

Frustration-induced exotic properties of magnetic molecules and low-dimensional antiferromagnets

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Oberseminar *Aktuelle Fragen der Theoretischen Physik*
Universität Stuttgart, May 9th 2006



文部科学省

In late 20th century people coming from



transport theory



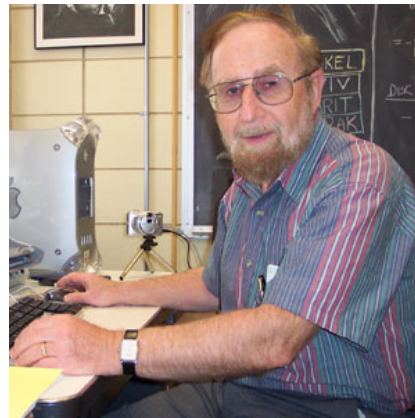
general relativity



nuclear physics



Zener diodes



were triggered by a “magnetic” enthusiast.

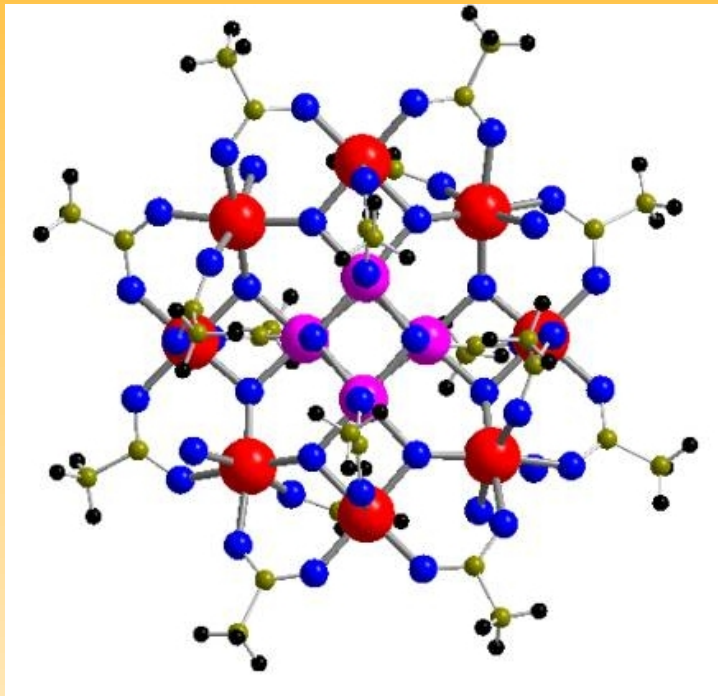
Meanwhile a big collaboration has been established

- K. Bärwinkel, H.-J. Schmidt, J. S., M. Allalen, M. Brüger, D. Mentrup, D. Müter, M. Exler, P. Hage, F. Hesmer, K. Jahns, F. Ouchni, R. Schnalle, P. Shechelovskyy, S. Torbrügge (U. Osnabrück);
- M. Luban, P. Kögerler, D. Vaknin (Ames Lab, USA);
J. Musfeldt (U. of Tennessee, USA); N. Dalal (Florida State, USA);
- A. Müller (U. Bielefeld); Chr. Schröder (FH Bielefeld);
- H. Nojiri (Tohoku University, Japan);
- R.E.P. Winpenny (Man U); L. Cronin (U. of Glasgow);
- J. Richter, J. Schulenburg, R. Schmidt (U. Magdeburg);
- S. Blügel, A. Postnikov (FZ Jülich); A. Honecker (Uni Braunschweig).
- E. Rentschler (U. Mainz); U. Kortz (IUB); A. Tennant (HMI Berlin).

... and various general results could be achieved

1. The suspects: magnetic molecules
2. The thumbscrew: Heisenberg model
3. Extension of Lieb, Schultz, and Mattis: k -rule for odd rings
4. Rotational bands in antiferromagnets
5. Giant magnetization jumps in frustrated antiferromagnets
6. Magnetization plateaus and susceptibility minima
7. Enhanced magnetocaloric effect
8. Hysteresis without anisotropy
9. A special triangular molecule-based spin tube

The beauty of magnetic molecules I

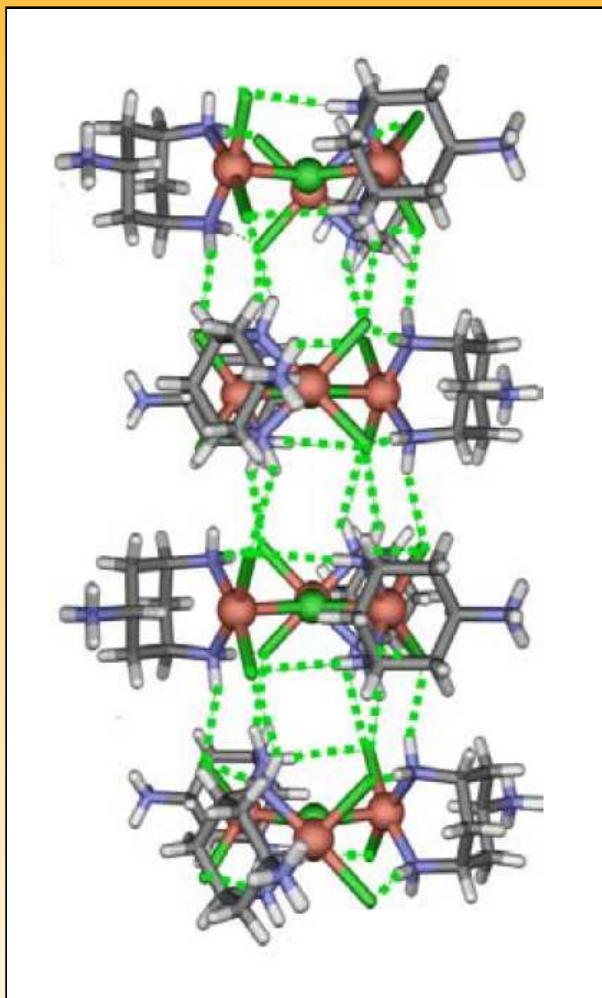


Mn₁₂

- Inorganic or organic macro molecules, e.g. polyoxometalates, where paramagnetic ions such as Iron (Fe), Chromium (Cr), Copper (Cu), Nickel (Ni), Vanadium (V), Manganese (Mn), or rare earth ions are embedded in a host matrix;
- Pure organic magnetic molecules: magnetic coupling between high spin units (e.g. free radicals);
- Single spin quantum number $1/2 \leq s \leq 7/2$;
- Intermolecular interaction relatively small, therefore measurements reflect the thermal behaviour of a single molecule.

Magnetism goes Nano, Ed. Stefan Blügel, Thomas Brückel, and Claus M. Schneider, FZ Jülich, Institute of Solid State Research, Lecture Notes **36** Jülich 2005

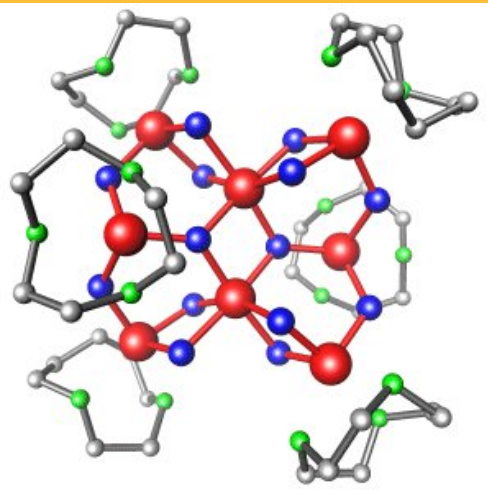
The beauty of magnetic molecules II



- Dimers (Fe_2), tetrahedra (Cr_4), cubes (Cr_8);
- Rings, especially iron rings (Fe_6 , Fe_8 , Fe_{10} , ...);
- Complex structures (Mn_{12}) – drosophila of molecular magnetism;
- “Soccer balls”, more precisely icosidodecahedra (Fe_{30}) and other macro molecules;
- Chain like and planar structures of interlinked magnetic molecules, e.g. triangular Cu chain:

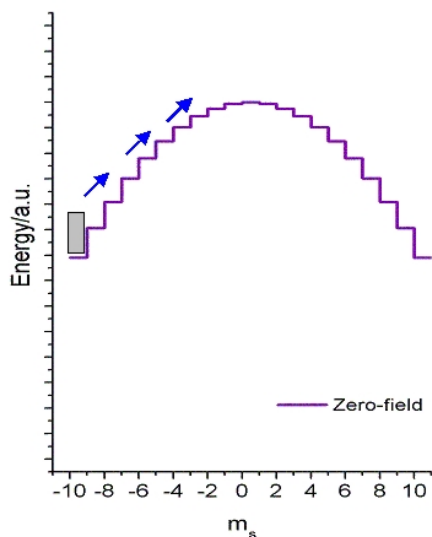
J. Schnack, H. Nojiri, P. Kögerler, G. J. T. Cooper, L. Cronin, Phys. Rev. B 70, 174420 (2004)

The beauty of magnetic molecules III



- Single Molecule Magnets (SMM): magnetic molecules with large ground state moment;
- Example: $S = 10$ for Mn_{12} or Fe_8 ;
- Anisotropy dominates approximate single-spin Hamiltonian:

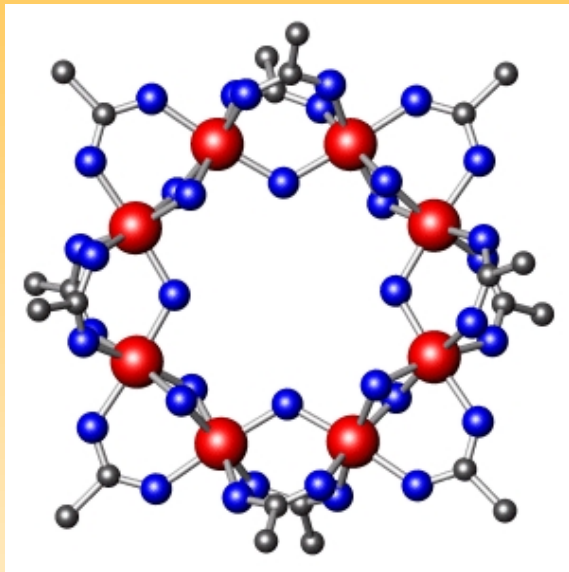
$$\underline{H} = -D\underline{S}_z^2 + \underline{H}', \quad [\underline{S}_z, \underline{H}'] \neq 0$$



- Single molecule shows: metastable magnetization, hysteresis, ground state magnetization tunneling, thermally and phonon assisted tunneling.
- Today's major efforts: improve stability of magnetization; investigate on surfaces.

The beauty of magnetic molecules IV

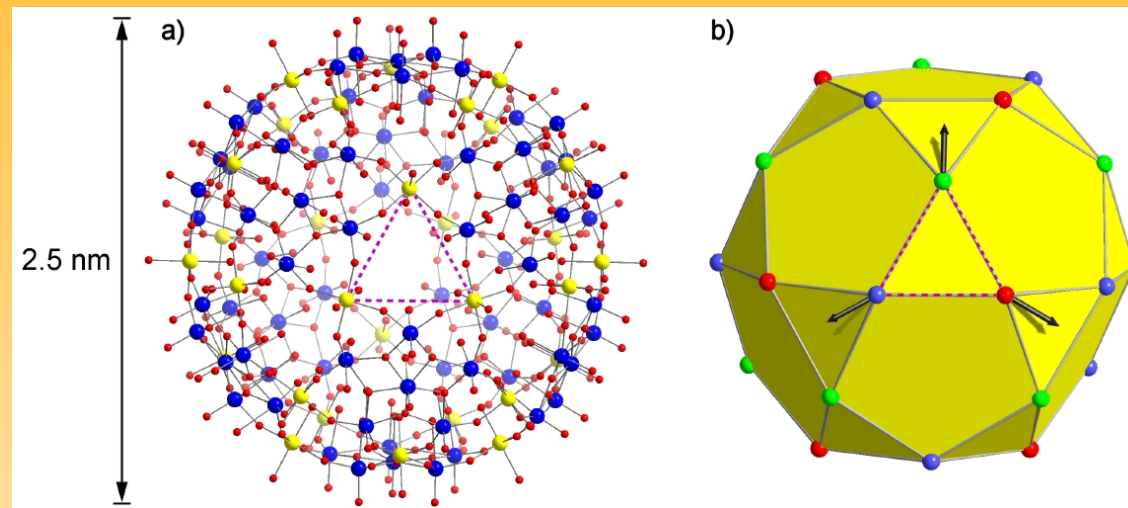
Why magnetic molecules?



Cr₈

- Interacting spin system largely decoupled from remaining degrees of freedom;
- Transition few-spin system \Rightarrow many-spin system, contribution to understanding of bulk magnetism;
- Transition quantum spin system ($s = 1/2$) \Rightarrow classical spin system ($s_{\text{Fe}} = 5/2$, $s_{\text{Gd}} = 7/2$);
- Easy to produce, single crystals with $> 10^{17}$ identical molecules can be synthesized and practically completely characterized;
- **Speculative applications: magnetic storage devices, magnets in biological systems, light-induced nano switches, displays, catalysts, qubits for quantum computers.**

{Mo₇₂Fe₃₀} – a molecular brother of the kagome lattice and an archetype of geometric frustration



- Giant magnetic Keplerate molecule;
- Structure: Fe - yellow, Mo - blue, O - red;
- Antiferromagnetic interaction mediated by O-Mo-O bridges (1).
- Classical ground state of {Mo₇₂Fe₃₀}: three sublattice structure, coplanar spins (2);
- Quantum mechanical ground state $S = 0$ can only be approximated, dimension of Hilbert space $(2s + 1)^N \approx 10^{23}$ (3).

(1) A. Müller *et al.*, Chem. Phys. Chem. **2**, 517 (2001) , (2) M. Axenovich and M. Luban, Phys. Rev. B **63**, 100407 (2001) , (3) M. Exler and J. Schnack, Phys. Rev. B **67**, 094440 (2003)

Hamiltonian and Tools

Model Hamiltonian – Heisenberg-Model

$$\tilde{H} = \sum_{i,j} \vec{\tilde{s}}(i) \cdot \mathbf{J}_{ij} \cdot \vec{\tilde{s}}(j) + \sum_{i,j} \vec{D}_{ij} \cdot [\vec{\tilde{s}}(i) \times \vec{\tilde{s}}(j)] + \mu_B B \sum_i^N g_i \tilde{s}_z(i)$$

Exchange/Anisotropy
Dzyaloshinskii-Moriya
Zeeman

Very often anisotropic terms are utterly negligible, then ...

$$\tilde{H} = - \sum_{i,j} J_{ij} \vec{\tilde{s}}(i) \cdot \vec{\tilde{s}}(j) + g \mu_B B \sum_i^N \tilde{s}_z(i)$$

Heisenberg
Zeeman

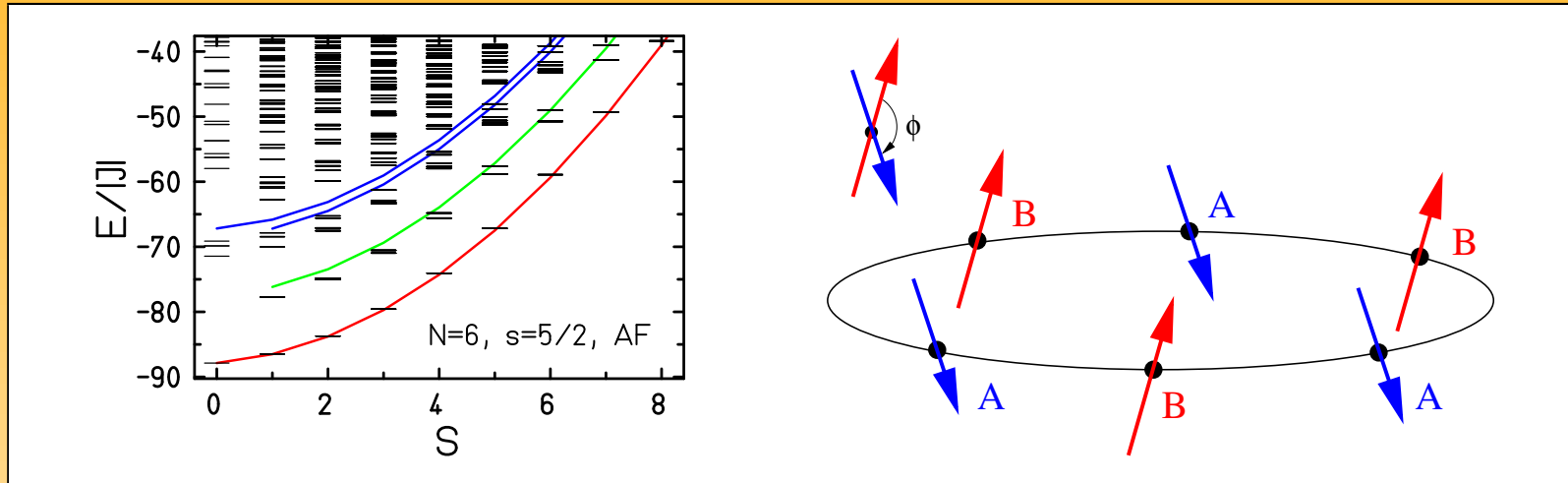
The Heisenberg Hamilton operator together with a Zeeman term are used for the following considerations; $J < 0$: antiferromagnetic coupling.

Our toolbox

- Analytical or numerically exact and complete diagonalization of the Hamiltonian: always superior, the complete thermodynamics is available;
- Lanczos diagonalization: some low-lying eigenstates in orthogonal subspaces;
- Density Matrix Renormalization Group technique (DMRG): often only the ground state for a given orthogonal subspace;
- Approximate Hamiltonians: good insight for certain systems;
- Classical spin dynamics: illuminating and “dangerous”;
- Spin coherent states: no real improvement compared to classical spin dynamics.

General Results

Rotational bands in antiferromagnets I

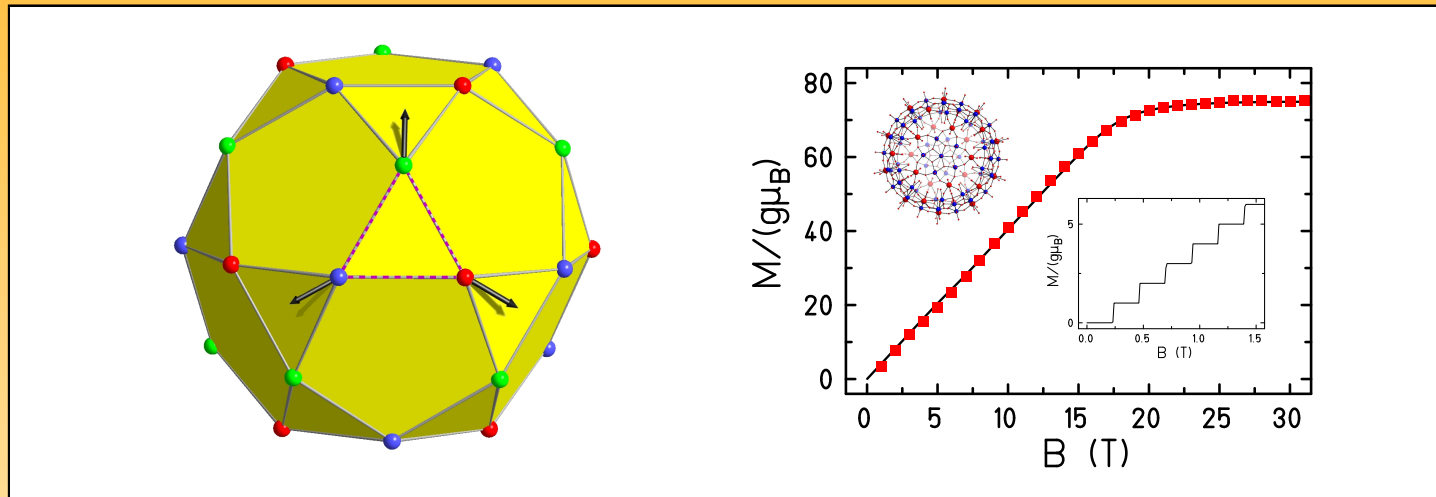


- Often minimal energies $E_{min}(S)$ form a rotational band: Landé interval rule (1);
- For bipartite systems (2,3): $\tilde{H}^{eff} = -2 J_{eff} \tilde{S}_A \cdot \tilde{S}_B$;
- Lowest band – rotation of Néel vector, second band – spin wave excitations (4).

(1) A. Caneschi *et al.*, Chem. Eur. J. **2**, 1379 (1996), G. L. Abbati *et al.*, Inorg. Chim. Acta **297**, 291 (2000)
 (2) J. Schnack and M. Luban, Phys. Rev. B **63**, 014418 (2001)
 (3) O. Waldmann, Phys. Rev. B **65**, 024424 (2002)
 (4) P.W. Anderson, Phys. Rev. B **86**, 694 (1952), O. Waldmann *et al.*, Phys. Rev. Lett. **91**, 237202 (2003).

Rotational bands in antiferromagnets II

Approximate Hamiltonian for $\{\text{Mo}_{72}\text{Fe}_{30}\}$



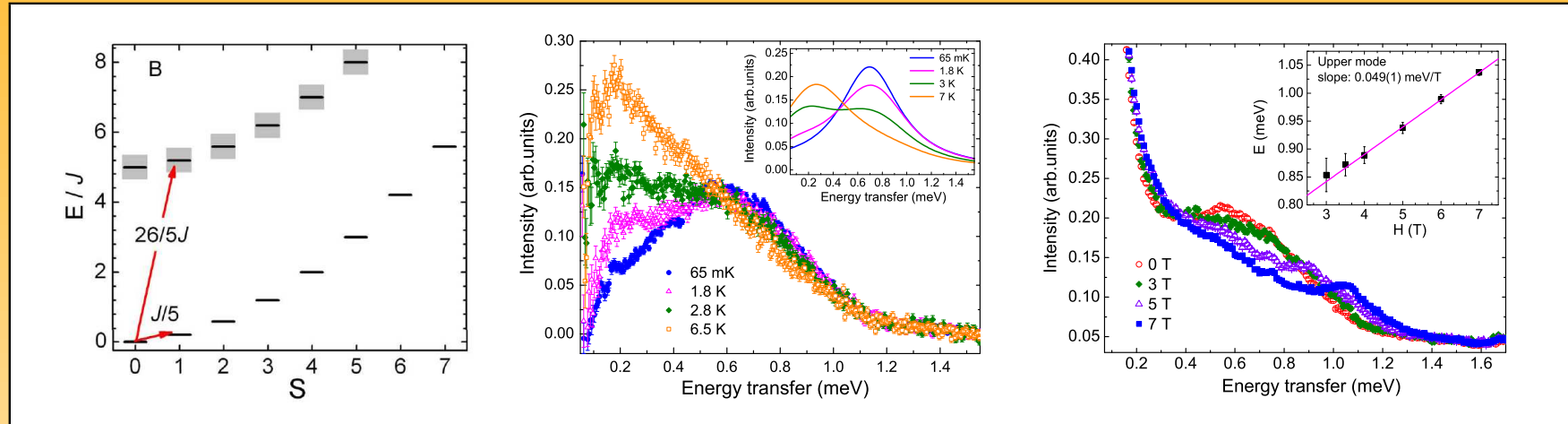
$$\underline{H} = -2J \sum_{(u < v)} \underline{\tilde{s}}(u) \cdot \underline{\tilde{s}}(v) \approx -2J_{\text{eff}} \left[\underline{\tilde{S}}_A \cdot \underline{\tilde{S}}_B + \underline{\tilde{S}}_B \cdot \underline{\tilde{S}}_C + \underline{\tilde{S}}_C \cdot \underline{\tilde{S}}_A \right] = \underline{H}^{\text{eff}}$$

Three sublattice system, classical 120° -ground state;
Good description of low-temperature magnetization.

J. Schnack, M. Luban, R. Modler, Europhys. Lett. **56**, 863 (2001)

Rotational bands in antiferromagnets III

Neutron scattering at $\{\text{Mo}_{72}\text{Fe}_{30}\}$



- INS shows broad peak at band separation, broad width is a sign of frustration, i.e. of the reduced significance of rotational bands.
- Thermal behavior understood; dependence on external field currently investigated.

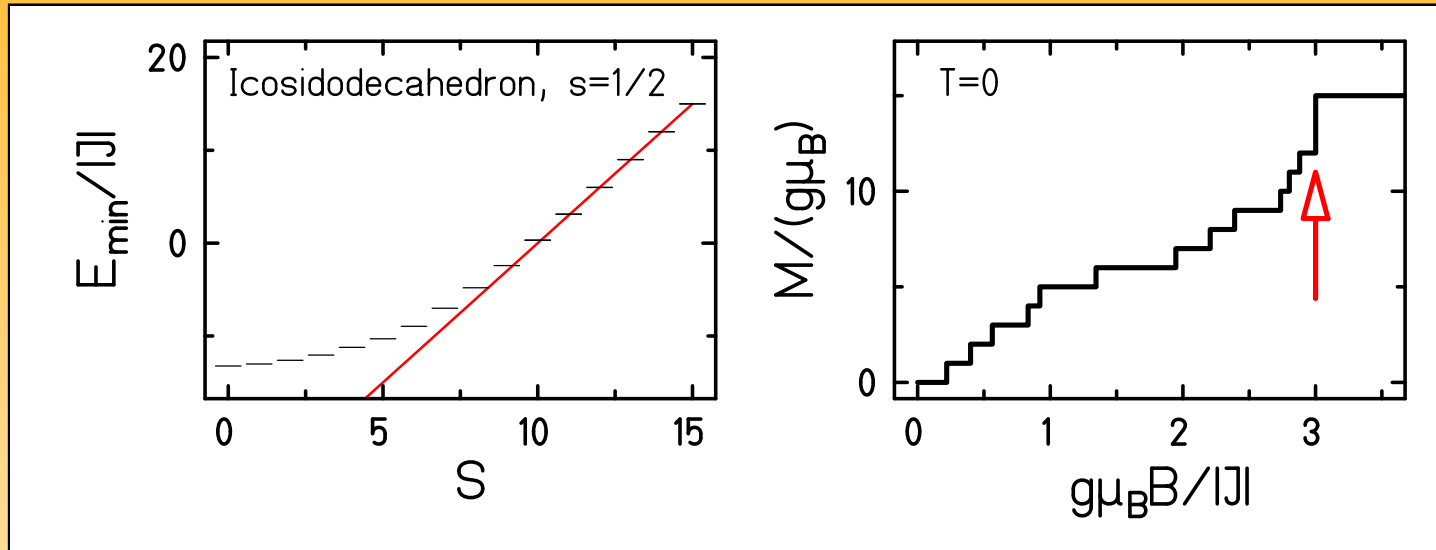
V. O. Garlea, S. E. Nagler, J. L. Zarestky, C. Stassis, D. Vaknin, P. Kögerler, D. F. McMorrow, C. Niedermayer, D. A. Tennant, B. Lake, Y. Qiu, M. Exler, J. Schnack, M. Luban, Phys. Rev. B **73**, 024414 (2006)

Surprise!

The parabola is straight . . .
. . . at least at the top end!

Giant magnetization jumps in frustrated antiferromagnets I

{Mo₇₂Fe₃₀}



- Close look: $E_{\min}(S)$ linear in S for high S instead of being quadratic (1);
- Heisenberg model: property depends only on the structure but not on s (2);
- Alternative formulation: independent localized magnons (3);

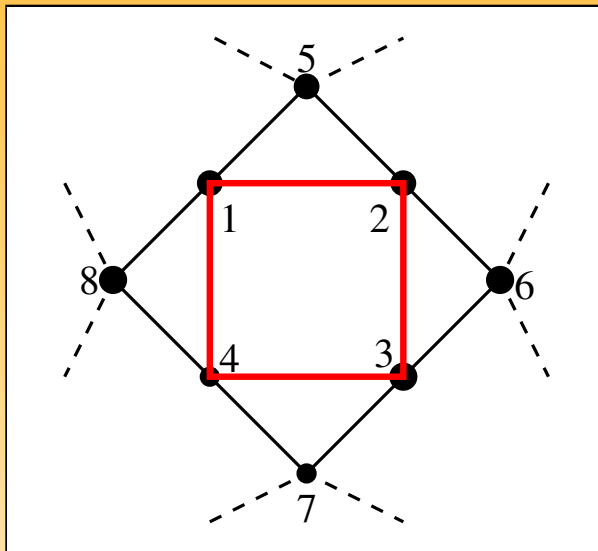
(1) J. Schnack, H.-J. Schmidt, J. Richter, J. Schulenburg, Eur. Phys. J. B **24**, 475 (2001)

(2) H.-J. Schmidt, J. Phys. A: Math. Gen. **35**, 6545 (2002)

(3) J. Schulenburg, A. Honecker, J. Schnack, J. Richter, H.-J. Schmidt, Phys. Rev. Lett. **88**, 167207 (2002)

Giant magnetization jumps in frustrated antiferromagnets II

Localized Magnons



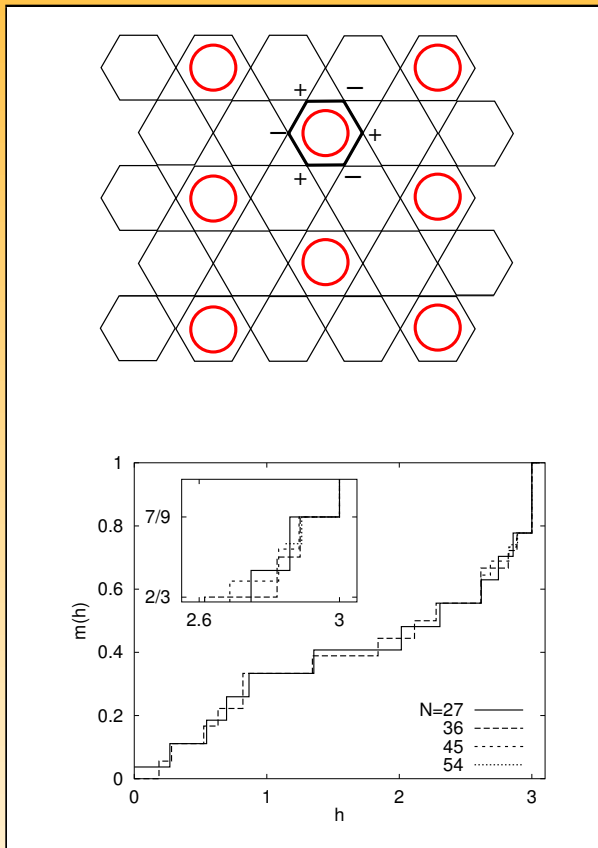
- $|\text{localized magnon}\rangle = \frac{1}{2} (|1\rangle - |2\rangle + |3\rangle - |4\rangle)$
- $|1\rangle = \tilde{s}^-(1) |\uparrow\uparrow\uparrow \dots\rangle$ etc.
- $\tilde{H} |\text{localized magnon}\rangle \propto |\text{localized magnon}\rangle$
- Localized magnon is state of lowest energy (1,2).

- Triangles trap the localized magnon, amplitudes cancel at outer vertices.

(1) J. Schnack, H.-J. Schmidt, J. Richter, J. Schulenburg, Eur. Phys. J. B **24**, 475 (2001)
 (2) H.-J. Schmidt, J. Phys. A: Math. Gen. **35**, 6545 (2002)

Giant magnetization jumps in frustrated antiferromagnets III

Kagome Lattice

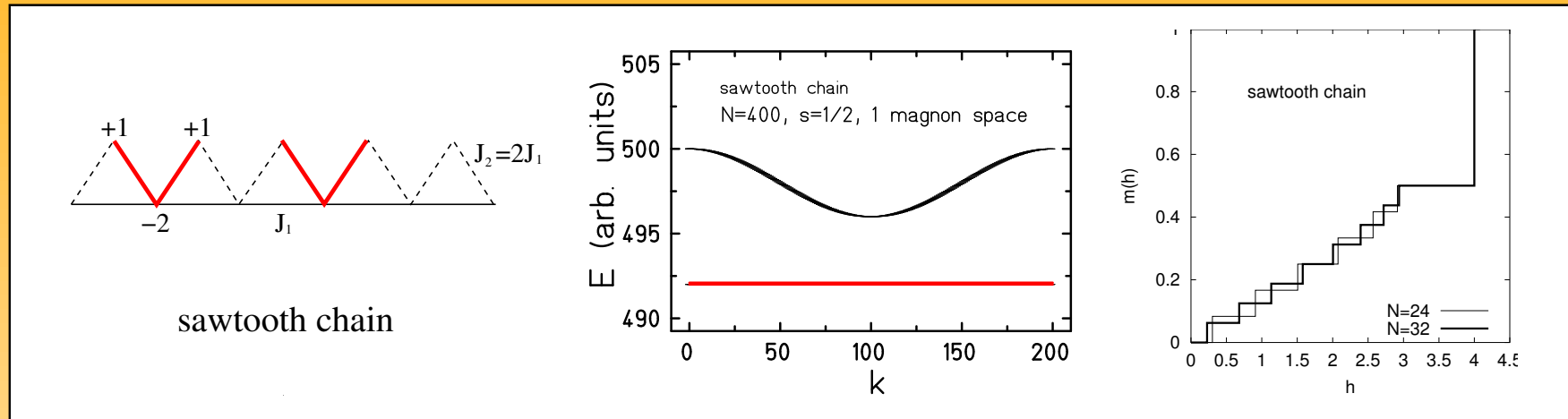


- Non-interacting one-magnon states can be placed on various lattices, e. g. kagome or pyrochlore;
- Each state of n independent magnons is the ground state in the Hilbert subspace with $M = Ns - n$;
Kagome: max. number of indep. magnons is $N/9$;
- Linear dependence of E_{\min} on M
 \Rightarrow magnetization jump;
- Jump is a macroscopic quantum effect!
- A rare example of analytically known many-body states!

J. Schulenburg, A. Honecker, J. Schnack, J. Richter, H.-J. Schmidt, Phys. Rev. Lett. **88**, 167207 (2002)

J. Richter, J. Schulenburg, A. Honecker, J. Schnack, H.-J. Schmidt, J. Phys.: Condens. Matter **16**, S779 (2004)

Condensed matter physics point of view: Flat band



- Flat band of minimal energy in one-magnon space, i. e. high degeneracy of ground state energy in $\mathcal{H}(M = Ns - 1)$;
- Localized magnons can be built from those eigenstates of the translation operator, that belong to the flat band;
- There is a relation to flat band ferromagnetism (H. Tasaki & A. Mielke), compare (1).

(1) A. Honecker, J. Richter, *Condens. Matter Phys.* **8**, 813 (2005)

Enhanced magnetocaloric effect I

Basics

$$\left(\frac{\partial T}{\partial B}\right)_S = -\frac{T}{C} \left(\frac{\partial S}{\partial B}\right)_T$$

(adiabatic temperature change)

- Heating or cooling in a varying magnetic field. Discovered in pure iron by E. Warburg in 1881.
- Typical rates: 0.5 ... 2 K/T.
- Giant magnetocaloric effect: 3 ... 4 K/T e.g. in $\text{Gd}_5(\text{Si}_x\text{Ge}_{1-x})_4$ alloys ($x \leq 0.5$).

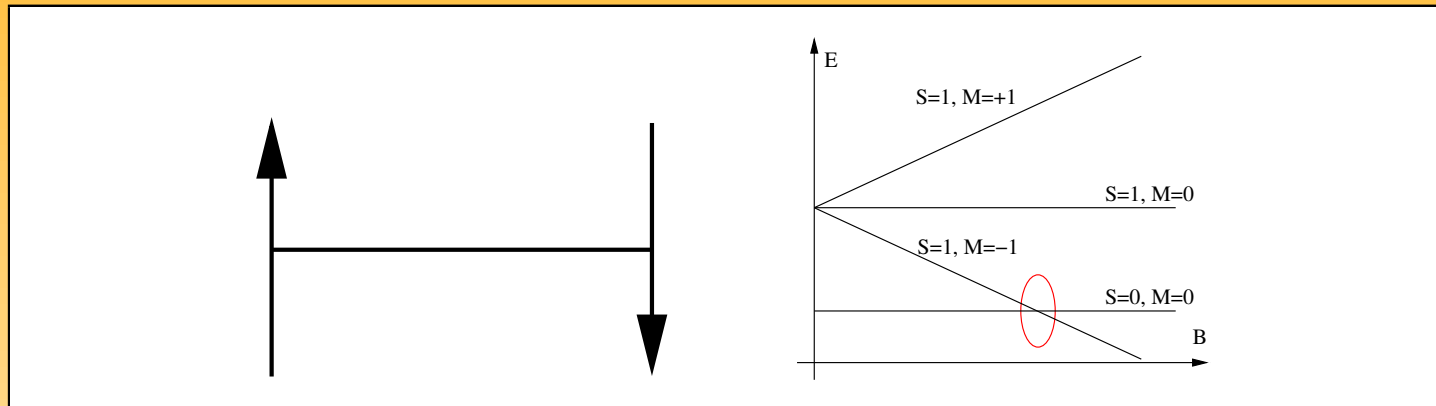
- MCE especially large at large isothermal entropy changes, i.e. at phase transitions (1), close to quantum critical points (2), or due to the condensation of independent magnons (3).

(1) V.K. Pecharsky, K.A. Gschneidner, Jr., A. O. Pecharsky, and A. M. Tishin, Phys. Rev. B **64**, 144406 (2001)

(2) Lijun Zhu, M. Garst, A. Rosch, and Qimiao Si, Phys. Rev. Lett. **91**, 066404 (2003)

(3) M.E. Zhitomirsky, A. Honecker, J. Stat. Mech.: Theor. Exp. **2004**, P07012 (2004)

Enhanced magnetocaloric effect II Simple af $s = 1/2$ dimer

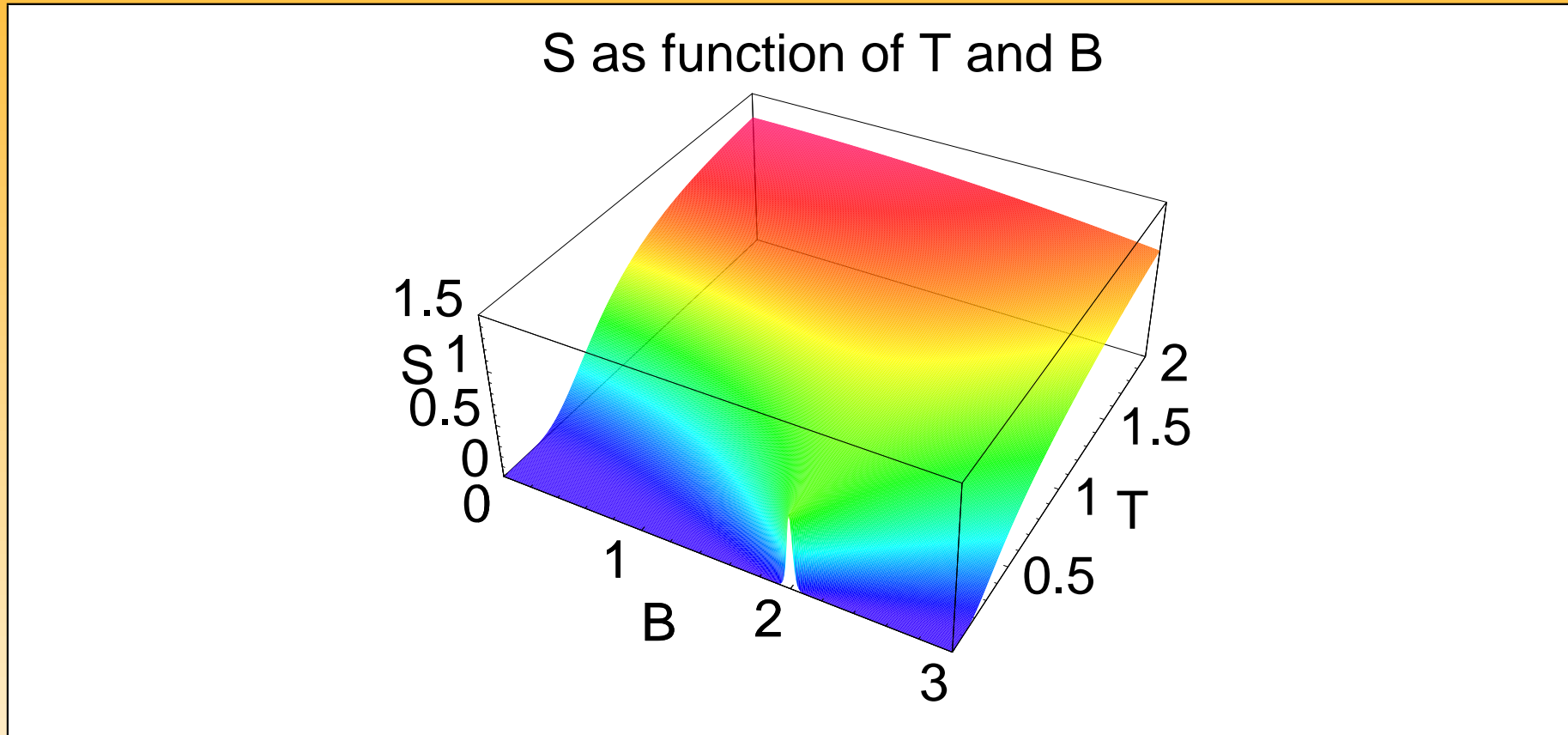


- Singlet-triplet level crossing causes a “quantum phase transition” (1) at $T = 0$ as a function of B .
- $M(T = 0, B)$ and $S(T = 0, B)$ not analytic as function of B .
- $C(T, B)$ varies strongly as function of B for low T .

(1) If you feel the urge to discuss the term “phase transition”, please let’s do it during the coffee break. I will bring Ehrenfest along with me.

Enhanced magnetocaloric effect III

Entropy of an $s = 1/2$ dimer

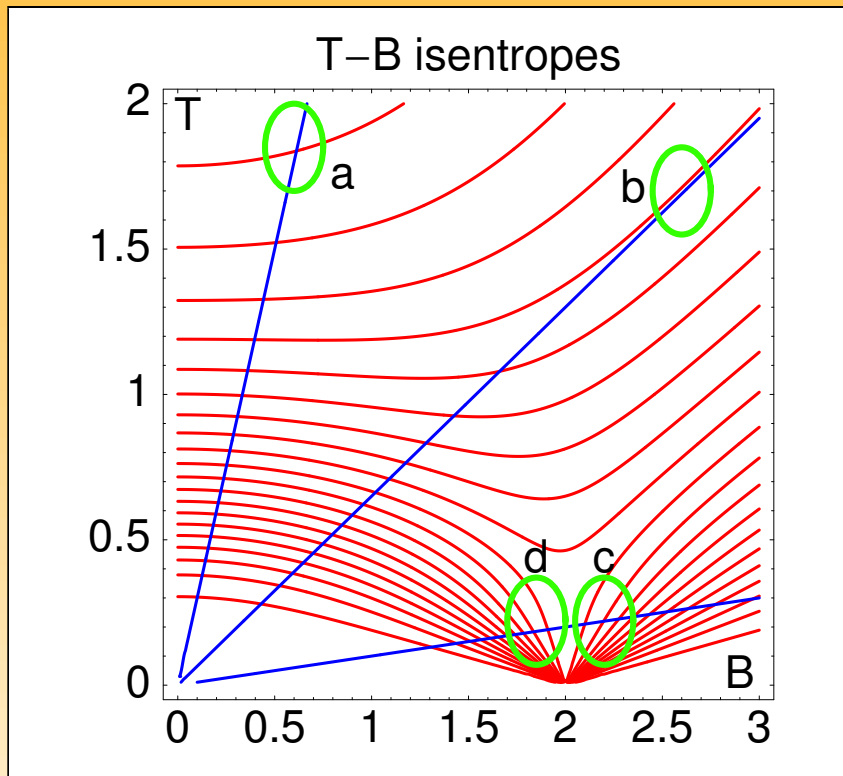


$S(T = 0, B) \neq 0$ at level crossing due to degeneracy

O. Derzhko, J. Richter, Phys. Rev. B **70**, 104415 (2004)

Enhanced magnetocaloric effect IV

Isentropes of af $s = 1/2$ dimer



blue lines: ideal paramagnet, red curves: af dimer

Magnetocaloric effect:

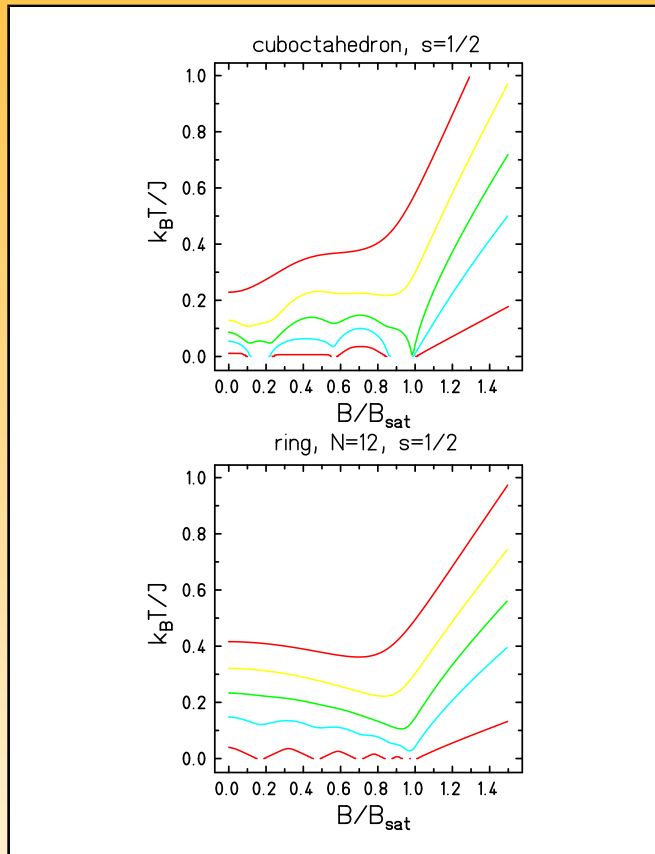
- (a) reduced,
- (b) the same,
- (c) enhanced,
- (d) opposite

when compared to an ideal paramagnet.

Case (d) does not occur for a paramagnet.

Enhanced magnetocaloric effect V

Two molecular spin systems

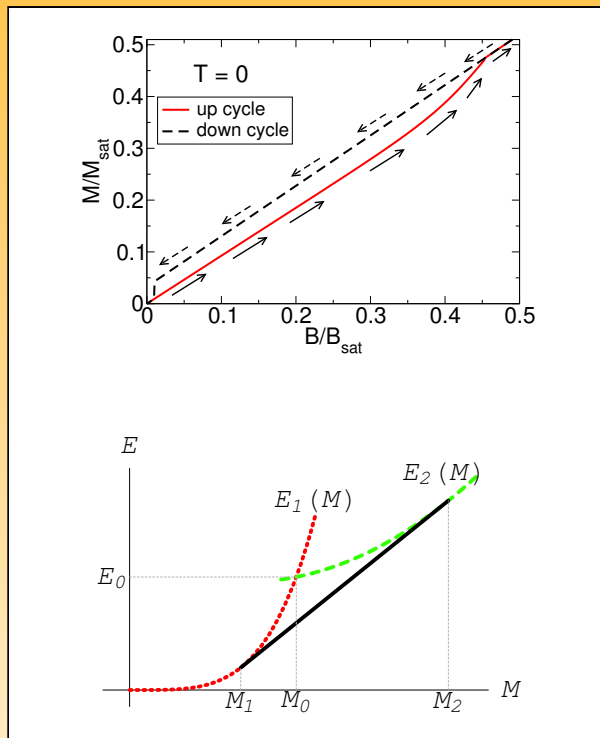


- Graphics: isentropes of the frustrated cuboctahedron and a $N = 12$ ring molecule;
- Cuboctahedron features independent magnons and extraordinarily high jump to saturation;
- Degeneracy and ($T = 0$)–entropy at saturation field higher for the cuboctahedron;
- Adiabatic (de-) magnetization more efficient for the frustrated spin system.

no more time option

Metamagnetic phase transition I

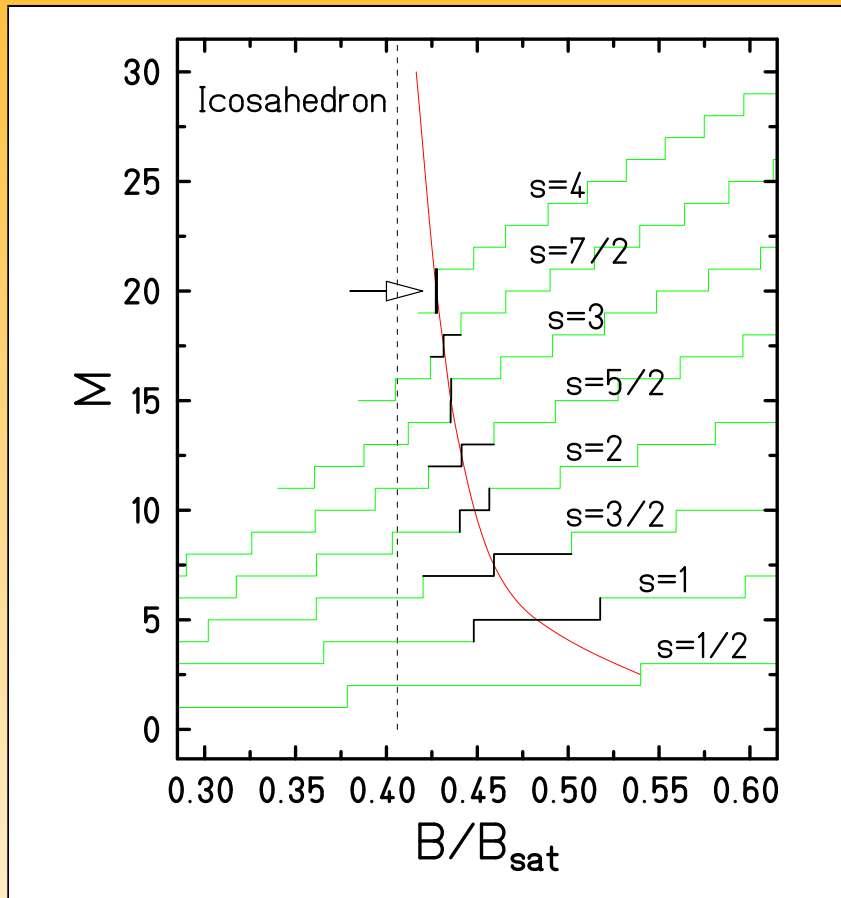
Hysteresis without anisotropy



- Normally hysteretic behavior of Single Molecule Magnets is an outcome of magnetic anisotropy.
- The classical AF Heisenberg Icosahedron exhibits a pronounced hysteresis loop.
- It shows a first order phase transition at $T = 0$ as function of B .
- The minimal energies are realized by two families of spin configurations.
- The overall minimal energy curve is not convex \Rightarrow magnetization jump.

C. Schröder, H.-J. Schmidt, J. Schnack, M. Luban, Phys. Rev. Lett. **94**, 207203 (2005)
 D. Coffey and S.A. Trugman, Phys. Rev. Lett. **69**, 176 (1992).

Metamagnetic phase transition II



- Quantum analog:
Non-convex minimal energy levels
⇒ magnetization jump of $\Delta M > 1$.
- Lanczos diagonalization for various s .
- True jump of $\Delta M = 2$ for $s = 4$.
- Polynomial fit in $1/s$ yields the classically observed transition field.

C. Schröder, H.-J. Schmidt, J. Schnack, M. Luban, Phys. Rev. Lett. **94**, 207203 (2005)

no more time option

Summary

Frustration can lead to exotic behavior.

And, the end is not in sight, . . .

... , however, this talk is at its end!

Thank you very much for your attention.

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Highlights. Tutorials. Who is who. DFG SPP 1137