Introduction to Computational Physics

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(To be discussed on Friday, 27.6. and Monday, 30.6.)

EXERCISE 7.1: **Gauss-Legendre Integration**
Employing the Gauss-Legendre implementation of the Numerical recipes `gauleg`, again calculate the area of the unit circle by computing

$$I \;=\; 2\int\limits_{-1}^{1} \sqrt{1-x^2}\,dx$$

for different number of integration intervals $n = 2^m$, $m = 1,\ldots,10$. Compare the achieved precision with the Romberg algorithm of exercise 6.1, and with the numerical integration routine `NIntegrate` implemented in Mathematica.

EXERCISE 7.2: **Gauss-Laguerre Integration**
Implement the algorithm for the Gauss-Laguerre quadrature formula into Mathematica, and again calculate $\Gamma(1/2)$, where Eulers Γ-function is defined by

$$\Gamma(z) \;=\; \int\limits_{0}^{\infty} e^{-t}t^{z-1}dt\,,$$

for various numbers of Gauss points. Compare your result with the corresponding algorithm of the Numerical Recipes `gaulag`, and discuss your findings.

EXERCISE 7.3: **Black Body Radiation**
With your implementation of the Gauss-Laguerre quadrature formula, calculate the integral

$$\int\limits_{0}^{\infty} \frac{x^3}{e^x-1}\,dx\,,$$

which arises in the context of Planck's black body radiation, for different number of Gauss points. Compare with the exact result $\pi^4/15$.

EXERCISE 7.4: **Integration of Data**
Integrate the data set `spline.dat` of exercise 4.1 using on the one hand the trapezoidal rule for half the data points, the full data set, and with the Gauss-Legendre algorithm for various numbers of Gauss points, employing in addition the cubic spline interpolation to calculate the function values at the roots of the Legendre polynomial. Compare the results for the different approaches.