Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions

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Abstract

Equilibration of observables in **closed quantum systems** that are described by a **unitary time evolution** is a meanwhile well-established phenomenon apart from a few equally well-established exceptions. Here [1] we report the surprising theoretical observation that integrable as well as non-integrable spin rings with nearest-neighbor or long-range isotropic Heisenberg interaction not only equilibrate but moreover also synchronize the directions of the expectation values of the individual spins. We highlight that this differs from spontaneous synchronization in quantum dissipative systems. We observe mutual synchronization of local spin directions in closed systems under unitary time evolution. Contrary to dissipative systems, this synchronization is independent of whether the interaction is ferro- or antiferromagnetic. In our numerical simulations, we investigate the free induction decay of an ensemble of up to $N = 25$ quantum spins with $s = 1/2$ each by solving the time-dependent Schrödinger equation numerically exactly. Our findings are related to, but not fully explained by conservation laws of the system. The synchronization is very robust against for instance random fluctuations of the Heisenberg couplings and inhomogeneous magnetic fields. Synchronization is not observed with strong enough symmetry-breaking interactions such as the dipolar interaction. We also compare our results to closed-system classical spin dynamics which does not exhibit phase synchronization due to the lack of entanglement. For classical spin systems the fixed magnitude of individual spins effectively acts like additional N conservation laws.

Introduction

In this work we discuss an observation that rests on decoherence, equilibration, entanglement and conservation laws. We study the free induction decay (FID) of quantum spins that are arranged on a ring-like geometry with nearest-neighbor as well as long-range isotropic Heisenberg interactions. For the overwhelming majority of investigated cases the initial product state of single-spin states entangles, i.e. turns into a superposition of product states, and thereby equilibrates at the level of single-spin observables. Our most striking observation is that expectation values of all individual spin vectors synchronize with respect to their orientation. In a FID setting this means that their various individual rotations about the common field axis synchronize and align in the course of time. In a co-rotating frame they simply align. The Hamiltonian of our spin model (in the ring like case) reads

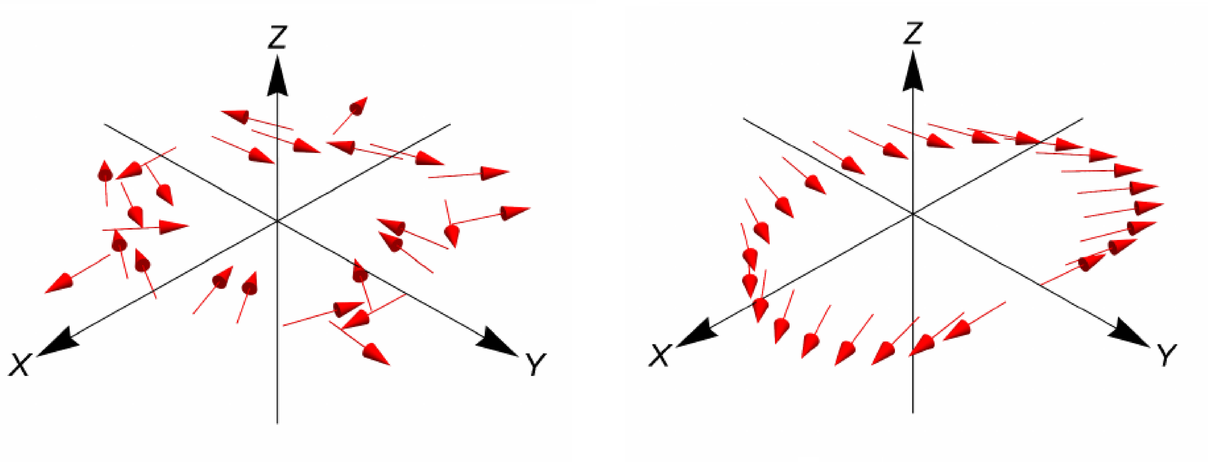
$$\underset{\sim}{H} = -\sum_{j=1}^{N} J_j \underset{\sim}{\vec{s}}_j \cdot \underset{\sim}{\vec{s}}_{j+1} - \sum_{j=1}^{N} h_j \underset{\sim}{s}_j^z \,, \qquad (1)$$

where the first sum corresponds to the isotropic Heisenberg model and the second sum denotes the Zeeman term. Operators are marked by a tilde, the Heisenberg interactions are denoted by J_j, local magnetic fields are given by h_j, and periodic boundary conditions $\underset{\sim}{\vec{s}}_{N+1} = \underset{\sim}{\vec{s}}_1$ are applied.

Furthermore, we define the transverse magnetization

$$M_{\text{trans}} := \sqrt{\langle \underset{\sim}{S}^x \rangle^2 + \langle \underset{\sim}{S}^y \rangle^2} = \sqrt{\left(\sum_j \langle \underset{\sim}{s}_j^x \rangle\right)^2 + \left(\sum_j \langle \underset{\sim}{s}_j^y \rangle\right)^2} \,. \qquad (2)$$

Here $\langle \underset{\sim}{S}^x \rangle$ denotes the expectation value with respect to a specified many-body state. We interpret (2) as the net magnetization precessing in the xy-plane. In case of Hamiltonian (1) this is also a conserved quantity if the local magnetic fields are all the same $h_j \equiv h \; \forall j$.



(a) random directions (b) 180 degrees

Figure 1: Visualization of some initial product states studied in this work. The arrows correspond to the single-spin expectation values. We refer the right state as $|\,\psi_B\,\rangle$, see [1].

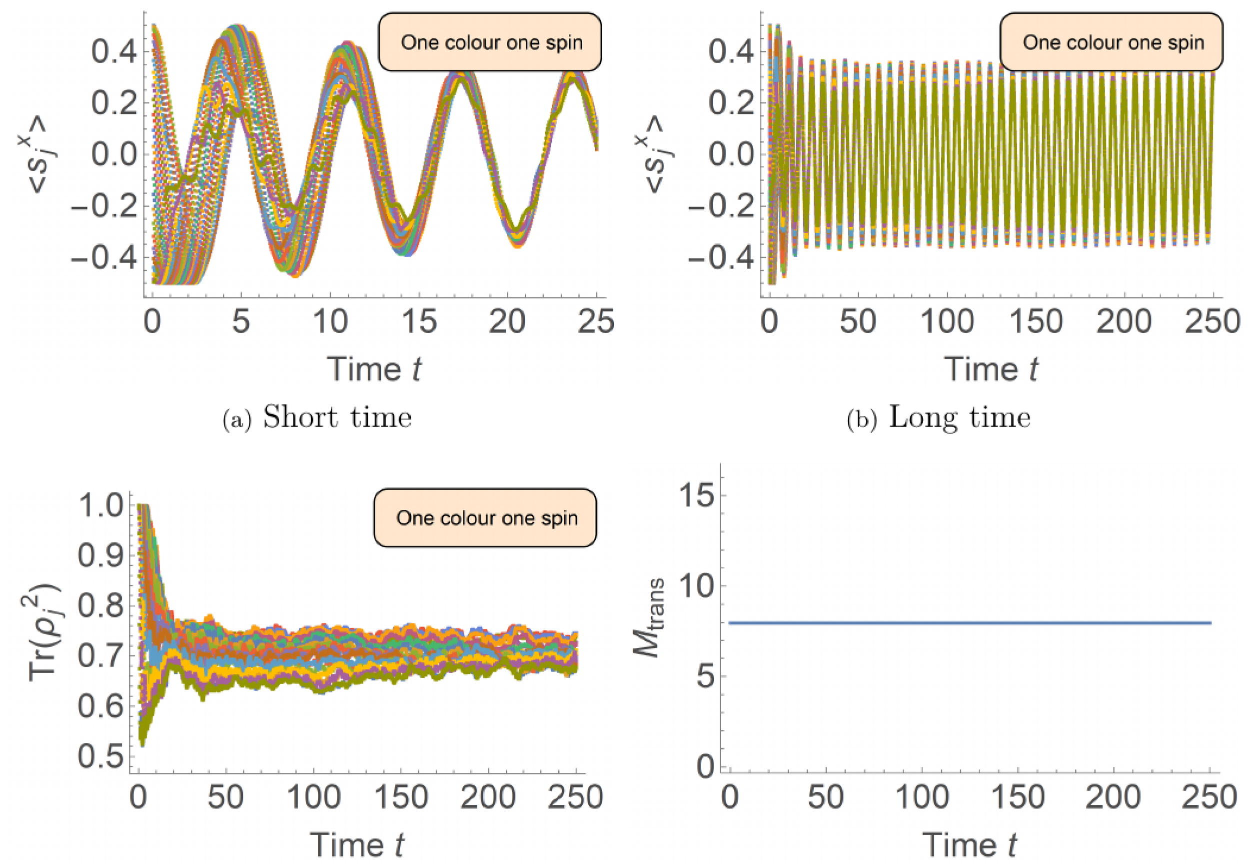
As initial many-body states we choose product states of the form

$$|\,\psi(t=0)\,\rangle = \bigotimes_{j=1}^{N} \frac{1}{\sqrt{2}} \left(|\uparrow\rangle + e^{i\theta_j} |\downarrow\rangle \right) \,, \qquad (3)$$

for which the expectation values of individual spins are oriented in the xy-plane and point in a direction that depends on θ_j. Fig. 1 is a visualization of such states.

Synchronization

The following calculations represent our main finding. Fig. 2 shows a time evolution for initial state $|\,\psi_B\,\rangle$. Initially the individual spin expectation values are spread out by 180 degrees, but during time evolution they align.



(a) Short time (b) Long time

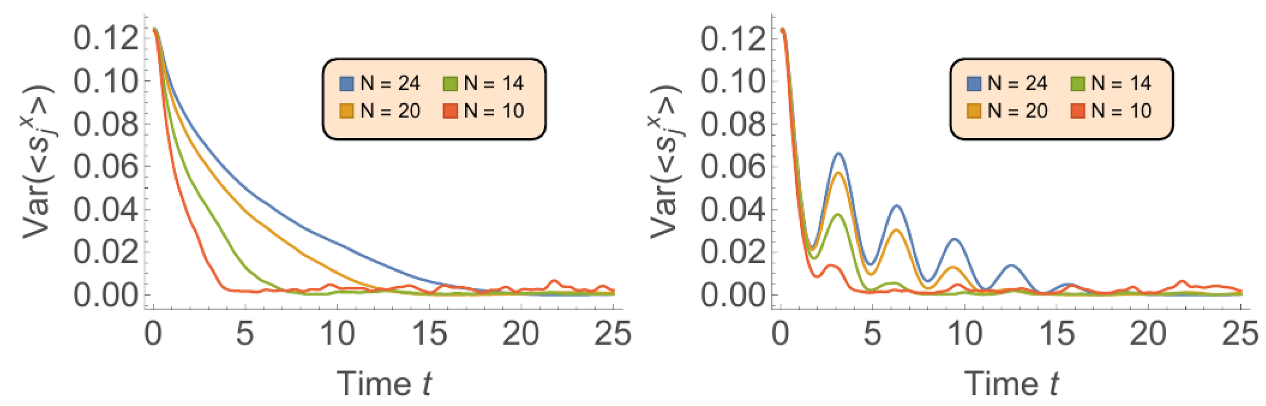
(c) Purity (d) Transverse magnetization

Figure 2: Time evolution of initial state $|\,\psi_B\,\rangle$ w.r.t. Hamiltonian (1) with isotropic Heisenberg interactions and $J_j \in [1.6, 2.4]$, $h_j = -1 \; \forall j$, $N = 25$ [1].

Figure 3 shows the variance of the expectation values of individual spin operators, defined as

$$\mathrm{Var}(<\underset{\sim}{s}_j^x>)(t) := \frac{1}{N}\sum_{j=1}^{N}\left(<\underset{\sim}{s}_j^x> - \frac{<S^x>}{N}\right)^2 \qquad (4)$$

for different system sizes N. That the variance decays to zero, compare 3, expresses precisely that the spins align until they point in the same direction. This process takes the longer the larger the system is. The synchronisation, i.e. the alignment of directions, also takes place in the absence of a magnetic field, as can be seen in 3(a).



(a) $h_j = 0 \; \forall j$ (b) $h_j = -1 \; \forall j$

Figure 3: Time evolution of initial state $|\,\psi_B\,\rangle$ w.r.t. Hamiltonian (1) with isotropic Heisenberg interactions and $J_j = 2 \; \forall j$ without (a) and with magnetic field (b), compare [1].

Summary

As a conclusion we can say first of all that the conservation of M_{trans} not just slows down the FID, but prevents the free induction from decaying if the Hamiltonian only contains isotropic Heisenberg interactions and the Zeeman terms of all spins are equal. Furthermore, we demonstrate in detail the interesting phenomenon that the single-spin vector expectation values align in the course of time almost independent of how they are initialized in the xy-plane. It does not matter if the initial state is a product state of the form (3) or if the spins start in an entangled state. For the process of synchronization the magnetic field is not necessary, it only induces a (collective) rotation of all spins about the field axis. The Heisenberg interactions cause an equilibration process under the constraint of conserved quantities. We show that after entanglement is maximised (under constraints of conserved quantities) and equilibration is completed the spins stay synchronized and fluctuate the less the larger the system is. Moreover, we show that the timescale of synchronization is independent of the width Δ of the variation of Heisenberg couplings J_j. In addition, we discuss that the synchronization of spin expectation values is very robust. It is robust regarding

- the initial state
- long and short range Heisenberg interactions
- larger spins than s= 1/2
- fluctuations in the individual magnetic fields h_j
- fluctuations in the individual coupling constants J_j

Further on, we provide examples of transient synchronization for systems where symmetries are broken, because the time scale of synchronization is shorter than that of equilibration. Systems with anisotropic XYZ interactions belong to this set if they are still close to the isotropic Heisenberg case, or if the symmetry is broken by means of an inhomogeneous magnetic field. Finally, we show that spins do not synchronize for interactions that are strongly anisotropic such as dipolar interactions.

References

[1] Patrick Vorndamme, Heinz-Jürgen Schmidt, Christian Schröder, and Jürgen Schnack. Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions. *New J. Phys.* 23, 083038 (2021), doi: 10.1088/1367-2630/ac18df.