Theory pictures of nuclei and nuclear reactions

GSI student program 1997

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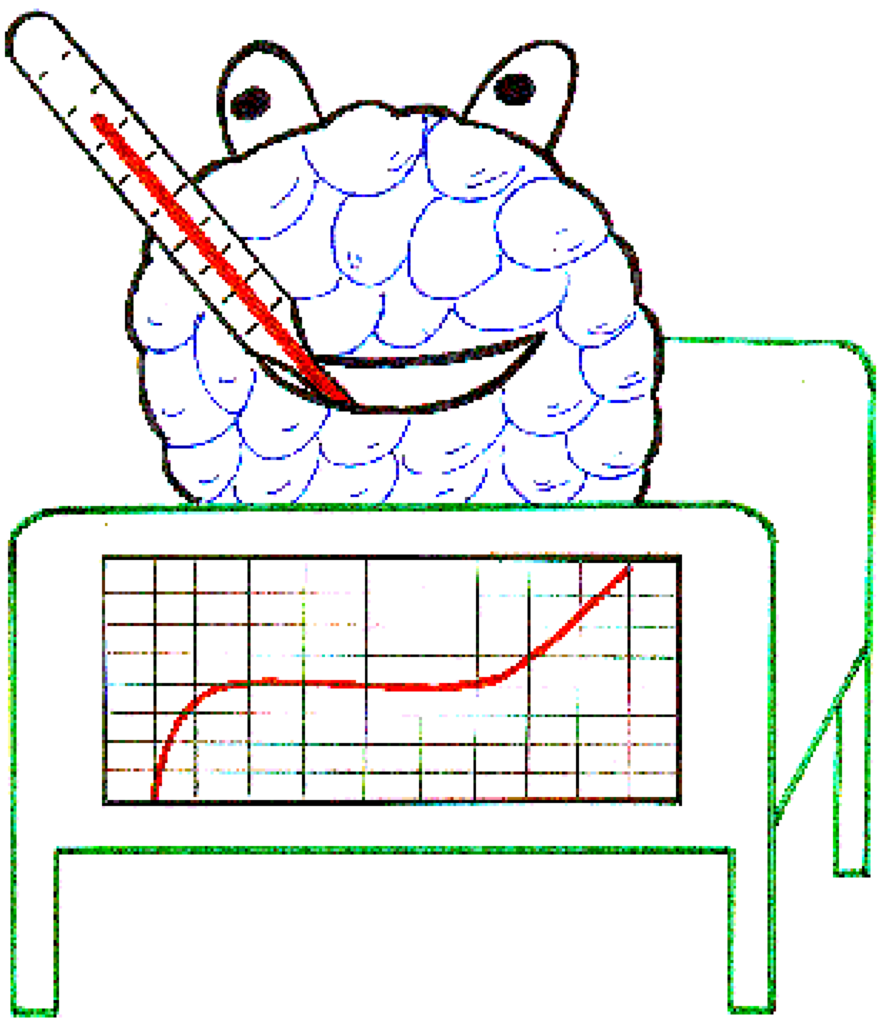


Questions of interest in nuclear many–body physics

1. ground states and properties of nuclei, nuclei far from stability (large proton to neutron ratio)

2. small exitations, collective phenomena like vibrations

3. reactions:

   - fusion e.g. production of superheavies
   - deeply inelastic (dissipative) reactions
   - fragmentation ($E^* \approx 5A\mathrm{MeV} \ldots 10A\mathrm{MeV} \Longleftrightarrow E_{lab} \approx 35A\mathrm{MeV} \ldots$ several $100A\mathrm{MeV}$); nuclear liquid–gas phase–transition
   - relativistic and ultrarelativistic reactions; particle production, QGP

Some typical objects and methods

- problems: many–body system, unknown nuclear interaction, relativity, non-equilibrium; observation almost only in reactions

- single–particle states, many–body states, symmetries, second quantization: creation and annihilation operators

- densities, observables like energy, charge radius, moments

- Schrödinger equation and approximations:
  Ritz variational principle, time–dependent variational principle, (time–dependent) Hartree–Fock, Fermionic Molecular Dynamics

- thermodynamic quantities like temperature, partition function, phase transitions

Many–body states

A–body states are states in the A–body Hilbert space. The easiest A–body states are just products of single–particle states with the correct symmetry.

1. product states of A single–particle states for distinguishable particles

$$| \Psi \rangle = | \psi_1 \rangle \otimes | \psi_2 \rangle \otimes \cdots \otimes | \psi_A \rangle$$

2. antisymmetric product states (Slater determinants) of A single–particle states for identical fermions

$$\begin{aligned} | \Psi \rangle &= \underset{\sim}{A} (| \psi_1 \rangle \otimes | \psi_2 \rangle \otimes \cdots \otimes | \psi_A \rangle) \\ &= \sum_\pi \mathrm{sgn}(\pi) \, | \psi_{\pi(1)} \rangle \otimes \cdots \otimes | \psi_{\pi(A)} \rangle \\ | \Psi; (i,j) \rangle &= - | \Psi; (j,i) \rangle \quad \text{Pauli principle} \end{aligned}$$

3. symmetric product states of A single–particle states for identical bosons

$$\begin{aligned} | \Psi \rangle &= \underset{\sim}{S} (| \psi_1 \rangle \otimes | \psi_2 \rangle \otimes \cdots \otimes | \psi_A \rangle) \\ &= \sum_\pi | \psi_{\pi(1)} \rangle \otimes \cdots \otimes | \psi_{\pi(A)} \rangle \\ | \Psi; (i,j) \rangle &= | \Psi; (j,i) \rangle \end{aligned}$$

In order to go beyond this product states one for instance uses superpositions of product states.

Single–particle states

Nuclei are built of protons and neutrons which both are fermions with spin $\frac{1}{2}$. Since protons and neutrons have approximately the same mass and their interaction is symmetric they might be understood as one particle, the nucleon, which has an additional quantum number, the isospin. The isospin is an operator like the spin, the nucleon is of isospin $\frac{1}{2}$.

$$|\,\psi_1\,\rangle \quad = \quad |\,\text{spatial}\,\rangle \otimes \;|\,\text{spin}\,\rangle \otimes \;|\,\text{isospin}\,\rangle$$

$$|\,\text{spatial}\,\rangle = \sum_n c_n \;|\,n\,\rangle$$

e.g. harmonic oscillator, plane waves, coherent states

$$|\,\text{spin}\,\rangle = c_s(\frac{1}{2}\,)\;|\,m_s = \frac{1}{2}\;\rangle + c_s(-\frac{1}{2}\,)\;|\,m_s = -\frac{1}{2}\;\rangle$$

$$|\,\text{isospin}\,\rangle = c_t(\frac{1}{2}\,)\;|\,m_t = \frac{1}{2}\;\rangle + c_t(-\frac{1}{2}\,)\;|\,m_t = -\frac{1}{2}\;\rangle$$

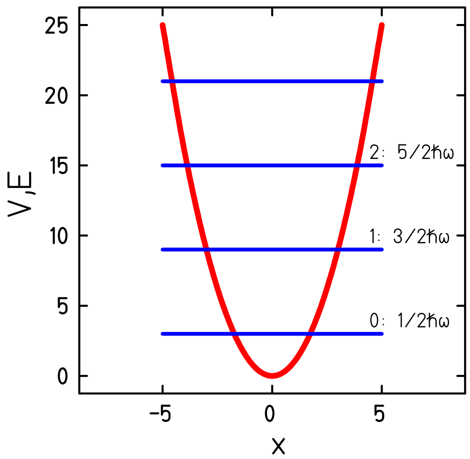
$m_t = \dfrac{1}{2}$ $\Leftrightarrow$ proton, $m_t = -\dfrac{1}{2}$ $\Leftrightarrow$ neutron

Ground states time–independent Schrödinger equation

$$
\begin{aligned}
\underset{\sim}{H} \mid \Psi_n \rangle &= E_n \mid \Psi_n \rangle \\
E_0 \leq E_1 &\leq E_2 \leq \cdots
\end{aligned}
$$

example: 3 non-interacting protons in a 1-dimensional harmonic oscillator potential:

$$
\begin{aligned}
\underset{\sim}{H} &= \underset{\sim}{h} \otimes \underset{\sim}{\mathbb{1}} \otimes \underset{\sim}{\mathbb{1}} + \underset{\sim}{\mathbb{1}} \otimes \underset{\sim}{h} \otimes \underset{\sim}{\mathbb{1}} + \underset{\sim}{\mathbb{1}} \otimes \underset{\sim}{\mathbb{1}} \otimes \underset{\sim}{h} \\
&= \underset{\sim}{h}(1) + \underset{\sim}{h}(2) + \underset{\sim}{h}(3) \\
\underset{\sim}{h} &= \frac{\underset{\sim}{k}^2}{2m} + \frac{1}{2}\, m\omega^2 \underset{\sim}{x}^2
\end{aligned}
$$



$$
\begin{aligned}
\mid \Psi_0 \rangle = \frac{1}{\sqrt{N}} \underset{\sim}{A} \quad \Big\{ \quad & (\mid 0 \rangle \otimes \mid m_s = \frac{1}{2} \rangle \otimes \mid m_t = \frac{1}{2} \rangle) \\
\otimes \quad & (\mid 0 \rangle \otimes \mid m_s = -\frac{1}{2} \rangle \otimes \mid m_t = \frac{1}{2} \rangle) \\
\otimes \quad & (\mid 1 \rangle \otimes \mid m_s = \frac{1}{2} \rangle \otimes \mid m_t = \frac{1}{2} \rangle) \Big\}
\end{aligned}
$$

Ground states
Ritz variational principle

The solution of the time–independent Schrödinger equation is not known analytically in almost all cases. Therefore approximation schemes are applied. A very basic one is the Ritz variational principle.

Trial state = ansatz for the state which is determined by some parameters $\{q_\mu\}$:

$$\mid Q \,\rangle \quad = \quad \frac{1}{\sqrt{N}} \underset{\sim}{A} \Big\{ \mid q_1 \,\rangle \otimes \cdots \otimes \mid q_A \,\rangle \Big\}$$

Energy:

$$\mathcal{H}(Q) \equiv \mathcal{H}(q_1, q_2, \dots) := \langle \, Q \mid \underset{\sim}{H} \mid Q \, \rangle$$

Ground state in the set of model states:
$\mid Q_{GS} \,\rangle \quad \Leftrightarrow \quad \mathcal{H}(Q_{GS})$ is minimum for all $\mid Q \,\rangle$

$$\forall q_\mu : \qquad \frac{\partial \, \mathcal{H}}{\partial \, q_\mu} \, (Q_{GS}) = 0$$

$$E_0 \leq \mathcal{H}(Q_{GS}) \qquad \textbf{!}$$

The Ritz variational principle is exact if varied in whole Hilbert space.

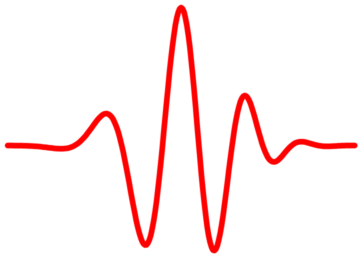
Ritz variational principle simple example

Let's consider the example of one particle in a 3-dimensional harmonic oscillator potential.

Hamiltonian:

$$\underset{\sim}{h} = \frac{\underset{\sim}{\vec{k}}^{\,2}}{2m} + \frac{1}{2}\, m\omega^2\, \underset{\sim}{\vec{x}}^{\,2}$$

Trial state $|\, q \,\rangle = |\, \vec{r}, \vec{p}, a \,\rangle$ omitting spin and isospin:



$$\langle\, \vec{x} \,|\, q \,\rangle = \exp\left\{ -\frac{(\vec{x} - \vec{r})^2}{2a} + i\vec{p}\cdot\vec{x} \right\}$$

Energy:

$$\begin{aligned} \mathcal{H}(q) \quad &\equiv \quad \mathcal{H}(\vec{r}, \vec{p}, a) := \frac{\langle\, q \,|\, \underset{\sim}{H} \,|\, q \,\rangle}{\langle\, q \,|\, q \,\rangle} \\ &= \quad \frac{1}{2m}\vec{p}^{\,2} + \frac{1}{2}\, m\omega^2 \vec{r}^{\,2} + \frac{1}{2m}\,\frac{3}{2a_R} + \frac{1}{2}\, m\omega^2\, \frac{3|a|^2}{2a_R} \end{aligned}$$

Ground state and energy in the model space:

$$\mathcal{H}(q_{GS}) = \mathcal{H}(\vec{r} = 0, \vec{p} = 0, a = \frac{1}{m\omega}) = \frac{3}{2}\omega$$

Ritz variational principle Hartree–Fock

The Hartree–Fock method is very important and basic in atomic, nuclear and solid state physics.

Trial state $|\,\Psi\,\rangle = \,|\,\{c_{ij}\}\,\rangle$:

$$
\begin{aligned}
|\,\Psi\,\rangle &= \frac{1}{\sqrt{N}}\,\underset{\sim}{A}\left(\,|\,\psi_1\,\rangle \otimes \,|\,\psi_2\,\rangle \otimes \cdots \otimes \,|\,\psi_A\,\rangle\right) \\
|\,\psi_i\,\rangle &= \sum_j c_{ij}\,|\,j\,\rangle \otimes \,|\,m_s(i)\,\rangle \otimes \,|\,m_t(i)\,\rangle
\end{aligned}
$$

Energy:

$$
\mathcal{H}(\Psi) \equiv \mathcal{H}(\{c_{ij}\}) := \langle\,\Psi\,|\,\underset{\sim}{H}\,|\,\Psi\,\rangle
$$

Ground state and energy in the set of model states:

$$
\forall c_{ij}: \qquad \frac{\partial\,\mathcal{H}}{\partial\,c_{ij}}\,(\Psi_{GS}) = 0
$$

The nice property of the Hartree–Fock method is that one can perform a unitary transformation from $\{\,|\,j\,\rangle\}$ to a new basis, so that the last equation becomes equivalent to solving a Schrödinger equation in one–body space.

Ritz variational principle
Fermionic Molecular Dynamics

The Fermionic Molecular Dynamics model was developed by Hans Feldmeier at GSI and is used for describing ground state properties of nuclei and reactions between them.

Trial state $|\, Q\, \rangle = |\, \{q_\mu\}\, \rangle$:

$$
\begin{aligned}
|\, Q\, \rangle &= \frac{1}{\sqrt{N}} \underset{\sim}{A} \left(|\, q_1\, \rangle \otimes |\, q_2\, \rangle \otimes \cdots \otimes |\, q_A\, \rangle \right) \\
\langle\, \vec{x}\, |\, q_i\, \rangle &= \exp\left\{ - \frac{(\, \vec{x} - \vec{r}_i\,)^2}{2\, a_i} + i \vec{x} \cdot \vec{p}_i \right\} \otimes |\, m_s(i)\, \rangle \otimes |\, m_t(i)\, \rangle
\end{aligned}
$$

$\vec{r}_i$: mean position, $\vec{p}_i$: mean momentum

a_i : complex width parameter

$m_s(i)$: spin, $m_t(i)$: isospin

Energy:

$$
\mathcal{H}(Q) \equiv \mathcal{H}(\vec{r}_1, \vec{p}_1, a_1; \vec{r}_2, \vec{p}_2, a_2; \ldots) := \langle\, Q\, |\, \underset{\sim}{H}\, |\, Q\, \rangle
$$

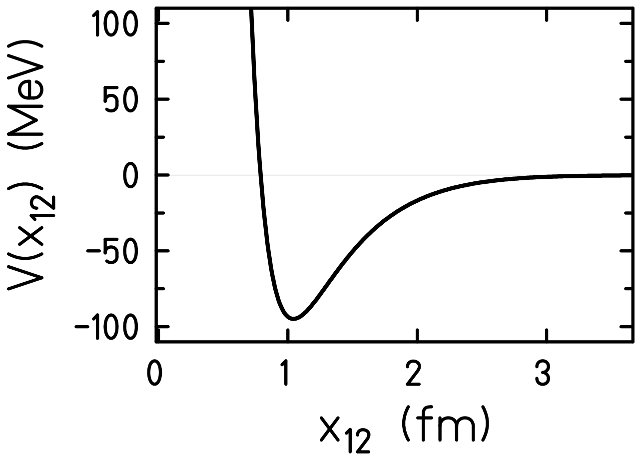
Ground state and energy in the set of model states:

$$
\forall q_\mu : \qquad \frac{\partial\, \mathcal{H}}{\partial\, q_\mu} \left(Q_{GS} \right) = 0
$$

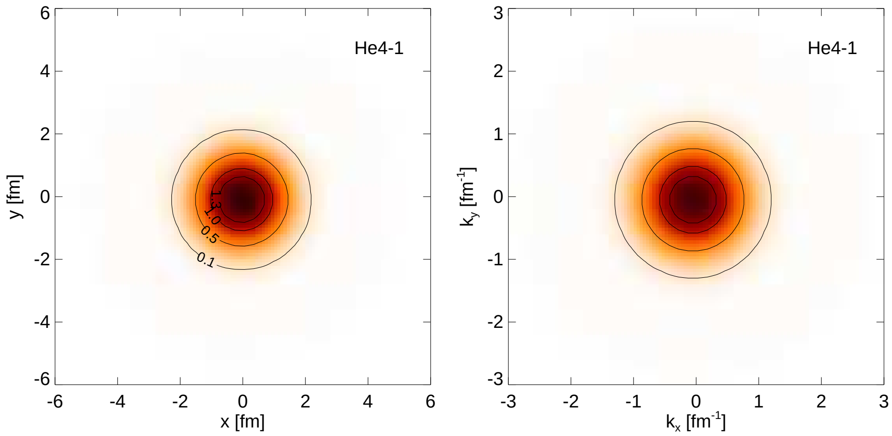
Ground states
Fermionic Molecular Dynamics

Hamiltonian:

$$\underset{\sim}{H} = \underset{\sim}{T} + \underset{\sim}{V} = \sum_{i=1}^{A} \underset{\sim}{t}(i) + \sum_{i<j}^{A} \underset{\sim}{v}(i,j)$$



Ground state density of ^{4}He:



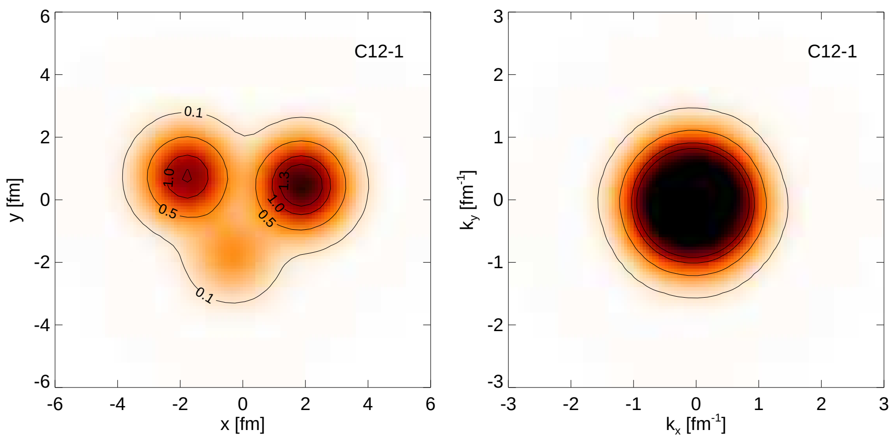
Densities

One–body density (also single–particle density):

$$
\begin{aligned}
\rho^{(1)}_x(\vec{x},\vec{x}\,') &\equiv \langle\, \vec{x}\,|\, \underset{\sim}{\rho}^{(1)}_x \,|\, \vec{x}\,'\,\rangle \\
&= \int \mathrm{d}^3x_2 \;\ldots \mathrm{d}^3x_A \langle\, Q\,|\, \vec{x}\,', \vec{x}_2, \ldots, \vec{x}_A\,\rangle\langle\, \vec{x}, \vec{x}_2, \ldots, \vec{x}_A\,|\, Q\,\rangle \\
&+ \int \mathrm{d}^3x_1\; \mathrm{d}^3x_3 \;\ldots \mathrm{d}^3x_A \langle\, Q\,|\, \vec{x}_1, \vec{x}\,', \vec{x}_3, \ldots\rangle\langle\, \vec{x}_1, \vec{x}, \vec{x}_3, \ldots\,|\, Q\,\rangle \\
&+ \ldots \\
&= A \int \mathrm{d}^3x_2 \;\ldots \mathrm{d}^3x_A \langle\, Q\,|\, \vec{x}\,', \vec{x}_2, \ldots, \vec{x}_A\,\rangle\langle\, \vec{x}, \vec{x}_2, \ldots, \vec{x}_A\,|\, Q\,\rangle \\
&= \langle\, Q\,|\, \underset{\sim}{\Psi}^\dagger(\vec{x}\,')\underset{\sim}{\Psi}(\vec{x})\,|\, Q\,\rangle \\
&= \sum_{k,l}^{A} \langle\, q_k\,|\, \vec{x}\,'\,\rangle\langle\, \vec{x}\,|\, q_l\,\rangle \mathcal{O}_{lk}\;, \qquad (\mathcal{O})^{-1}_{kl} = \langle\, q_k\,|\, q_l\,\rangle
\end{aligned}
$$

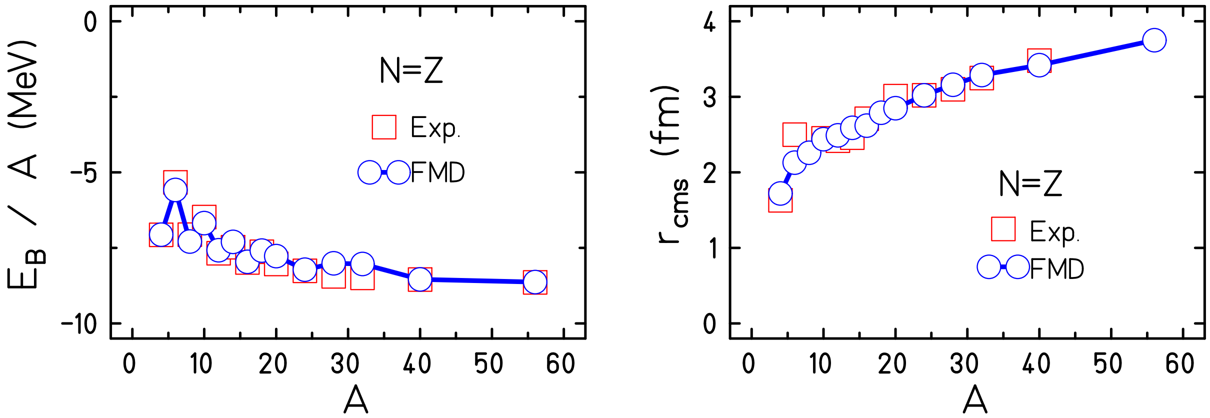
Momentum representation analogue.

Ground state density $\rho^{(1)}_x(\vec{x},\vec{x})$ and $\rho^{(1)}_k(\vec{k},\vec{k})$ of ^{12}C:

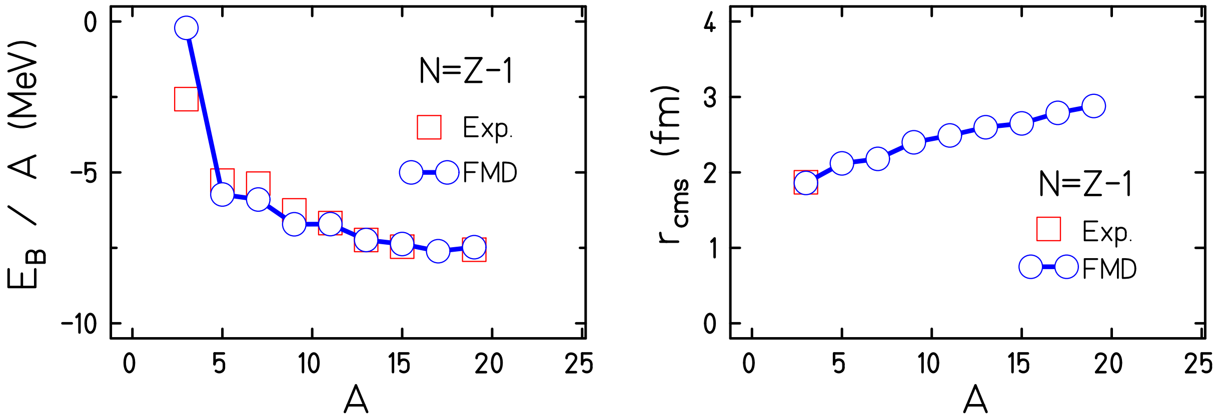


Ground state properties

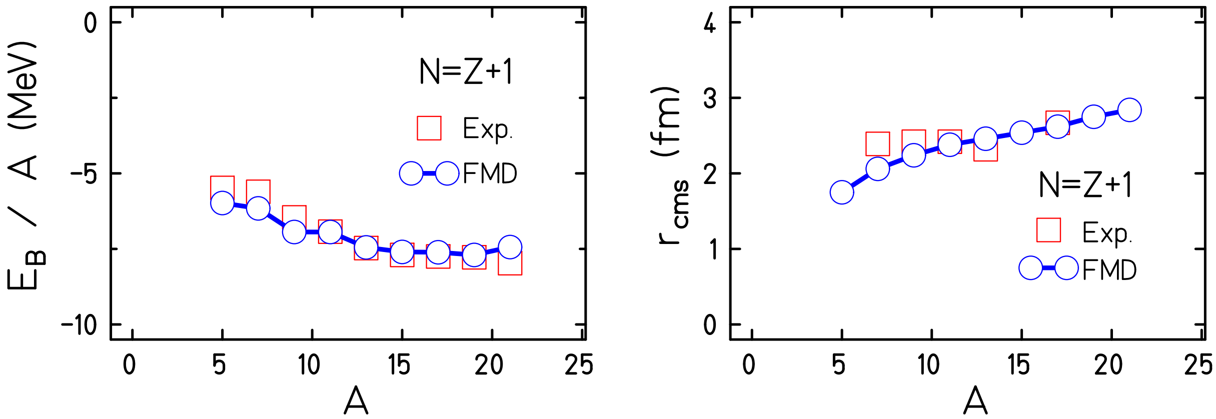
$N = Z$:



$N = Z - 1$:



$N = Z + 1$:



Dynamics time–dependent Schrödinger equation

$$i\frac{d}{dt}\,|\,\Psi(t)\,\rangle = \underset{\sim}{H}\,|\,\Psi(t)\,\rangle$$

Time–independent $\underset{\sim}{H}$ and eigenstates $|\,\Psi_n\,\rangle$ of $\underset{\sim}{H}$ known:

$$\begin{aligned}
|\,\Psi(t_0)\,\rangle &= \sum_n c_n(t_0)\,|\,\Psi_n\,\rangle \\
|\,\Psi(t)\,\rangle &= \exp\{-i\underset{\sim}{H}(t-t_0)\}\sum_n c_n(t_0)\,|\,\Psi_n\,\rangle \\
&= \sum_n c_n(t_0)\exp\{-i\underset{\sim}{E_n}(t-t_0)\}\;|\,\Psi_n\,\rangle
\end{aligned}$$

Example of a free wave packet omitting spin and isospin $|\,q(t_0)\,\rangle = |\,\vec{r}(t_0),\vec{p}(t_0),a(t_0)\,\rangle = |\,\vec{r},\vec{p},a\,\rangle$:

$$\begin{aligned}
\langle\,\vec{x}\,|\,q(t_0)\,\rangle &= \mathrm{e}^{-\frac{(\vec{x}-\vec{r})^2}{2a}+i\vec{p}\cdot\vec{x}} \\
|\,q(t_0)\,\rangle &= \int \mathrm{d}^3k\;a^{\frac{3}{2}}\,\mathrm{e}^{-\frac{a}{2}\vec{k}^{\,2}-i\vec{k}\cdot(\vec{r}+ia\vec{p})-\frac{a}{2}\vec{p}^{\,2}+i\vec{k}\cdot\vec{r}}\;|\,\vec{k}\,\rangle \\
|\,q(t)\,\rangle &= \mathrm{e}^{-\frac{i}{2m}\underset{\sim}{\vec{k}}^{2}(t-t_0)}\int \mathrm{d}^3k\;a^{\frac{3}{2}}\,\mathrm{e}^{-\frac{a}{2}\vec{k}^{\,2}-\cdots}\;|\,\vec{k}\,\rangle \\
&= \int \mathrm{d}^3k\;a^{\frac{3}{2}}\,\mathrm{e}^{-\frac{a+\frac{i}{m}(t-t_0)}{2}\vec{k}^{\,2}-\cdots}\;|\,\vec{k}\,\rangle
\end{aligned}$$

Dynamics time–dependent variational principle

TDVP:

$$0 \quad = \quad \delta \int_{t_1}^{t_2} \mathrm{d}t \, \langle\, Q(t) \,|\; i\frac{d}{dt} - \underset{\sim}{H} \;|\, Q(t) \,\rangle$$

Variation in the whole Hilbert space yields the Schrödinger equation:

$$\begin{aligned} 0 \quad &= \quad \int_{t_1}^{t_2} \mathrm{d}t \, \langle\, \delta Q(t) \,|\; i\frac{d}{dt} - \underset{\sim}{H} \;|\, Q(t) \,\rangle \\ &\quad + \int_{t_1}^{t_2} \mathrm{d}t \, \langle\, Q(t) \,|\; i\frac{d}{dt} - \underset{\sim}{H} \;|\, \delta Q(t) \,\rangle \\ 0 \quad &= \quad \left(i\frac{d}{dt} - \underset{\sim}{H} \right) \;|\, Q(t) \,\rangle \qquad \text{and h.c.} \end{aligned}$$

Dynamics
time–dependent variational principle

$$
\begin{aligned}
0 &= \delta \int_{t_1}^{t_2} \mathrm{d}t \, \langle\, Q(t) \,|\; i\frac{d}{dt} - \underset{\sim}{H} \;|\, Q(t) \,\rangle \\
&= \delta \int_{t_1}^{t_2} \mathrm{d}t \; \mathcal{L}(Q(t), \dot{Q}(t), t) \\
&= \delta \int_{t_1}^{t_2} \mathrm{d}t \; \Big(\mathcal{L}_0(Q(t), \dot{Q}(t), t) - \mathcal{H}(Q(t), t) \Big)
\end{aligned}
$$

Euler–Lagrange–equations for the parameters $Q(t) = \{q_\nu(t)\}$ (no explicit time dependence assumed):

$$
\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{q}_\nu} - \frac{\partial \mathcal{L}}{\partial q_\nu} = 0 \qquad , \qquad \nu = 1, 2, \ldots
$$

Equivalent formulation:

$$
\sum_\nu \mathcal{A}_{\mu\nu}(Q)\; \dot{q}_\nu = -\frac{\partial}{\partial q_\mu} \;\langle\, Q(t) \,|\, \underset{\sim}{H} \,|\, Q(t) \,\rangle
$$

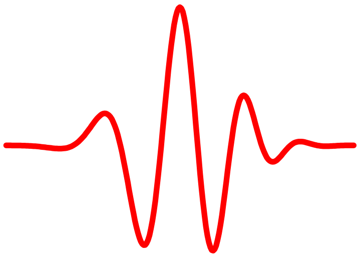
$$
\mathcal{A}_{\mu\nu}(Q) = \frac{\partial^2 \mathcal{L}_0}{\partial \dot{q}_\mu \partial q_\nu} - \frac{\partial^2 \mathcal{L}_0}{\partial \dot{q}_\nu \partial q_\mu}
$$

TDVP
simple example

Freely moving wave packet, hamiltonian:

$$\underset{\sim}{h} = \frac{\underset{\sim}{\vec{k}}^{\,2}}{2m}$$

Trial state $|\, q(t) \,\rangle = |\, \vec{r}(t), \vec{p}(t), a(t) \,\rangle$ omitting spin and isospin:



$$\langle\, \vec{x} \,|\, q(t) \,\rangle = \exp\left\{ -\frac{(\vec{x} - \vec{r}(t))^2}{2a(t)} + i\vec{p}(t)\cdot\vec{x} \right\}$$

Energy:

$$\mathcal{H}(q(t)) = \frac{1}{2m}\vec{p}^{\,2}(t) + \frac{1}{2m}\frac{3}{2a_R(t)}$$

Equations of motion for the parameters:

$$\sum_\nu \mathcal{A}_{\mu\nu}(q)\dot{q}_\nu = -\frac{\partial\,\mathcal{H}(q(t))}{\partial q_\mu} \quad \Leftrightarrow \quad \frac{d}{dt}\,\vec{p}(t) = 0$$

$$\frac{d}{dt}\,\vec{r}(t) = \frac{\vec{p}(t)}{m}$$

$$\frac{d}{dt}\,a(t) = \frac{i}{m}$$

Time–dependent Hartree–Fock

The time–dependent Hartree–Fock method uses the same ansatz as the time–independent Hartree–Fock method but with time–dependent coefficients.

Trial state $\mid \Psi(t) \,\rangle = \,\mid \{c_{ij}(t)\} \,\rangle$:

$$
\begin{aligned}
\mid \Psi(t) \,\rangle &= \frac{1}{\sqrt{N(t)}} \underset{\sim}{A} \left(\mid \psi_1(t) \,\rangle \otimes \,\mid \psi_2(t) \,\rangle \otimes \cdots \otimes \,\mid \psi_A(t) \,\rangle \right) \\
\mid \psi_i(t) \,\rangle &= \sum_j c_{ij}(t) \mid j \,\rangle \otimes \,\mid m_s(i) \,\rangle \otimes \,\mid m_t(i) \,\rangle
\end{aligned}
$$

Energy:

$$
\mathcal{H}(\Psi(t)) \quad \equiv \quad \mathcal{H}(\{c_{ij}(t)\}) := \langle \, \Psi(t) \mid \underset{\sim}{H} \mid \Psi(t) \,\rangle
$$

Equatins of motion:

$$
\frac{d}{dt} \frac{\partial \mathcal{L}}{\partial \dot{c}_{ij}} - \frac{\partial \mathcal{L}}{\partial c_{ij}} = 0 \qquad , \qquad \forall i, j
$$

The nice property of the Hartree–Fock method is again that one can perform a unitary transformation from $\{ \mid j \,\rangle \}$ to a new basis, so that the last equation becomes equivalent to solving a time–dependent Schrödinger equation in one–body space; that's possible numerically.

TDVP
Fermionic Molecular Dynamics

Trial state $|\,Q(t)\,\rangle = \,|\,\{q_\mu(t)\}\,\rangle$:

$$|\,Q(t)\,\rangle \quad = \quad \frac{1}{\sqrt{N(t)}}\,\underset{\sim}{A}\left(\,|\,q_1(t)\,\rangle \otimes \,|\,q_2(t)\,\rangle \otimes \cdots \otimes \,|\,q_A(t)\,\rangle\right)$$

$$\langle\,\vec{x}\,|\,q_i(t)\,\rangle \quad = \quad \mathrm{e}^{-\frac{(\,\vec{x}-\vec{r}_i(t)\,)^2}{2\,a_i(t)}+i\vec{x}\cdot\vec{p}_i(t)} \otimes |\,m_s(i)\,\rangle \otimes |\,m_t(i)\,\rangle$$

Energy:

$$\begin{aligned}
\mathcal{H}(Q(t)) \quad &\equiv \quad \mathcal{H}(\vec{r}_1(t),\vec{p}_1(t),a_1(t);\vec{r}_2(t),\vec{p}_2(t),a_2(t);\ldots) \\
&:= \quad \langle\,Q(t)\,|\,\underset{\sim}{H}\,|\,Q(t)\,\rangle \\
&= \quad \langle\,\underset{\sim}{T}+\underset{\sim}{V}\,\rangle = \langle\,\sum_{i=1}^{A}\underset{\sim}{t}(i) + \sum_{i<j}^{A}\underset{\sim}{v}(i,j)\,\rangle
\end{aligned}$$

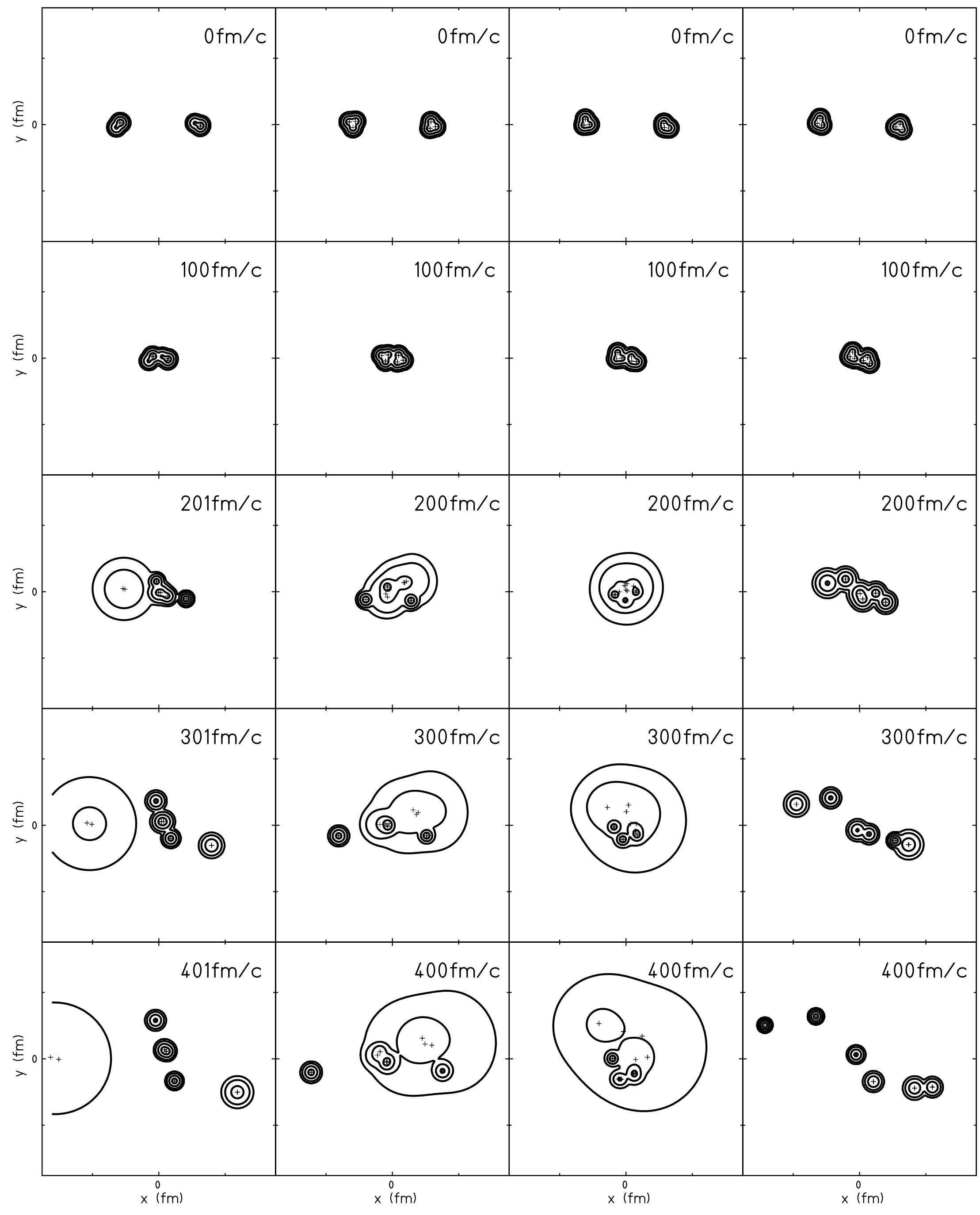
Equations of motion:

$$\sum_\nu \mathcal{A}_{\mu\nu}(Q)\;\dot{q}_\nu = -\frac{\partial}{\partial q_\mu}\;\langle\,Q(t)\,|\,\underset{\sim}{H}\,|\,Q(t)\,\rangle$$

$$\mathcal{A}_{\mu\nu}(Q) = \frac{\partial^2\mathcal{L}_0}{\partial\dot{q}_\mu\partial q_\nu} - \frac{\partial^2\mathcal{L}_0}{\partial\dot{q}_\nu\partial q_\mu}$$

Fermionic Molecular Dynamics

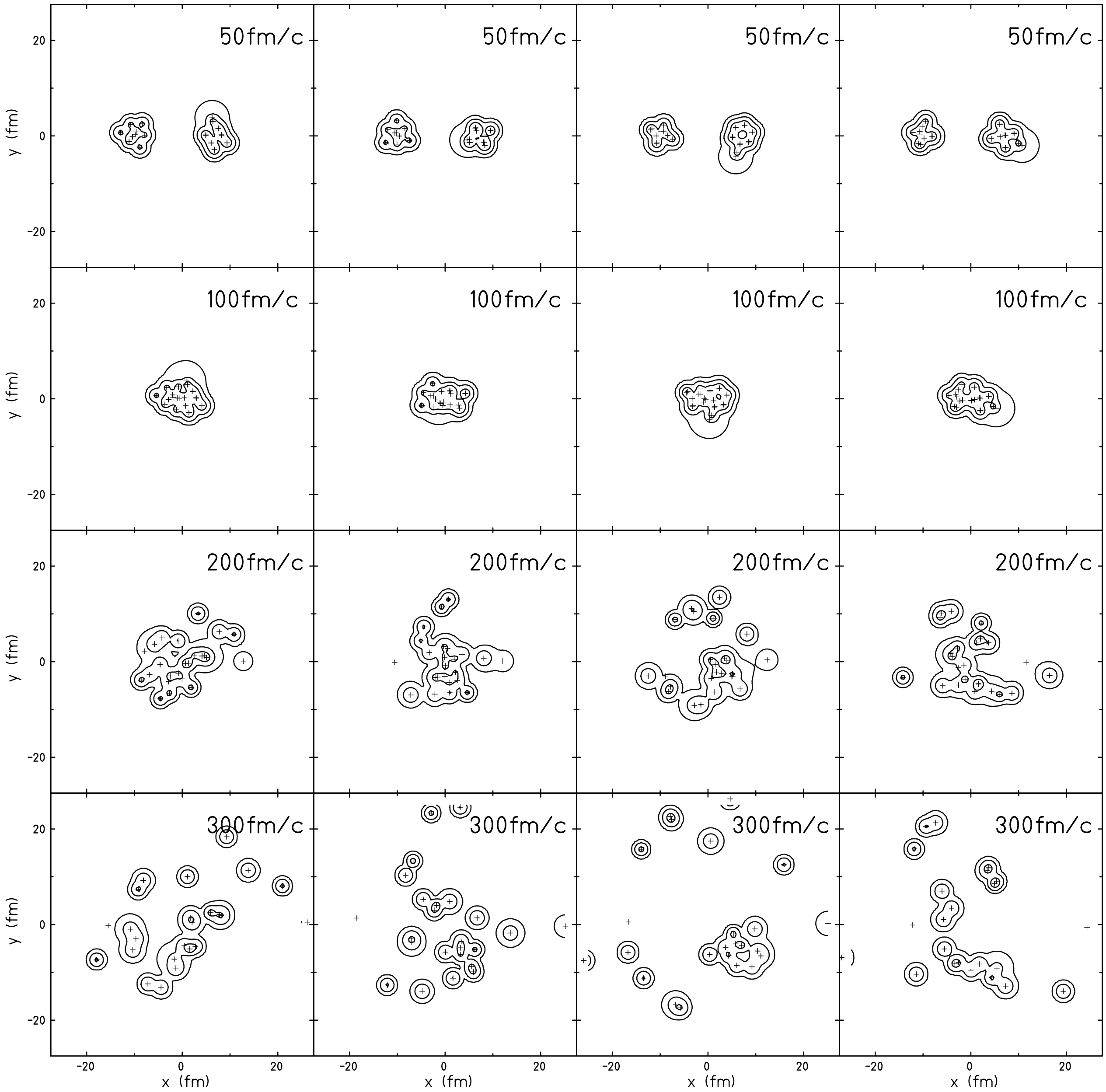
$\rho_x^{(1)}(\vec{x},\vec{x})$ for ^{12}C–^{12}C at $E_{Lab} = 28.7A$MeV:



$b = 0.25$fm $b = 0.75$fm $b = 1.25$fm $b = 1.75$fm

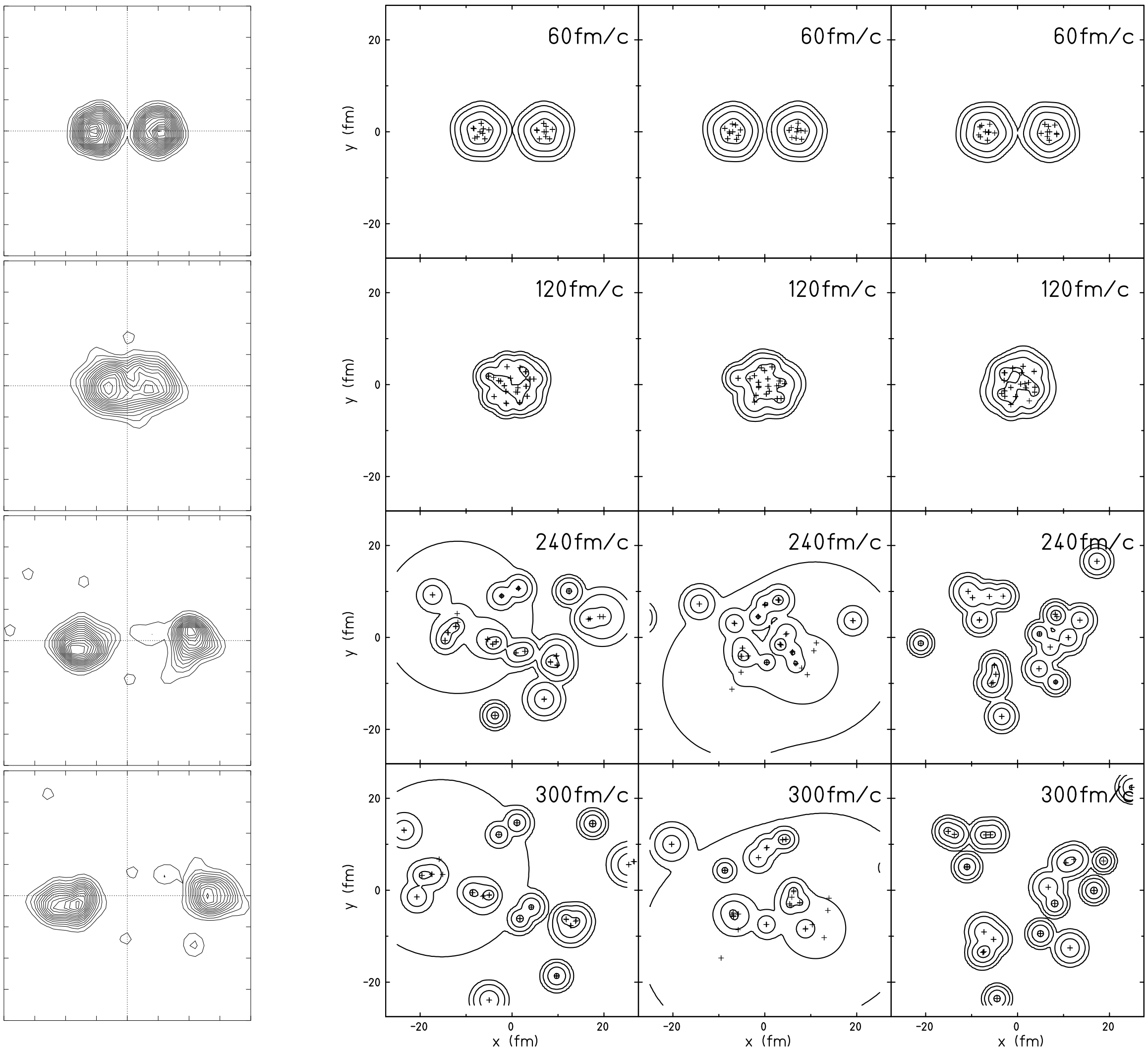
Fermionic Molecular Dynamics

$\rho_x^{(1)}(\vec{x},\vec{x})$ for ^{19}F–^{27}Al at $E_{Lab} = 32A$MeV and $b = 0.5$fm:



Fermionic Molecular Dynamics

$\rho_x^{(1)}(\vec{x}, \vec{x})$ for ^{40}Ca–^{40}Ca at $E_{Lab} = 35A$MeV and $b = 0.0$fm:



Left column Antisymmetrized Molecular Dynamics.

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