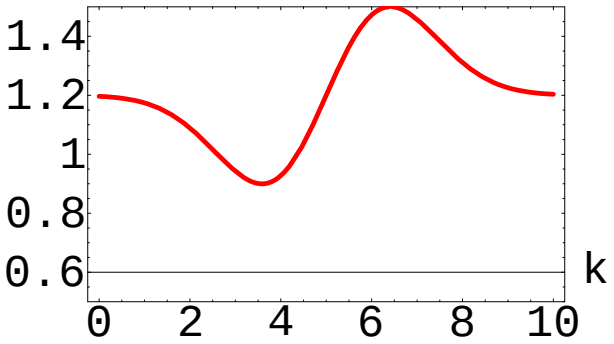
Ausbreitung einer skalaren Welle im Medium, Phasen– und Gruppengeschwindigkeit

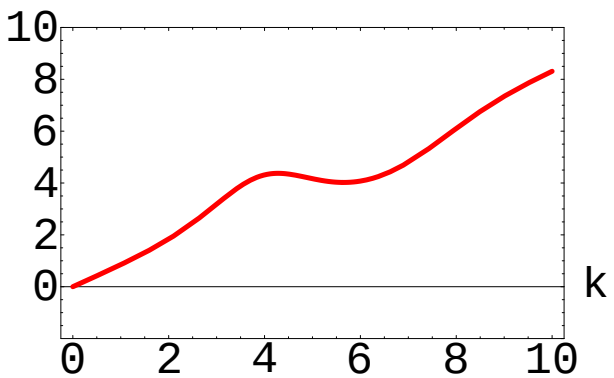
■ Brechungsindex

```
c = 1; η = 0.35; k0 = 5;

n[k_] = 1.2 + η (k - k0) e^-((k-k0)^2/4);

Plot[n[k], {k, 0, 10}, PlotRange -> {0.5, 1.5},
  AxesLabel -> {"k", " "}];
```



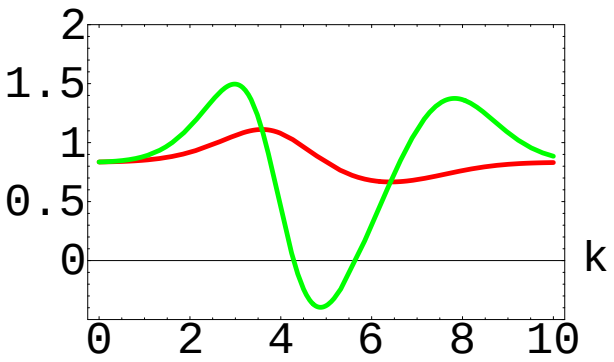
■ Dispersionsrelation

```
ω[k_] = c k / n[k];

Plot[ω[k], {k, 0, 10}, PlotRange -> {-2, 10},
  AxesLabel -> {"k", " "}];
```



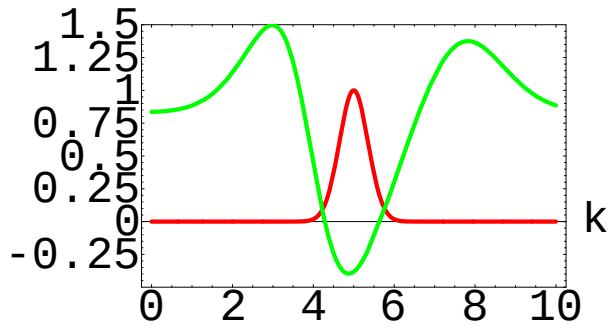
■ Phasen– und Gruppengeschwindigkeit

```
v_p[k_] = ω[k] / k;

v_g[k_] = ∂_k ω[k];

Plot[{v_p[k], v_g[k]}, {k, 0, 10}, PlotRange -> {-0.5, 2},
 AxesLabel -> {"k", " "}];
```



■ spektrale Verteilung der Welle

```
g[k_] = e^(-(k-k_0)^2 / 0.25);

Plot[{g[k], v_g[k]}, {k, 0, 10}, PlotRange -> {-0.5, 1.5},
 AxesLabel -> {"k", " "}];
```



■ skalare Welle

```
φ[x_, t_] = NIntegrate[g[k] e^(i (k x - ω[k] t)) , {k, 0, 4, 5, 6, 20}];

Plot[Re[φ[x, 0]], {x, -10, 10},
 PlotRange -> {-2.5, 2.5}, AxesLabel -> {"x", "Re[φ]"}];
```

```
NIntegrate::inum :
 Integrand 2.31952283024357×10^-16 <<20>>^(1. I (<<1>>)) is not numerical at {k} = {2}.

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```
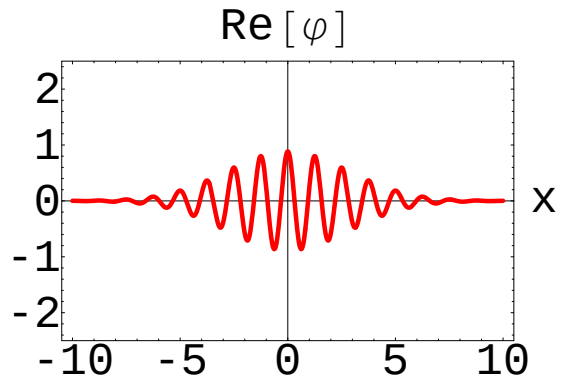
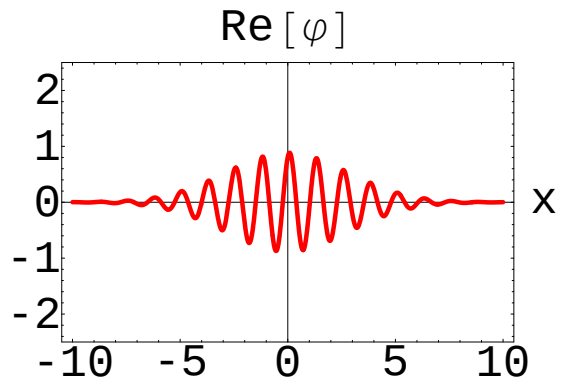



■ Zeitentwicklung

```
For[i = 1, i < 101, i++,
  t = 0.1 i;
  Plot[Re[φ[x, t]], {x, -10, 10}, PlotPoints → 40,
     PlotRange -> {-2.5, 2.5}, AxesLabel -> {"x", "Re[φ]"}]];
```