

# Large-scale numerical investigations of the antiferromagnetic Heisenberg icosidodecahedron

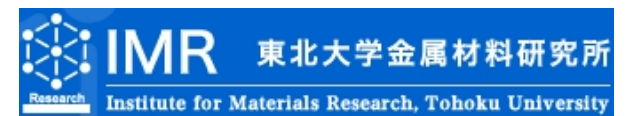
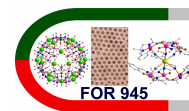
Jörg Ummethum, Andreas Läuchli, Jürgen Schnack

Department of Physics – University of Bielefeld – Germany

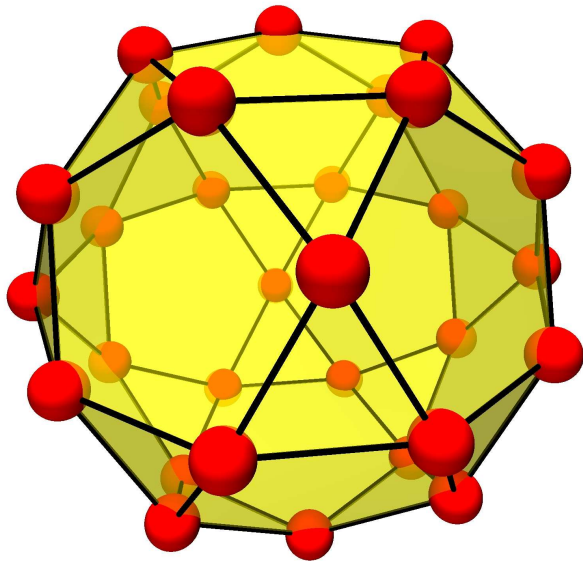
<http://obelix.physik.uni-bielefeld.de/~schnack/>

MA 40.5

DPG-Frühjahrstagung 2013, Regensburg



# The icosidodecahedron

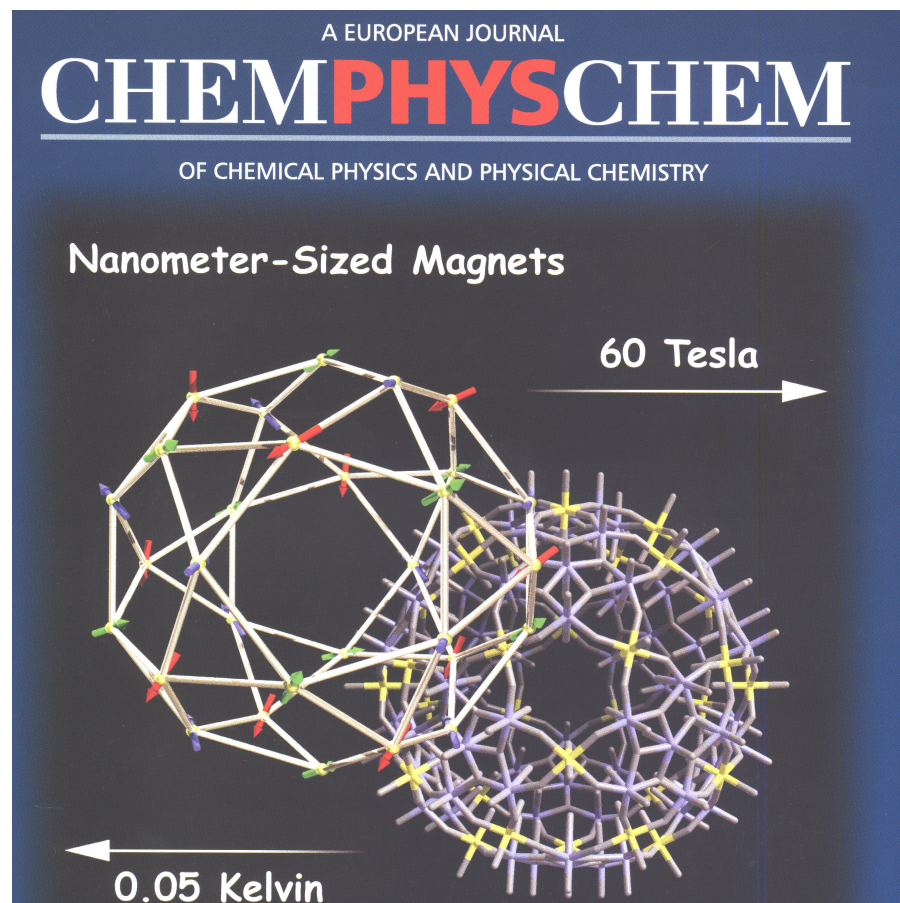


– History –

Celebrating 10+ years of

Searching in the dark!

# In 2001 (we thought) we solved the problem!



A. Müller *et al.*, Chem. Phys. Chem. **2**, 517 (2001).



# Again in 2001 (we thought) we solved the problem!

EUROPHYSICS LETTERS

15 December 2001

*Europhys. Lett.*, **56** (6), pp. 863–869 (2001)

## Quantum rotational band model for the Heisenberg molecular magnet $\{\text{Mo}_{72}\text{Fe}_{30}\}$

J. SCHNACK<sup>1</sup>, M. LUBAN<sup>2</sup> and R. MODLER<sup>2</sup>

<sup>1</sup> *Universität Osnabrück, Fachbereich Physik - D-49069 Osnabrück, Germany*

<sup>2</sup> *Ames Laboratory & Department of Physics and Astronomy, Iowa State University  
Ames, Iowa 50011, USA*

(received 27 April 2001; accepted in final form 4 October 2001)

PACS. 75.10.Jm – Quantized spin models.

PACS. 75.50.Xx – Molecular magnets.

J. Schnack, M. Luban, and R. Modler, *Europhys. Lett.* **56**, 863 (2001).

The following solutions were so good,  
they all appeared in Phys. Rev. Lett.!

# In 2002 (we thought) we solved the problem!

VOLUME 88, NUMBER 16

PHYSICAL REVIEW LETTERS

22 APRIL 2002

## Macroscopic Magnetization Jumps due to Independent Magnons in Frustrated Quantum Spin Lattices

J. Schulenburg,<sup>1</sup> A. Honecker,<sup>2</sup> J. Schnack,<sup>3</sup> J. Richter,<sup>1</sup> and H.-J. Schmidt<sup>3</sup><sup>1</sup>*Institut für Theoretische Physik, Universität Magdeburg, P.O. Box 4120, D-39016 Magdeburg, Germany*<sup>2</sup>*Institut für Theoretische Physik, TU Braunschweig, Mendelssohnstrasse 3, D-38106 Braunschweig, Germany*<sup>3</sup>*Universität Osnabrück, Fachbereich Physik, Barbarastrasse 7, D-49069 Osnabrück, Germany*

(Received 29 August 2001; published 8 April 2002)

For a class of frustrated spin lattices including the Kagomé lattice we construct exact eigenstates consisting of several independent, localized one-magnon states and argue that they are ground states for high magnetic fields. If the maximal number of local magnons scales with the number of spins in the system, which is the case for the Kagomé lattice, the effect persists in the thermodynamic limit and gives rise to a macroscopic jump in the zero-temperature magnetization curve just below the saturation field. The effect decreases with increasing spin quantum number and vanishes in the classical limit. Thus it is a true macroscopic quantum effect.

DOI: 10.1103/PhysRevLett.88.167207

PACS numbers: 75.10.Jm, 75.45.+j, 75.50.Ee, 75.60.Ej

J. Schulenburg *et al.*, Phys. Rev. Lett. **88**, 167207 (2002).

# In 2005 (we thought) we solved the problem!

PRL 94, 017205 (2005)

PHYSICAL REVIEW LETTERS

week ending  
14 JANUARY 2005

## Competing Spin Phases in Geometrically Frustrated Magnetic Molecules

Christian Schröder\*

*Department of Electrical Engineering and Computer Science, University of Applied Sciences Bielefeld, D-33602 Bielefeld, Germany  
and Ames Laboratory, Ames, Iowa 50011, USA*

Hiroyuki Nojiri

*Department of Physics, Okayama University, Tsushimanaka 3-1-1, Okayama, 700-8530 Japan*

Jürgen Schnack and Peter Hage

*Fachbereich Physik, Universität Osnabrück, D-49069 Osnabrück, Germany*

Marshall Luban and Paul Kögerler

*Ames Laboratory & Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA  
(Received 18 May 2004; revised manuscript received 27 November 2004; published 11 January 2005)*

We had to *discuss* the experimental data somewhat. ⇒ !!!

C. Schröder *et al.*, Phys. Rev. Lett. **94**, 017205 (2005).

# In 2008 even the editor thought that the problem was solved!

PHYSICAL REVIEW B 77, 224409 (2008)



## Multiple nearest-neighbor exchange model for the frustrated magnetic molecules $\{\text{Mo}_{72}\text{Fe}_{30}\}$ and $\{\text{Mo}_{72}\text{Cr}_{30}\}$

Christian Schröder\*

*Department of Electrical Engineering and Computer Science, University of Applied Sciences Bielefeld, D-33602 Bielefeld, Germany  
and Ames Laboratory, Ames, Iowa 50011, USA*

Ruslan Prozorov, Paul Kögerler,<sup>†</sup> Matthew D. Vannette, Xikui Fang, and Marshall Luban  
*Ames Laboratory & Department of Physics and Astronomy, Iowa State University, Ames, Iowa 50011, USA*

Akira Matsuo and Koichi Kindo

*Institute for Solid State Physics, University of Tokyo, Kashiwanoha 5-1-5, Kashiwa, Chiba 277-8581, Japan*

Achim Müller and Ana Maria Todea

*Fakultät für Chemie, Universität Bielefeld, D-33501 Bielefeld, Germany*

(Received 27 December 2007; revised manuscript received 9 May 2008; published 4 June 2008)

C. Schröder *et al.*, Phys. Rev. B **77**, 224409 (2008).

# Finally in 2010: Hare Krishna!

PERSPECTIVE

www.rsc.org/dalton | Dalton Transactions

## Structure-related frustrated magnetism of nanosized polyoxometalates: aesthetics and properties in harmony†

Paul Kögerler,<sup>\*a</sup> Boris Tsukerblat<sup>b</sup> and Achim Müller<sup>c</sup>

*Received 1st June 2009, Accepted 1st September 2009*

*First published as an Advance Article on the web 12th October 2009*

DOI: 10.1039/b910716a

P. Kögerler, B. Tsukerblat, and A. Müller, Dalton Trans. **39**, 1 (2010).



This world could be  
so nice and harmonic . . .



... except:  
IT IS NOT!

# So, what is the problem?

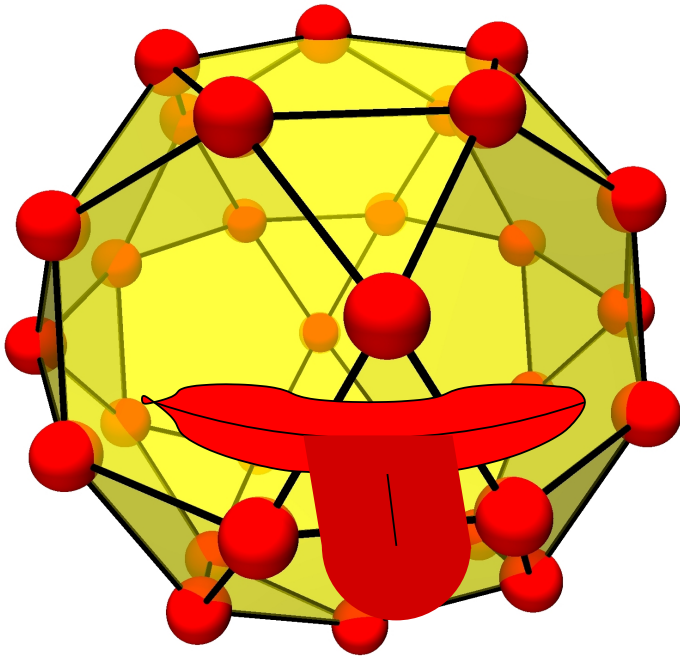
Well, we all thought that  
 $\{ (\text{Mo/W})_{72}(\text{Fe/Cr/V})_{30} \}$  is

“The Kagome on a sphere”.

## But it is not.

I. Rousochatzakis, A. M. Läuchli, and F. Mila, Phys. Rev. B **77**, 094420 (2008).

# Contents for you today



1. History ✓
2. Expectations
3. The Reality
4. Speculation: random  $J$  (CS)
5.  $\{W_{72}V_{30}\}$

# Expectations

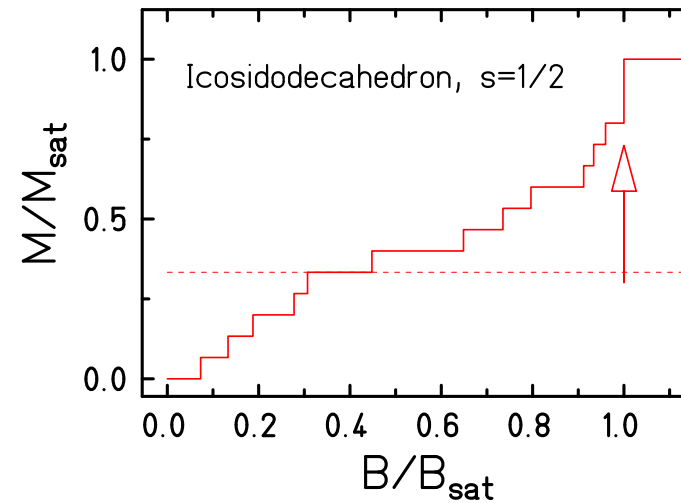
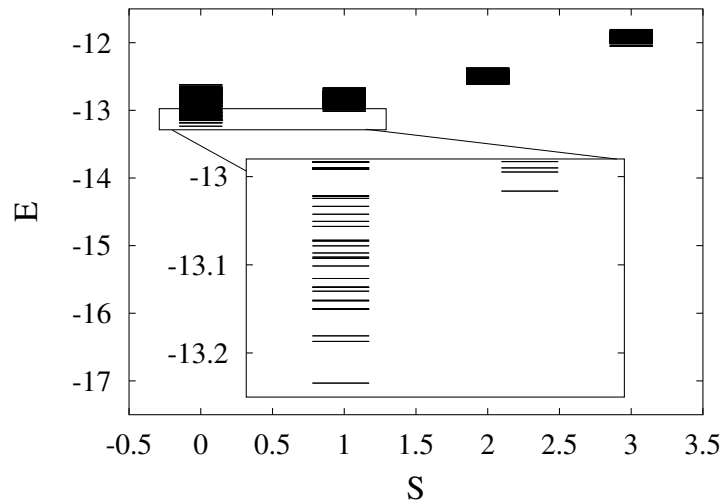
Icosidodecahedron

+

Heisenberg model

=

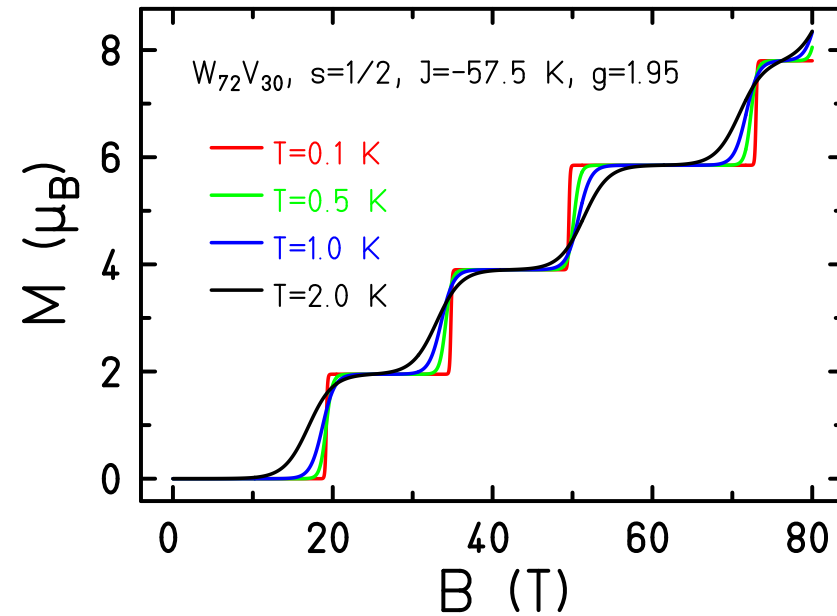
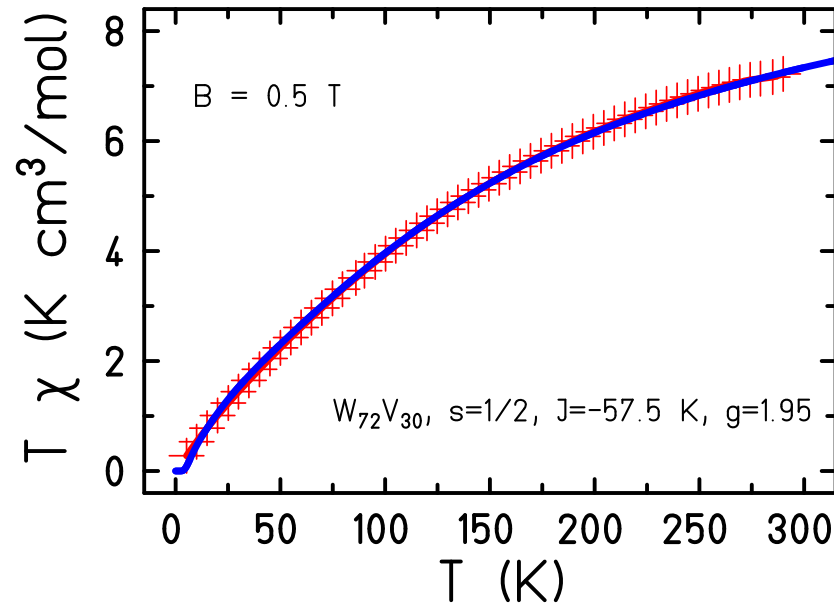
# Expectations: Singlets, Plateau, and Jump



- Many singlet levels below the first triplet;
- Plateau at  $\mathcal{M} = \mathcal{M}_{\text{sat}}/3$ ;
- Unusually large jump to saturation.

R. Schmidt, J. Schnack, and J. Richter, *J. Magn. Magn. Mater.* **295**, 164 (2005).  
 J. Schnack, *Dalton Trans.* **39**, 4677 (2010).

# Expectation: $\{W_{72}V_{30}\}$



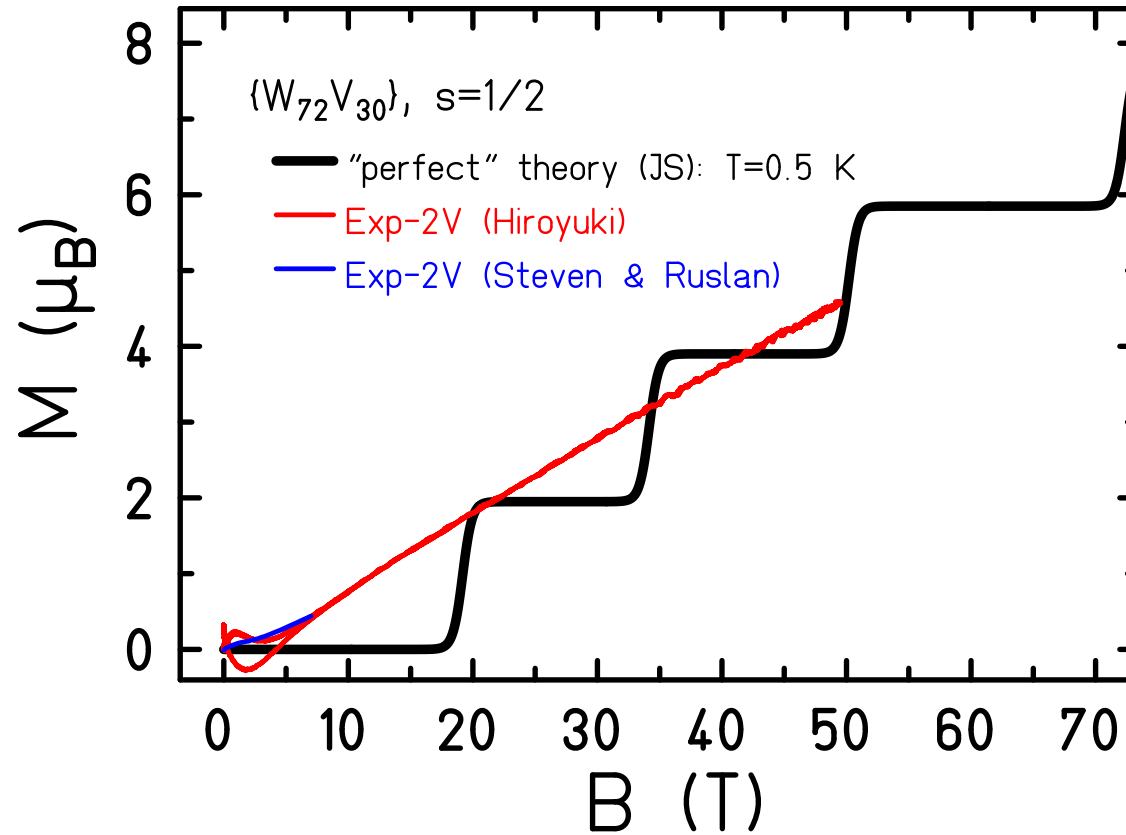
- Perfect fit of susceptibility (FTLM);
- Expect magnetization steps in high-field measurements.

exp. data: A. M. Todea, A. Merca, H. Bögge, T. Glaser, L. Engelhardt, R. Prozorov, M. Luban, A. Müller, Chem. Commun., 3351 (2009).  
 J. Schnack, arXiv:1012.4980



# Reality

# Reality: magnetization curve without steps



# Speculations

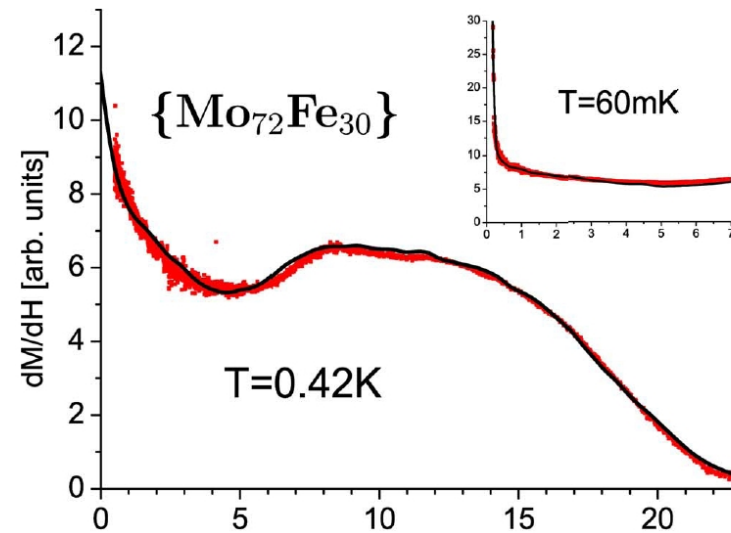
## Speculation I: random $J$

$$\underline{H} = \underbrace{-2 \sum_{i < j} J_{ij} \vec{s}_{\sim}(i) \cdot \vec{s}_{\sim}(j)}_{\text{Heisenberg}} + \underbrace{g \mu_B B \sum_i^N s_{z\sim}(i)}_{\text{Zeeman}}$$

Idea of Christian Schröder:  $J_{ij}$  fluctuate ( $J$ -strain), therefore, are chosen according to a random distribution.

C. Schröder *et al.*, Phys. Rev. B **77**, 224409 (2008).

# Speculation I: random $J$

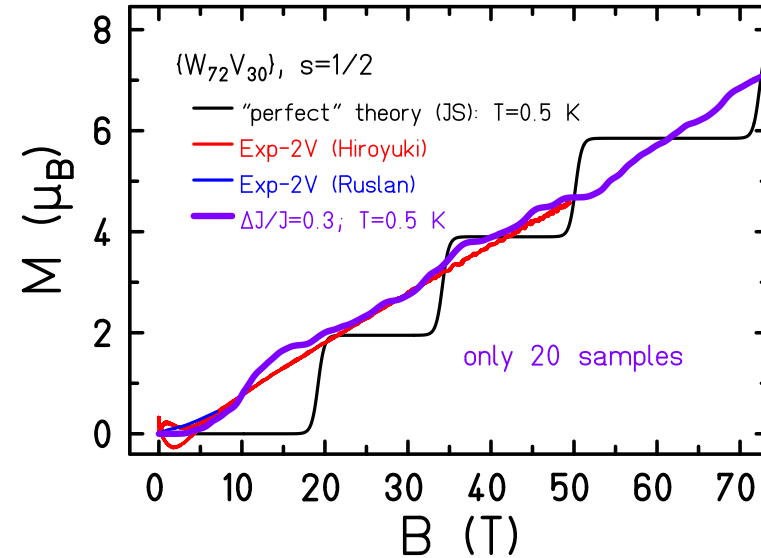
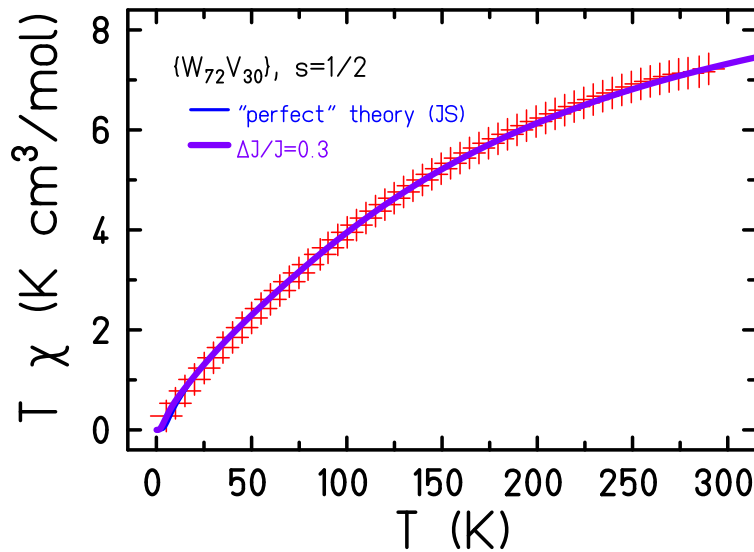


- Classical spin dynamics with random  $J$  for  $\{\text{Mo}_{72}\text{Fe}_{30}\}$  and  $\{\text{Mo}_{72}\text{Cr}_{30}\}$ ;
- Absolutely astonishing accuracy.

C. Schröder *et al.*, Phys. Rev. B **77**, 224409 (2008).

{W<sub>72</sub>V<sub>30</sub>}

# {W<sub>72</sub>V<sub>30</sub>}

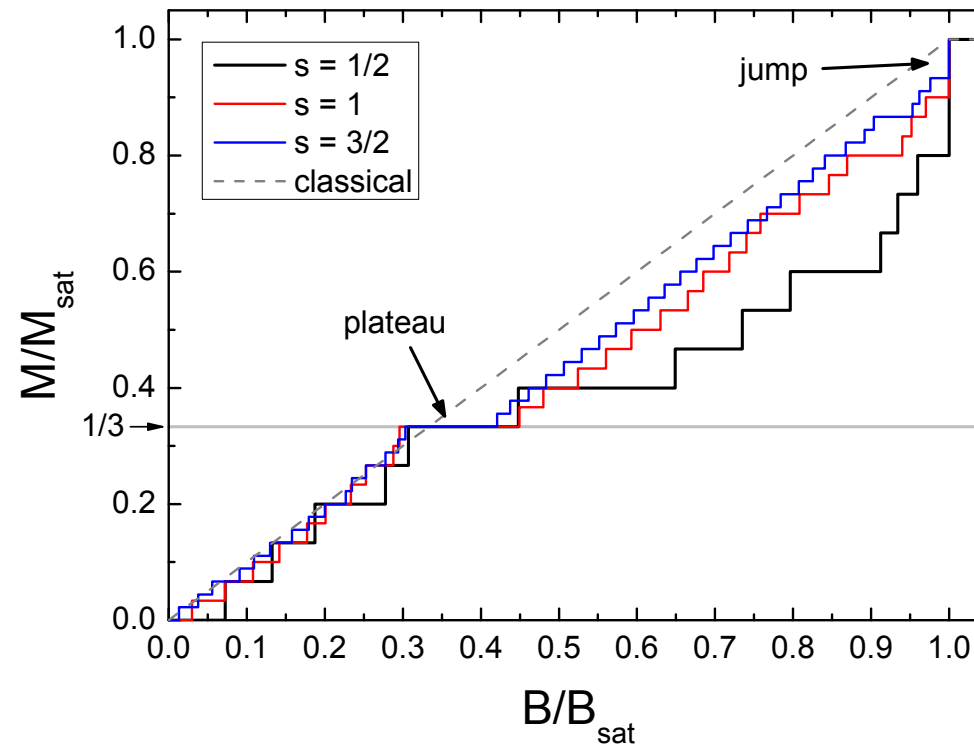


- {W<sub>72</sub>V<sub>30</sub>} is the MOST symmetric Keplerate (AM).
- Nevertheless, it needs a  $J$  variation of about 30%.
- Full FTLM quantum calculations, 15 hours on 510 cores for a single parameter set.



# Outlook

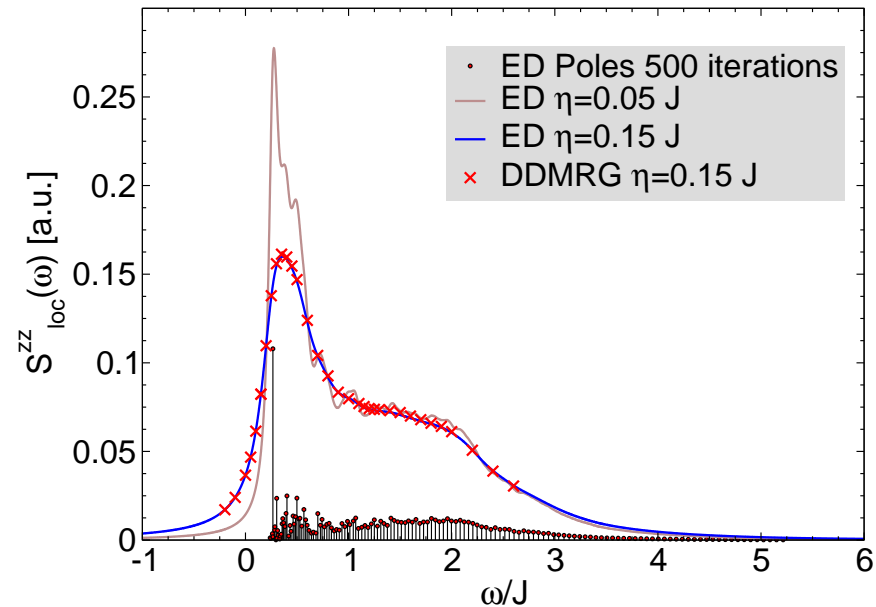
# DMRG & FTLM provide full details I



- DMRG yields ground states + very few low-lying states in orthogonal subspaces.
- Magnetization curve for  $T = 0$ , resonance energies for spectroscopy.

(1) J. Ummethum, J. Schnack, and A. Laeuchli, J. Magn. Magn. Mater. **327** (2013) 103

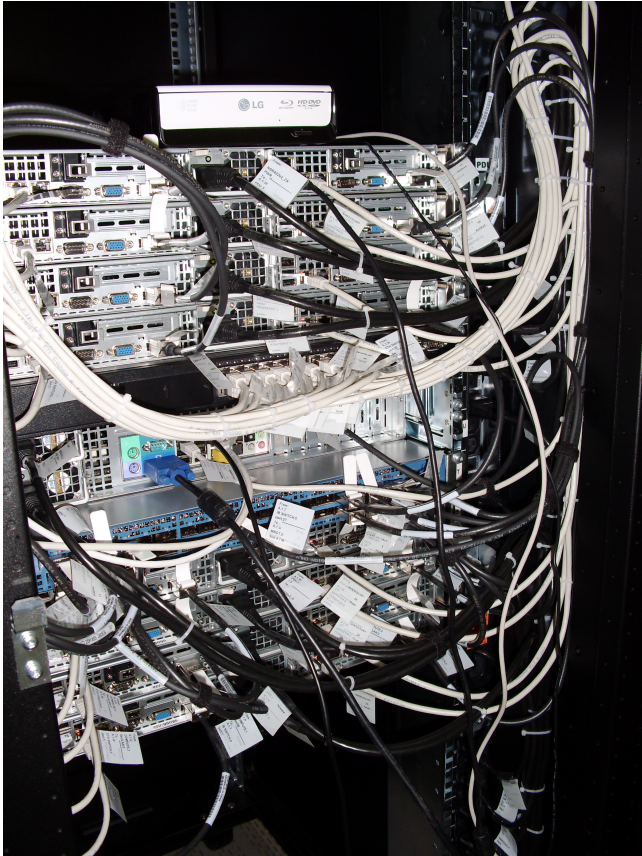
## DMRG & FTLM provide full details II



- $V_{30}$ :  $S_{loc}^{zz}(\omega) = -\frac{1}{\pi} \text{Im} \langle \text{GS} | S_0^z \frac{1}{\omega - (H - E_{\text{GS}}) + i\eta} S_0^z | \text{GS} \rangle$
- Aim: better understanding of low-lying levels, **INS**.

(1) J. Ummethum, J. Schnack, and A. Laeuchli, J. Magn. Magn. Mater. **327** (2013) 103

# Summary



- The Keplerate molecules remain my hobby.
- Finite-temperature Lanczos is perfect to deal with larger systems and to go beyond Heisenberg models.
- *J*-strain should be visible elsewhere!
- If you have no explanation, try Dzyaloshinskii-Moriya! This is our next step.

## Many thanks to my collaborators worldwide

- T. Glaser, Chr. Heesing, M. Höck, N.B. Ivanov, J. Korenke, H.T. Langwald, S. Leiding, A. Müller, Chr. Schröder, J. Ummethum, O. Wendland (Bielefeld)
- K. Bärwinkel, H.-J. Schmidt, M. Neumann (Osnabrück)
- M. Luban (Ames Lab, USA); P. Kögerler (Aachen, Jülich, Ames); R.E.P. Wimpenny, E.J.L. McInnes (Man U, UK); L. Cronin, M. Murrie (Glasgow, UK); E. Brechin (Edinburgh, UK); H. Nojiri (Sendai, Japan); A. Postnikov (Metz, France); M. Evangelisti (Zaragosa, Spain)
- J. Richter, J. Schulenburg (Magdeburg); A. Honecker (Göttingen); U. Kortz (Bremen); A. Tennant, B. Lake (HMI Berlin); B. Büchner, V. Kataev, H.-H. Klauß (Dresden); P. Chaudhuri (Mühlheim); J. Wosnitza (Dresden-Rossendorf); J. van Slageren (Stuttgart); R. Klingeler (Heidelberg); O. Waldmann (Freiburg)

Thank you very much for your  
attention.

The end.

Molecular Magnetism Web

[www.molmag.de](http://www.molmag.de)

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