Magnon pairing in quantum spin nematic

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Outline

- Introduction
- Bound magnon states: from 1D chain to frustrated square lattice
- Condensate of bound magnon pairs: spin nematic state
Spin Liquids

- ‘Traditional’ quantum magnetism: ordered magnetic states, spin chains, ladders,…

- Modern theories: possibility of spin liquids in arbitrary dimension; short-range RVB, long-range RVB, $Z_2$, $U(1)$, topological,…

- Recent experiments: triangular organics, (hyper)kagome, $^3$He monolayers,

(anything else exotic?)
3 + 1 states of Matter

<table>
<thead>
<tr>
<th>Matter</th>
<th>Magnetic Matter</th>
<th>Defining property</th>
</tr>
</thead>
<tbody>
<tr>
<td>gas</td>
<td>paramagnet</td>
<td>dilute</td>
</tr>
<tr>
<td>liquid</td>
<td>spin-liquid</td>
<td>isotropic</td>
</tr>
<tr>
<td>solid</td>
<td>ordered magnets</td>
<td>anisotropic</td>
</tr>
<tr>
<td>liquid crystal</td>
<td>spin-nematic*</td>
<td>partial symmetry breaking</td>
</tr>
</tbody>
</table>

*(a.k.a. multipolar states) described by tensor order parameters, searched for the past 40 years, were rarely observed so far*
Spin nematics in literature and in nature

- Magnets with \( S \geq 1 \) and biquadratic exchange
  on site tensor

\[
\langle S_i^\alpha S_i^\beta \rangle \neq \delta^{\alpha\beta}
\]

- 'Preformed' CEF multipoles: \( |J, m\rangle \rightarrow |\Gamma^{(l)}\rangle \)
  such that

\[
\langle \Gamma^{(l)}_k | J^\alpha | \Gamma^{(l)}_n \rangle = 0 \text{ , while } \langle \Gamma^{(l)}_k | J^\alpha J^\beta | \Gamma^{(l)}_n \rangle \neq 0
\]

- Spin-nematic order from strong correlations in a system of magnetic dipoles

\[K(S_i \cdot S_j)^2\]

Blume & Hseich (1969)

\[\text{NpO}_2, \text{Ce}_x\text{La}_{1-x}\text{B}_6\]
Multipoles: symmetry classification

- zero magnetic field, irreps of $SO(3)$, classification in $L$
  - Monopole: $L = 0$, $\rho$
  - Dipole: $L = 1$, $d^\alpha$
  - Quadrupole: $L = 2$, $Q^{\alpha\beta}$
  - Octupole: $L = 3$, $T^{\alpha\beta\gamma}$

- finite magnetic fields, irreps of $SO(2)$, classif. in $L_z$, spin nematic
  - $L_z = \pm 2$
Magnon BEC in applied field

- **Bosons**
  - $U(1)$ gauge invariance
  - Condensate wavefunction $\langle \psi(r) \rangle$
  - Chemical potential $\mu$

- **Spins**
  - $SO(2)$ rotation symmetry
  - Staggered magnetization
    \[ M_j^\dagger = \langle S_j^x + iS_j^y \rangle \]
  - Magnetic field $H$

![Diagram](image-url)
Magnon BEC: quest for new physics

- change bosons: add flavor, valley index, triplet polarization,…

- change kinetic energy: localized magnons in flat bands

- change potential energy: add attraction, competing ferro- and antiferromagnetic bonds
Pair superfluidity in attractive Bose gases

- pair condensation versus density collapse
  Valatin (1958), Nozières (1982)

- constrained Bose-Hubbard model, $n_i \leq 2$
  Bonnes & Wessel (2011)
Frustrated ferromagnets

- chain materials LiCuVO$_4$, Li$_2$CuO$_2$, Li$_2$ZrCuO$_4$...

- layered square-lattice afms

90° Cu-O-Cu bonds yield FM n.n. exchange $J_1<0$ with AF n.n.n. $J_2>0$

by A. Tsirlin
Magnon pairing in strong fields

- fully polarized state \( |0\rangle = |\uparrow\uparrow\uparrow\rangle \)

- single spin-flips (magnons) \( |\psi_1\rangle = \sum f_i S_i^- |0\rangle \) unstable at \( H_{s1} \)

\[ \epsilon_k = H + \frac{1}{2} (J_k - J_0) \]

- pair of spin flips \( |\psi_2\rangle = \sum f_{ij} S_i^- S_j^- |0\rangle \) unstable at \( H_{s2} = H_{s1} + \frac{1}{2} E_B \)

\[ (\epsilon_2 - \epsilon_{k/2+q} - \epsilon_{k/2-q}) f_k(q) = \frac{1}{2} \sum_p (J_{p+q} + J_{p-q} - J_{k/2+q} - J_{k/2-q}) f_k(p) \]
Magnon pairs in 1D

- frustrated spin chain
- bound magnon pairs with $k = \pi$ and

$$E_B = J_1^2 / (J_2 + |J_1|)$$

- numerical studies


Magnon pairs in LiCuVO$_4$

- weakly coupled chains
  
  \[ J_1 = -1.6 \text{meV}, \quad J_2 = 3.6 \text{meV}, \quad J_5 \sim -0.4 \text{meV} \]


- pair condensation

\[ H_{s2} = 47.1 \text{T} \quad \text{and} \quad H_{s1} = 46.5 \text{T} \]

Tsunetsugu & MZ (2010)
Magnon pairs in 2D: $J_1$-$J_2$ SAFM

- bound pairs with $k=(0,0)$

\[ f_k(q) \propto (\cos q_x - \cos q_y) \]

\[ E_B = 2je^{-\pi j} \]

Magnon pairs in 2D

- antiferromanetic chains coupled by frustrating ferromagnetic bond

- frustration is essential as it suppresses the kinetic energy of single spin flips
Condensate of magnon pairs

- Bogolyubov theory via the coherent boson states

\[ \langle \psi | a_0 | \psi \rangle \neq 0 \Rightarrow |\psi\rangle = e^{\varphi a_0^+} |0\rangle \quad (a_0 |\psi\rangle = \psi |\psi\rangle) \]

- Single-magnon condensate = canted AF state

\[ |\psi\rangle = \exp(\varphi S^-_Q) |0\rangle \quad \langle \psi | S^-_i | \psi \rangle \propto e^{iQr_i} \]

  ► dipolar symmetry

\[ C^z_\pi |\psi\rangle = - |\psi\rangle \]

- Coherent pair condensate

\[ |\Delta\rangle \approx \exp(\Delta \sum f_{ij} S^-_i S^-_j) |0\rangle \]

  ► nematic symmetry

\[ C^z_\pi |\Delta\rangle = |\Delta\rangle \]
Spin correlations

- absence of the sub-lattice magnetization \( \langle \Delta | S_i^- | \Delta \rangle \equiv 0 \)

- exponential decay in transverse and longitudinal channels

\[
\langle \Delta | S_i^- S_j^+ | \Delta \rangle \approx |\Delta|^2 \sum_l f_{il}^* f_{lj} \quad \langle \Delta | \partial S_i^z \partial S_j^z | \Delta \rangle \approx |\Delta|^2 |f_{ij}|^2
\]

- quadrupolar (nematic) order parameter

\[
Q_{ij}^{\alpha\beta} = \frac{1}{2} \langle S_i^\alpha S_j^\beta + S_i^\beta S_j^\alpha \rangle
\]

\[
Q_{ij}^{xx} + iQ_{ij}^{xy} = \frac{1}{2} \langle S_i^+ S_j^+ \rangle \approx \frac{\Delta}{2} f_{ij}
\]
Self-consistent BCS-type theory

- bosonization via Holstein-Primakoff transformation
- mean-field averages
  \[ \Delta_{ij} = \langle a_i a_j \rangle \quad \text{and} \quad n_r = \langle a_i^+ a_{i+r} \rangle \]
- quasiparticle excitations: unpaired magnons

\[
\epsilon_{k/2+q} = \sqrt{A_q^2 - B_q^2} - \sum_r J(r) \left( \frac{1}{2} - n - n_r \right) \sin qr \sin \frac{1}{2} kr \\
A_q = H - \sum J(r) \left( \frac{1}{2} - n - n_r \right) \cos qr \cos \frac{1}{2} kr \\
B_q = \sum J(r) \Delta_r \cos qr
\]
Self-consistent theory: results

- energy-field diagram
- magnon excitations

▶ for LiCuVO$_4$: $H_{s2} = 47T$ and $H_c = 44T$

Tsunetsugu & MZ, EPL 92, 37001 (2010)
New ultra-high-field phase in LiCuVO$_4$

- pulsed field experiments

- ultra-high field phase
  - Svistov et al (2011)

- the ultra high-field phase is a transverse spin-nematic state
New high-field phases in LiCuVO$_4$

- ‘not so’ high-field phase: longitudinal SDW in isotropic AFM?
  
  Buetgen et al (2008,10)

- relation to the transverse spin-nematic at higher fields?
Longitudinal SDW in LiCuVO$_4$

- neutron diffraction

Masuda et al (2011)

$Q = \frac{1}{2} (1 - 2m^z)$
2D spin-nematic?

- spin-nematic phase in frustrated SAFM BaCdVO(PO$_4$)$_2$? Tsirlin et al (2008)

The high-field region is easily achievable in a Lab: $H_s = 4.2$ T; various experimental techniques may be applied.
Summary

- magnon pair condensate = spin nematic (quadrupole) state

- smoking gun for spin-nematic: hidden-order transitions, i.e. without evident order parameters

- dynamical properties: collective quadrupole mode in longitudinal channel?

- instabilities in the spin nematic phase: longitudinal IC state in LiCuVO$_4$

- nematic LRO in BaCdVO(PO$_4$)$_2$ and other frustrated 2D FMs
Magnon pairing ar a root to spin-nematic

Thank you!