Introduction to Computational Physics

Matthias Jamin

(To be discussed on Friday, 30.5. and Monday, 2.6.)

EXERCISE 3.1: **Polynomial Interpolation**
From the given four values of the Bessel function $J_0(z)$:

$$\begin{aligned} J_0(4) &= -0.3971498098638473722866\,, & J_0(5) &= -0.1775967713143383043474\,, \\ J_0(6) &= 0.1506452572509969316623\,, & J_0(7) &= 0.3000792705195555966503\,, \end{aligned}$$

calculate $J_0(5.5)$ by polynomial interpolation with the Numerical Recipes algorithm given on the CP web-page. (Try to reproduce how this algorithm works!) Compute the exact result from the series expansion

$$J_0(z) \;=\; \sum_{k=0}^{\infty} \frac{(-1)^k}{(k!)^2} \left(\frac{z}{2}\right)^{2k} ,$$

and check the precision of the interpolation. Compute the root of $J_0(z)$ closest to $z = 5.5$, and determine the value of the interpolating polynomial at the exact location of the root. Employ the two additional points

$$J_0(3) = -0.2600519549019334376242\,, \qquad J_0(8) = 0.1716508071375539060909\,,$$

and investigate by how much the interpolation for $J_0(5.5)$ improves. Also compare to the linear interpolation just using the points $J_0(5)$ and $J_0(6)$.

EXERCISE 3.2: **Tridiagonal Linear Systems**
Show that the solution for the tridiagonal linear system

$$\begin{pmatrix} b_1 & c_1 & & & & \\ a_2 & b_2 & c_2 & & & \\ & a_3 & b_3 & c_3 & & \\ & & & \ddots & & \\ & & & a_{n-1} & b_{n-1} & c_{n-1} \\ & & & & a_n & b_n \end{pmatrix} \begin{pmatrix} x_1 \\ x_2 \\ x_3 \\ \vdots \\ x_{n-1} \\ x_n \end{pmatrix} = \begin{pmatrix} r_1 \\ r_2 \\ r_3 \\ \vdots \\ r_{n-1} \\ r_n \end{pmatrix}$$

is given by

$$x_{n-j} \;=\; \frac{1}{\beta_{n-j}} \left(\rho_{n-j} - c_{n-j}\, x_{n-j+1}\right), \qquad j \;=\; 1,\ldots,n-1$$

where

$$\beta_j \;=\; b_j - \frac{a_j}{\beta_{j-1}}\, c_{j-1} \qquad \text{and} \qquad \rho_j \;=\; r_j - \frac{a_j}{\beta_{j-1}}\, \rho_{j-1}\,, \qquad j \;=\; 2,\ldots,n$$

and the initial conditions are $x_n \;=\; \rho_n/\beta_n$ as well as $\beta_1 = b_1$ and $\rho_1 = r_1$.
Implement the solution in your numerical language of choice.