Introduction to Computational Physics

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(To be discussed on Friday, 20.6. and Monday, 23.6.)

EXERCISE 6.1: **Romberg Integration**

Implement the algorithm for Romberg integration into Mathematica.

With that algorithm calculate the area of the unit circle by computing:

$$I \;=\; 2\int\limits_{-1}^{1} \sqrt{1-x^2}\,dx\,.$$

Investigate the convergence of the algorithm by calculating the integral for different number of integration intervals $n = 2^m$, $m = 0,\ldots,10$, and different Romberg order k up to $k = 3$, where $k = 0$ corresponds to the extended trapezoidal integration rule.

Calculate the ratio $R_m \equiv (S_{m,0} - S_{m-1,0})/(S_{m+1,0} - S_{m,0})$ for different m, and discuss the influence of that ratio for the convergence of the Romberg algorithm. (Hint: the expectation for this ratio would be $R_m \approx 4$.)

Calculate the same integral after performing the substitution $x = \cos\theta$. Again investigate the convergence and discuss your findings.

EXERCISE 6.2: **Improper integrals**

Calculate $\Gamma(1/2)$ to a precision of 10^{-12}, where Eulers Γ-function is defined by

$$\Gamma(z) \;=\; \int\limits_{0}^{\infty} e^{-t} t^{z-1} dt\,,$$

with the Romberg implementation of exercise 6.1, after applying a suitable substitution.

EXERCISE 6.3: **Elliptic Integrals**

The time T of one period for a mathematical pendulum of length l is found to be

$$T \;=\; 4\sqrt{\frac{l}{g}}\,K\!\left(\sin\frac{\theta_0}{2}\right),$$

where $K(z)$ is the elliptic integral of the first kind:

$$K(z) \;=\; \int\limits_{0}^{\pi/2} \frac{d\varphi}{\sqrt{1-z^2\sin^2\varphi}}\,.$$

Compute $K(z)$ with the Romberg implementation of exercise 6.1 for $0 < \theta_0 < \pi$ to a precision of 10^{-12}. Compare your result with the Mathematica implementation `EllipticK[`z^2`]`.