Universität Bielefeld	Symmetrien in der Physik	Prof. Dr. Jürgen Schnack
Fakultät für Physik	WS 2014/2015	jschnack@uni-bielefeld.de

Problem sheet 15

15.1 Isospin formalism

a. We consider fermions of two kinds which are modeled by the following creation and annihilation operators

$$\underset{\sim}{a}_p^\dagger,\ \underset{\sim}{a}_p,\ \underset{\sim}{a}_n^\dagger,\ \underset{\sim}{a}_n\ . \tag{1}$$

The subscript p labels protons and n labels neutrons.

How do the commutation relations for these operators look like?

b. With the help of the fermionic operators (1) one can define the following operators

$$\underset{\sim}{\tau}_+ = \underset{\sim}{a}_p^\dagger \underset{\sim}{a}_n \quad , \quad \underset{\sim}{\tau}_- = \underset{\sim}{a}_n^\dagger \underset{\sim}{a}_p \tag{2}$$

$$\underset{\sim}{\tau}_0 = \frac{1}{2}\left[\underset{\sim}{a}_p^\dagger \underset{\sim}{a}_p - \underset{\sim}{a}_n^\dagger \underset{\sim}{a}_n\right] \quad , \quad \underset{\sim}{N} = \underset{\sim}{a}_p^\dagger \underset{\sim}{a}_p + \underset{\sim}{a}_n^\dagger \underset{\sim}{a}_n = 1\ . \tag{3}$$

Show that $\underset{\sim}{\tau}_+$, $\underset{\sim}{\tau}_-$ and $\underset{\sim}{\tau}_0$ fulfill the commutator algebra of angular momenta.

15.2 Preparation for the exam

a. What is a symmetry about? How does one model a symmetry? What is the benefit for physics?

b. You should know EVERYTHING about angular momenta. And you should be able to couple angular momenta.

c. Redo all exercises.

d. One-dimensional systems are so easy that they naturally qualify for exams.

e. The Wigner-Eckart theorem is of fundamental importance! The version with Clebsch-Gordan coefficients more insightful. Why?

f. One should know dipole selection rules.

g. You should know a few fundamental things about groups, but I won't ask special stuff (such as center etc.). You should know for instance what conjugate classes are, how many irreducible representations exist and what a *basis function generating machine* is.

h. Please don't forget classical physics, e.g. invariance of the Lagrange function or the electric field of a radially symmetric charge distribution.