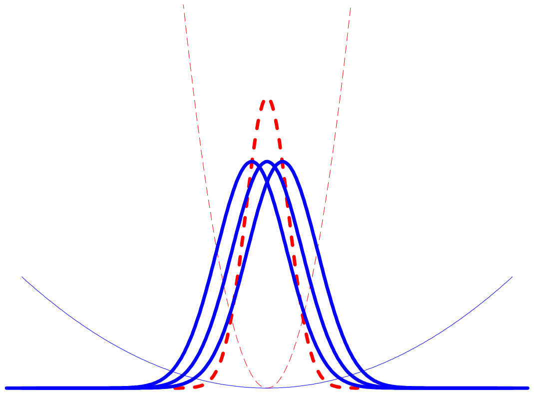
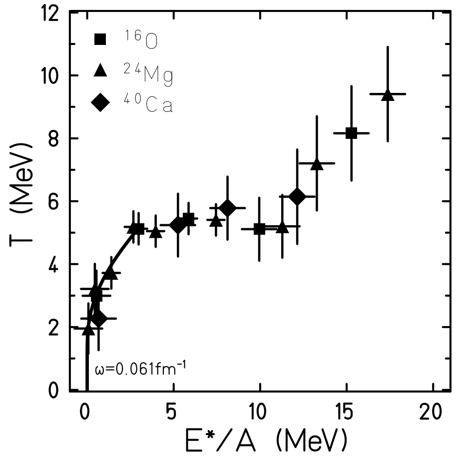
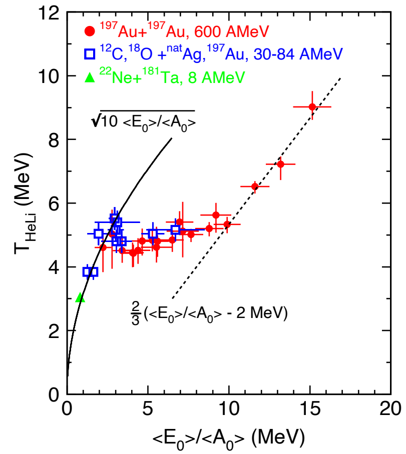
Fakultät für Physik

Something we took for granted 30 years ago ...









$$\delta \int_{t_1}^{t_2} \mathrm{d}t \, \langle \, Q(t) \, | \, i\frac{d}{dt} - \underset{\sim}{H} \, | \, Q(t) \, \rangle = 0 \; , \quad | \, Q(t) \, \rangle = \; | \, system \, \rangle \otimes \; | \, thermometer \, \rangle$$

TDVP[1], symplectic dynamics, caloric curve, nuclear liquid-gas phase transition [2], more on equilibration [3]

[1] H. Feldmeier and J. Schnack, Rev. Mod. Phys. **72**, 655 (2000).

[2] J. Schnack and H. Feldmeier, Phys. Lett. B **409**, 6 (1997).

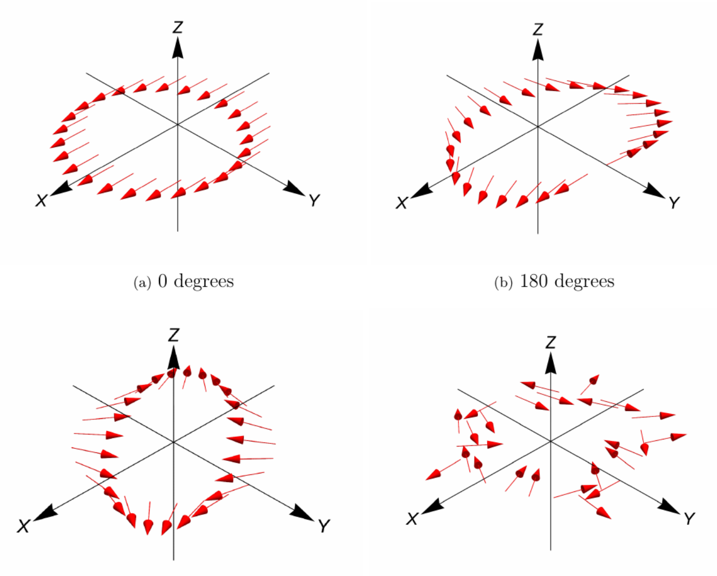
[3] J. Schnack and H. Feldmeier, Nucl. Phys. A **601**, 181 (1996).

Context

Investigation of relaxation, equilibration,
and possibly thermalization
during unitary time-evolution,
i.e. of a closed system.

Movie 1

Synchronization I – Setting



(a) 0 degrees (b) 180 degrees (c) 360 degrees (d) random directions

- System of N spins (mostly $s = 1/2$);
- Unitary time evolution with Hamitonian $\underset{\sim}{H}$;
- Zeeman term included, field along z–direction;
- Initial state, e.g. product state, with single spin expectation values in x–y–plane;
- What do you expect?

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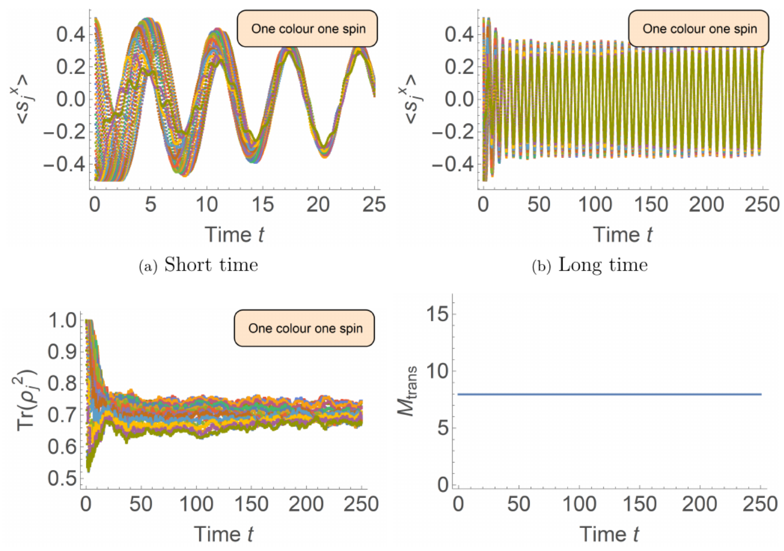
Synchronization II – Heisenberg case

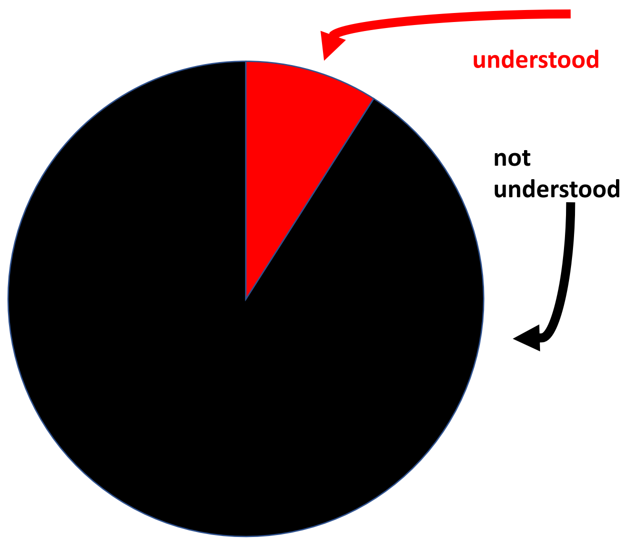


(a) Short time (b) Long time (c) Puritys (d) Transverse magnetization

Time evolution of initial state $\mid \psi_B \,\rangle$ w.r.t. Hamiltonian (1) with isotropic Heisenberg interactions and $J_j \in [1.6, 2.4]$, $h_j = -1 \;\forall j$, $N = 25$.

- $\underset{\sim}{H} = -\sum_{j=1}^{N} J_j \,\underset{\sim}{\vec{s}}_j \cdot \underset{\sim}{\vec{s}}_{j+1} - \sum_{j=1}^{N} h_j \,\underset{\sim}{s}_j^z$ (1);
- $\forall j : h_j = h$: total spin and transverse magnetization conserved; $M_{\text{trans}} := \sqrt{\langle\, \underset{\sim}{S}^x \,\rangle^2 + \langle\, \underset{\sim}{S}^y \,\rangle^2}$;
- Not entangled: purity $Tr\left(\underset{\sim}{\rho}_j^2\right) = 1$, maximally entangled: purity $Tr\left(\underset{\sim}{\rho}_j^2\right) = 0.5$;
- Let go with random J_j!
- What do you expect?

Synchronization III – our understanding



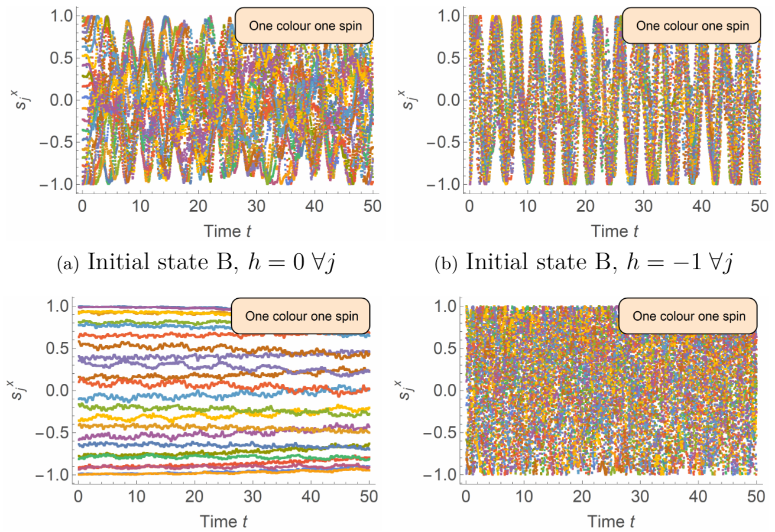
- We understand the case where all $J_j = J$ and all $h_j = h$, i.e. all spins equivalent!
- Total spin and transverse magnetization are conserved.
- $\Rightarrow$ If one assumes local equilibration to a state compatible with the conserved quantities, then all spins need to have the same vector expectation value $\langle \underset{\sim}{\vec{S}} \rangle /N$. Analytical proof by Peter.
- BUT: Synchronization is observed for the vast majority of all initial states and Heisenberg Hamiltonians that we investigated so far.

What about similar systems of classical spins?

Movie 2

(Christian Schröder)

Synchronization IV – classical Heisenberg case



(a) Initial state B, $h = 0\ \forall j$ (b) Initial state B, $h = -1\ \forall j$

(c) Initial state C, $h = 0\ \forall j$ (d) Initial state D, $h = 0\ \forall j$

Time evolution of initial states A, dots, D w.r.t. classical Hamiltonian (1) with isotropic Heisenberg interactions and $J_j \in [1.6, 2.4]$, $h_j = -1\ \forall j$, $N = 24$.

- $H = -\sum_{j=1}^{N} J_j \vec{s}_j \cdot \vec{s}_{j+1} - \sum_{j=1}^{N} h_j s_j^z$ (1);
- Classical spins do not synchronize in a closed system. Never!
- Classical spins have N additional conserved quantities, the length of the classical spins. The synchronized state is not on the energy shell.
- Classical spins cannot entangle.

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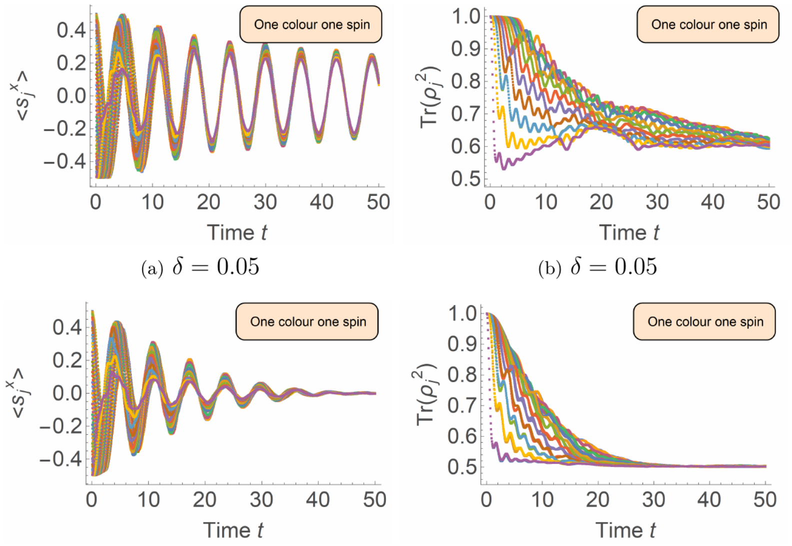
What about other systems

in the zoo of spin Hamiltonians?

Movie 3

(Guess what happens to the purity!)

Synchronization V – loss of symmetries

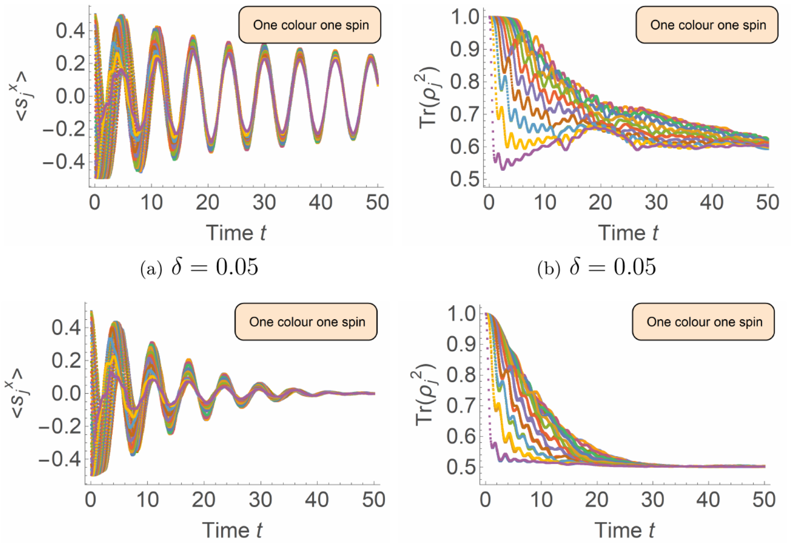


(a) $\delta = 0.05$ (b) $\delta = 0.05$
(c) $\delta = 0.2$ (d) $\delta = 0.2$

Time evolution of initial state $\mid \psi_B \rangle$ w.r.t. Hamiltonian (2) with for two values of δ, and $N = 24$, $J = 2$, $h = -1$.

- $\underset{\sim}{H}_{XYZ} = -J \sum_{j=1}^{N} \underset{\sim}{s}_j^x \underset{\sim}{s}_{j+1}^x - (J-\delta) \sum_{j=1}^{N} \underset{\sim}{s}_j^y \underset{\sim}{s}_{j+1}^y - (J-2\delta) \sum_{j=1}^{N} \underset{\sim}{s}_j^z \underset{\sim}{s}_{j+1}^z - h \sum_{j=1}^{N} \underset{\sim}{s}_j^z$ (2);
- Hamiltonians with less symmetries down to none;
- What do you expect?

Synchronization V – loss of symmetries



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- Hamiltonians with less symmetries down to none;
- What do you expect?
  Transient synchronization and decay to zero!

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Summary

Heisenberg systems (SU(2) symmetry) appear to exhibit robust synchronization of single-spin expectation values under unitary time evolution.

Classical Heisenberg as well as quantum spin systems without SU(2) symmetry do not synchronize or in a transient way at most.

Thank you very much for your attention.


Patrick Vorndamme


Christian Schröder


Heinz-Jürgen Schmidt


Jürgen Schnack

The end.

Observation of phase synchronization and alignment . . .

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