Non-equilibration and synchronization in isotropic Heisenberg models

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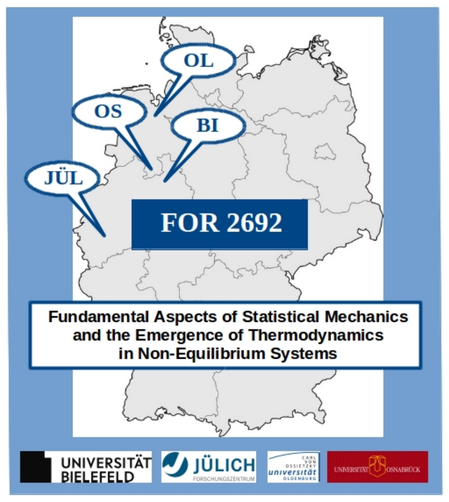

Abstract

Isotropic, but otherwise largely arbitrary Heisenberg models in the presence of a homogeneous magnetic field are considered, including various integrable, non-integrable, as well as disordered examples, and not necessarily restricted to one dimension or short-range interactions. Taking for granted that the non-equilibrium initial condition and the spectrum of the field-free model satisfy some very weak requirements, expectation values of generic observables are analytically shown to exhibit permanent long-time oscillations, thus ruling out equilibration. If the model (but not necessarily the initial condition) is translationally invariant, the long-time oscillations are moreover shown to exhibit synchronization in the long run, meaning that they are invariant under arbitrary translations of the observable. Analogous long-time oscillations are also recovered for temporal correlation functions when the system is already at thermal equilibrium from the outset, thus realizing a so-called time crystal [1].

General framework

We consider a Heisenberg model on an arbitrary (not necessarily one-dimensional) lattice, whose sites are labeled by i. We denote by Λ the set of all possible lattice sites, and by κ their total number:

$$H \;:=\; H_0 + h\,S^z\,,\, H_0 := \sum_{i,j\in\Lambda} J_{ij}\,\vec{s}_i\cdot\vec{s}_j\;. \tag{1}$$

Due to SU(2) symmetry the eigenvectors of H_0 can be chosen as simultaneously eigenvectors of S^z and $\vec{S}^2 := (S^x)^2+(S^y)^2+(S^z)^2$, i.e.,

$$H_0|n,l\rangle \;=\; E_n^0\,|n,l\rangle\;,\; S^z|n,l\rangle = l\,|n,l\rangle\;,\; \vec{S}^2|n,l\rangle = L_n(L_n+1)\,|n,l\rangle\;,\; L_n>0\;. \tag{2}$$

It follows that

$$H\,|n,l\rangle \;=\; E_{nl}\,|n,l\rangle\;,\; E_{nl} := E_n^0 + l\,h\;. \tag{3}$$

With respect to any pure or mixed initial state $\rho(0)$, the expectation value of any observable A at time t is given by

$$\langle A\rangle_t := \mathrm{Tr}\{\rho(t)A\} = \sum_{mnkl} \rho_{mn}^{k,l} A_{nm}^{l,k}\, e^{i(E_n^0-E_m^0+[l-k]h)t}\;, \tag{4}$$

where the sum is tacitly restricted to indices m,n,k,l within their admitted range, and

$$\rho_{mn}^{k,l} \;:=\; \langle m,k|\rho(0)|n,l\rangle\;,\; A_{nm}^{l,k} := \langle n,l|A|m,k\rangle\;. \tag{5}$$

Going over from the summation index l to $\nu := l-k$ yields

$$\langle A\rangle_t \;=\; \sum_{\nu} f_\nu(t)\,e^{i\nu ht}\;,\; f_\nu(t) := \sum_{mn} e^{i(E_n^0-E_m^0)t} \sum_k \rho_{mn}^{k,k+\nu} A_{nm}^{k+\nu,k}\;, \tag{6}$$

with $f_{-\nu}(t) = f_\nu^*(t)$, hence (6) could be rewritten as a purely real Fourier series. Since the eigenvectors $|n,l\rangle$ are independent of h, the same property is inherited by the matrix elements in (5), and finally by the functions $f_\nu(t)$ in (6). In other words, the only h-dependence arises via the exponential factors on the left-hand side of (6).

Main results

For sufficiently large systems, the expectation values in (6) can be approximated very well by

$$\langle A\rangle_t \rightsquigarrow \mathcal{A}_t := \sum_{\nu} \bar{f}_\nu\, e^{i\nu ht} \quad,\; \bar{f}_\nu := {\sum_{mnk}}' \rho_{mn}^{k,k+\nu} A_{nm}^{k+\nu,k} \tag{7}$$

for the vast majority of all sufficiently late times t, where the prime symbol indicates that the summation is restricted to indices m and n with the property $E_m^0 = E_n^0$. In the generic case that all energies E_n^0 are pairwise different, this boils down to

$$\bar{f}_\nu = \sum_{nk} \rho_{nn}^{k,k+\nu} A_{nn}^{k+\nu,k}\;. \tag{8}$$

$\langle n,l|\rho(0)|n,l\rangle$ can be identified with the population of the energy eigenstate $|n,l\rangle$ by the initial state $\rho(0)$. Likewise,

$$p_{\max} := \max_{n,l}\langle n,l|\rho(0)|n,l\rangle \propto \exp\{-\mathcal{O}(\kappa)\} \tag{9}$$

thus amounts to the maximal level population; it is independent of h.

For pairs of indices (m,n) with $E_m^0 \neq E_n^0$, γ_{mn}^0 denotes the degeneracy of the energy gap $E_m^0 - E_n^0$. Then, the maximal energy gap degeneracy is defined as $\gamma^0 := \max_{m,n}\gamma_{mn}^0$. γ_{mn}^0 and γ^0 refer, as indicated by the superscript "0", to the unperturbed system, and as such are independent of h.

Indicating the temporal average over an interval $[0,T]$ by the symbol $\langle\cdot\rangle_T$, the mean square deviation of the true expectation values from the auxiliary function obeys for all sufficiently large times T the inequality

$$\Big\langle\big[\langle A\rangle_t - \mathcal{A}_t\big]^2\Big\rangle_T \leq \gamma^0\,(2s\kappa+1)^2\,\Delta_A^2\,p_{\max}\;, \tag{10}$$

where s is the single-spin quantum number and κ the size of the system. Furthermore, Δ_A is the measurement range of the observable A, i.e., the difference between the largest and smallest possible measurement outcomes, or equivalently, eigenvalues of A.

Numerical Examples

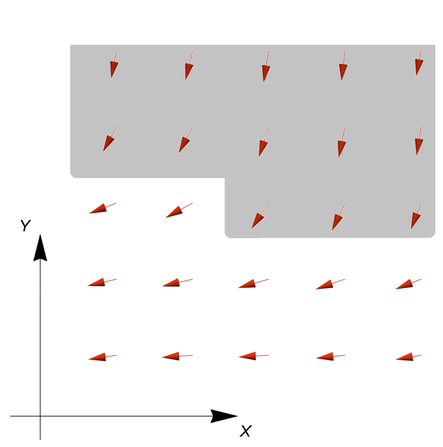

Figure 1: Red arrows: Visualization of the projections to the (x,y)-plane of the expectation values of the local spin vector operators $\vec{s}_i$ with respect to the initial state $\rho(0)$ for a square lattice model. The grey and white regions indicate our choice of two regions Λ_1 and Λ_2. As initial condition $\rho(0)$ we choose a pure state of the form $\rho(0) = |\psi\rangle\langle\psi|$ with $|\psi\rangle \propto e^{-\frac{\beta}{2}\tilde{H}}|\phi\rangle$ where $|\phi\rangle$ is a normalized random vector and $\tilde{H}$ differs by different directions of an applied field in regions Λ_1 and Λ_2.

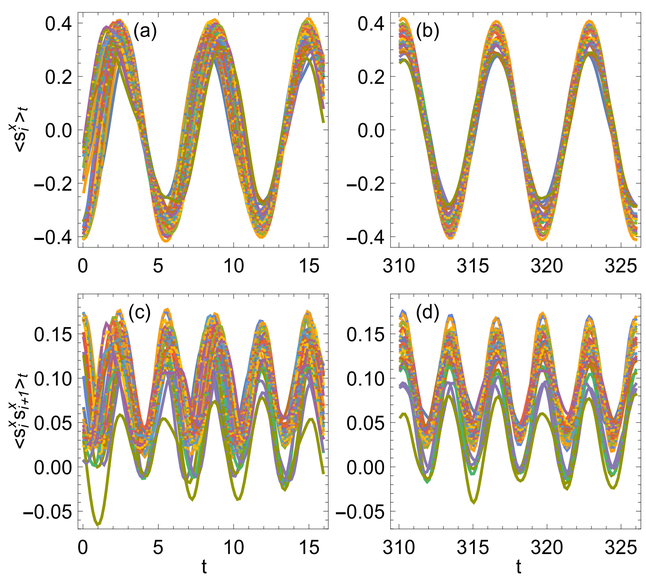

Figure 2: (a) and (b): Expectation values of the local observables s_i^x for early times (a) as well as for late times (b) by numerically solving the spin model for a 5×5 square lattice, **open boundary conditions,** and couplings $J=-2$. (c) and (d): Same, but for the observables $s_i^x s_{i+1}^x$.

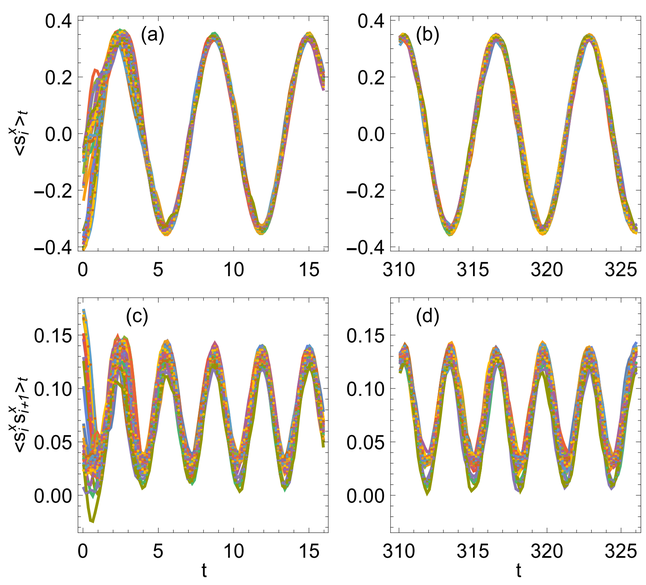

Figure 3: Same as in Fig. 2, but now for **periodic boundary conditions** in the 5×5 square lattice model with exactly the same the initial condition. Since all spins are equivalent, near perfect synchronization and periodicity is observed, compare also [2].

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References

[1] Peter Reimann, Patrick Vorndamme, Jürgen Schnack, Non-equilibration, synchronization, and time crystals in isotropic Heisenberg models, arXiv:2303.11148

[2] Patrick Vorndamme, Heinz-Jürgen Schmidt, Christian Schröder, and Jürgen Schnack. Observation of phase synchronization and alignment during free induction decay of quantum spins with Heisenberg interactions. *New J. Phys.* 23, 083038 (2021), doi: 10.1088/1367-2630/ac18df.