Introduction to Computational Physics

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(To be discussed on Friday, 4.7. and Monday, 7.7.)

EXERCISE 8.1: **Eulers Algorithms**
Implement the simple, modified and improved Euler algorithms into Mathematica. With these implementations, solve the differential equation

$$y' = y^2 + 1$$

in the interval $[0,1]$, with the initial condition $y(0) = 0$, and two step sizes $h = 0.10$ and $h = 0.05$. Compare the precision of the various results with each other and with the exact solution.

EXERCISE 8.2: **Conserved Quantities**
A mass point coupled to a spring is described by the differential equation

$$F = m\frac{dv}{dt} = -kx\,.$$

Calculate the position and velocity of the mass point with the simple and modified Euler algorithms in the interval $t = 0, \ldots, 6$ and the initial condition $v = 1$ at $t = 0$ and $x = 0$. For simplicity set $m = k = 1$ and use $\delta = 0.1$ as the step size. Investigate how well the energy $E = mv^2/2 + kx^2/2$ is conserved for both solutions.

Solve the same system with an algorithm that guarantees the conservation of the energy, and compare the accuracy of the solutions with each other and with the exact result.

EXERCISE 8.3: **Runge-Kutta Algorithm**
Implement the Runge-Kutta algorithm of 4th order with adaptive step size into Mathematica and solve the differential equation for a projectile in one dimension including friction

$$F = m\frac{dv}{dt} = mg - kv^2\,.$$

Use the values $m = 10\,\mathrm{g}$ and $k = 10^{-4}\,\mathrm{kg/m}$ as mass and friction coefficient respectively. Calculate the velocity of the mass point in the interval $0\,\mathrm{s} < t < 10\,\mathrm{s}$ with a precision of 4 digits if the particle is at rest for $t = 0$ at $x = 0$.

Compare your result with the exact solution if the friction is neglected and with the numerical solution obtained the the Mathematica function `NDSolve[]`.