

Nonequilibration, synchronization, and time crystals in isotropic Heisenberg models

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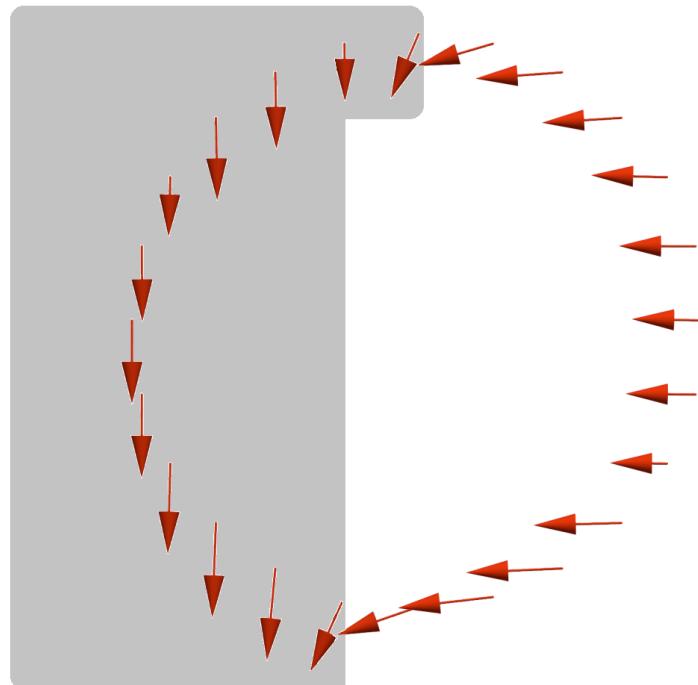
Context

Investigation of closed quantum systems
under unitary time-evolution:
What happens?

Relaxation, equilibration, thermalization?

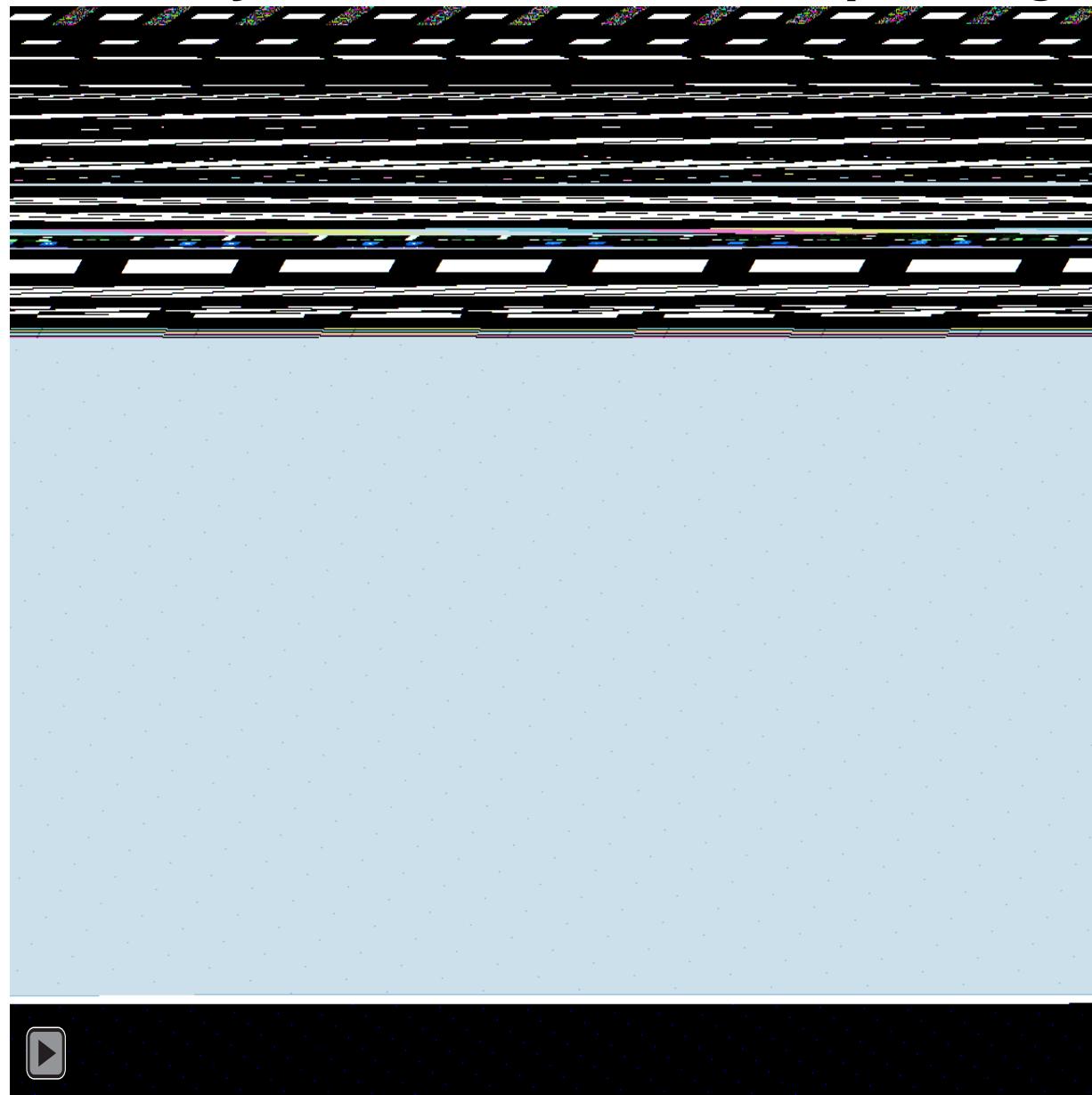
...

Guess what happens!



- System of N spins (e.g. $s = 1/2$);
- Heisenberg Hamiltonian and magnetic field along z -direction;
- Unitary time evolution of many-body state;
- Initial state, e.g. product state or pre-quench state, with single spin expectation values in x - y -plane;
- **What do you expect?**

Unitary time evolution of a spin ring



Synchronization – physical explanation

- The case where all spins are equivalent, e.g., due to some translational symmetry, can be easily rationalized.
- The total spin \vec{S} and its z -component S^z are conserved.
The total spin vector precesses.
- If one assumes equilibration to a state compatible with the conserved quantities, then all spins need to have the same vector expectation value $\langle \vec{S} \rangle / N$.
Thus synchronization occurs.
- Some details: equilibration means that equivalent observables assume the same value; for almost all late times; in a narrow band around some mean value.

P. Vorndamme, H.-J. Schmidt, Chr. Schröder, J. Schnack, *Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions*, New J. Phys. **23**, 083038 (2021)

Synchronization – Technical details I

Heisenberg model: SU(2) invariant; simultaneous eigenstates $|\nu\rangle$ with

$$(\tilde{H}_0 + \omega_L \tilde{S}^z) |\nu\rangle = (E_\nu^{(0)} + \omega_L M_\nu) |\nu\rangle, \quad \tilde{S}^2 |\nu\rangle = S_\nu(S_\nu + 1) |\nu\rangle, \quad \tilde{S}^z |\nu\rangle = M_\nu |\nu\rangle$$

For a general time evolution of an expectation value we obtain

$$\begin{aligned} \langle \tilde{A} \rangle_t &:= \text{tr}\{\tilde{\rho}(t)\tilde{A}\} = \sum_{\mu,\nu} \rho_{\mu\nu} A_{\nu\mu} e^{i(E_\nu^0 - E_\mu^0 + [M_\nu - M_\mu]\omega_L)t} = \sum_{\Delta M} f_{\Delta M}(t) e^{i\Delta M \omega_L t} \\ &\Rightarrow \sum_{\Delta M} \bar{f}_{\Delta M} e^{i\Delta M \omega_L t} \end{aligned}$$

Equilibration of Fourier coefficients $\bar{f}_{\Delta M}$ can be shown under rather general assumptions (1).

(1) P. Reimann, P. Vorndamme, J. Schnack, *Non-equilibration, synchronization, and time crystals in isotropic Heisenberg models*, Phys. Rev. Research **5**, 043040 (2023)

Synchronization – Technical details II

Fourier coefficients $\bar{f}_{\Delta M}$

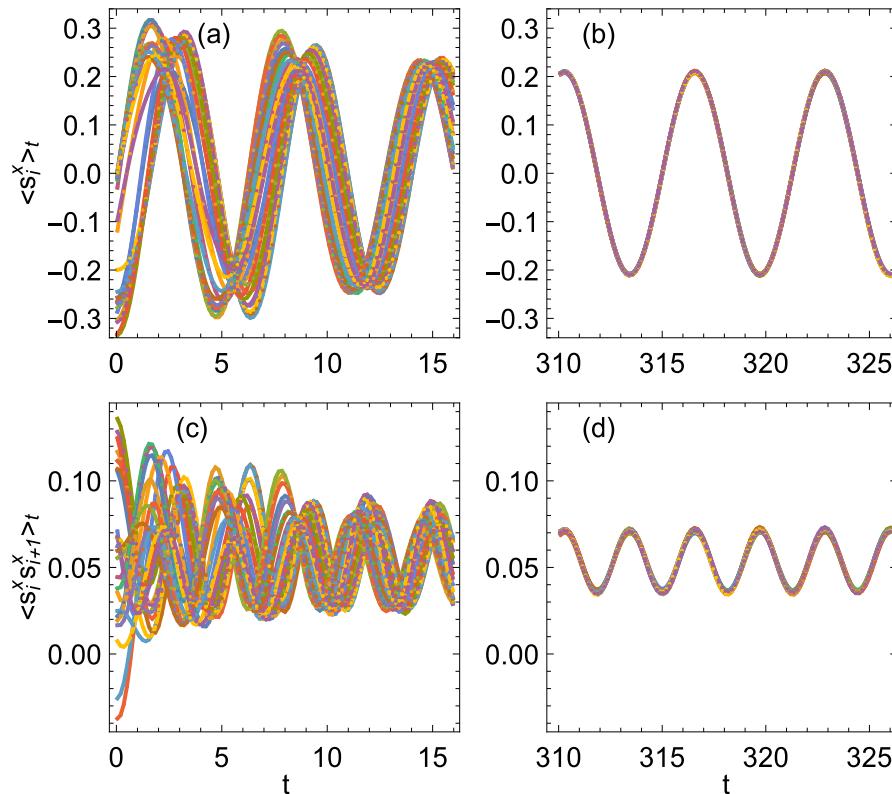
$$\bar{f}_{\Delta M} = \sum'_{\mu, \nu} \rho_{\mu\nu} A_{\nu\mu} \text{ with } E_\mu = E_\nu \text{ & } \Delta M = M_\nu - M_\mu$$

Operator \hat{A} determines through its matrix elements $A_{\nu\mu}$ which multiples ΔM of the Larmor frequency ω_L contribute (selection rules).

This is not just a rotating frame!

P. Reimann, P. Vorndamme, J. Schnack, *Non-equilibration, synchronization, and time crystals in isotropic Heisenberg models*, Phys. Rev. Research 5, 043040 (2023)

Synchronization – homogeneous spin ring, $N = 24$



- $\tilde{s}_i^x \propto (\tilde{s}_i^+ + \tilde{s}_i^-)$
thus $|\omega| = \omega_L$
- $\tilde{s}_i^x \tilde{s}_{i+1}^x \propto (\tilde{s}_i^+ \tilde{s}_{i+1}^+ + \tilde{s}_i^+ \tilde{s}_{i+1}^- + \tilde{s}_i^- \tilde{s}_{i+1}^+ + \tilde{s}_i^- \tilde{s}_{i+1}^-)$
thus $|\omega| = 2\omega_L$

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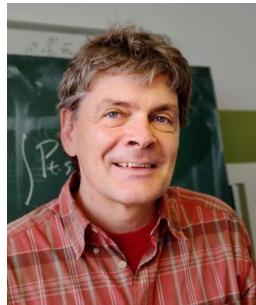
Summary

Heisenberg systems (SU(2) symmetry) exhibit robust synchronization of expectation values of equivalent operators under unitary time evolution.

This holds independently of N , s , geometry, and initial state.

P. Reimann, P. Vorndamme, J. Schnack, *Non-equilibration, synchronization, and time crystals in isotropic Heisenberg models*, Phys. Rev. Research **5**, 043040 (2023)

Thank you very much for your attention.



Peter Reimann



Patrick Vorndamme



Jürgen Schnack

The end.

Observation of phase synchronization and alignment ...



P. Reimann, P. Vorndamme, J. Schnack, *Non-equilibration, synchronization, and time crystals in isotropic Heisenberg models*, Phys. Rev. Research **5** (2023) 043040