The kagome antiferromagnet: a chiral spin liquid ?

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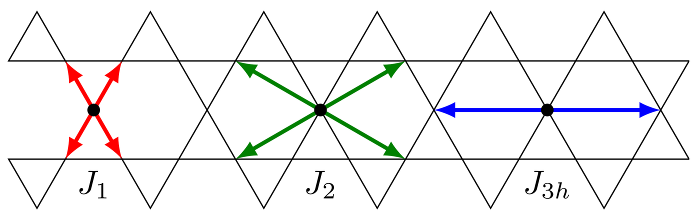
Classical limit of spin liquids: regular orders

- In 2D frustrated spin systems, specific phases without any symmetry breaking even at $T=0$ can exist. They are called *spin liquids* (SL's) and belong to the exotic phases underlined by Anderson in the 70's. Low value of spins ($S=1/2$, 1) favorizes them.
- The spin 1/2 *kagome* lattice with Heisenberg interactions is a promising candidate of SL.

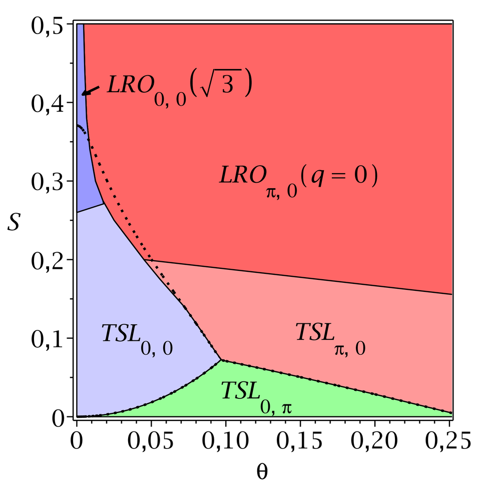
$$H=\sum_{i,j} J_{ij}\,\mathbf{S}_i\cdot\mathbf{S}_j.$$



- In the classical limit ($S\to\infty$), $\mathbf{S}_i$ is a unit vector and long-range spin order appears.
- The *Schwinger boson mean-field theory* interpolates between quantum and classical models. κ is a continuous parameter similar to $2S$. $a^\dagger$ ($b^\dagger$) creates a $\uparrow$ ($\downarrow$) spin 1/2 boson.

$$\begin{cases} 2S_i^z = a_i^\dagger a_i - b_i^\dagger b_i \\ S_i^+ = a_i^\dagger b_i \end{cases} \qquad \widehat{A}_{ij}=(a_ib_j-b_ia_j)/2$$

$$H=\sum_{i,j} J_{ij}\,\mathbf{S}_i\cdot\mathbf{S}_j=\frac{1}{2}\sum_{i,j} J_{ij}\left(S^2-2\widehat{A}_{ij}^\dagger\widehat{A}_{ij}\right) \sim \sum_{i,j} J_{ij}\left(|\mathcal{A}_{ij}|^2-\widehat{A}_{ij}^\dagger\mathcal{A}_{ij}-\widehat{A}_{ij}\mathcal{A}_{ij}^*\right)+\sum_i \lambda_i(\kappa-\widehat{n}_i)$$

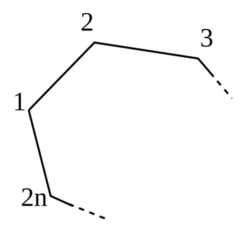


- Typical SBMFT phase diagram (kagome with $J_1 = 1$ and Dzyaloshinskii-Moriya interaction of strength θ)

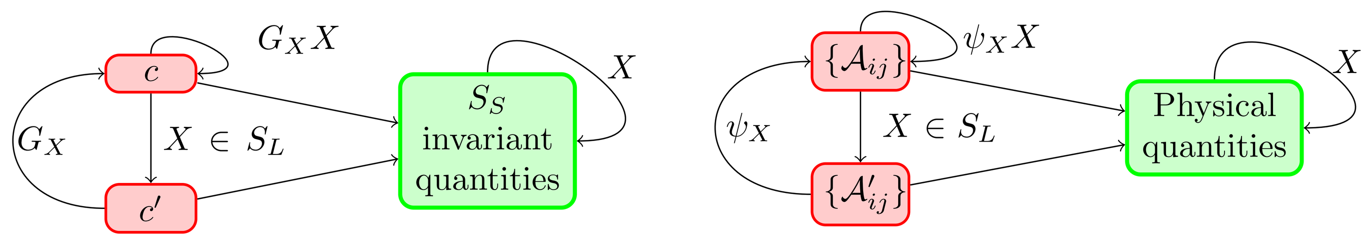
  -LRO: long range orders for $\kappa>\kappa_c$,
  -TSL : topological spin liquids fof $\kappa<\kappa_c$

  There are different SL phases, each associated to different LRO's. What are the symmetry properties of these LRO's ?
  $\rightarrow$ definition of regular orders (RO's) as classical limits of SL's.

Definition and properties of a regular order

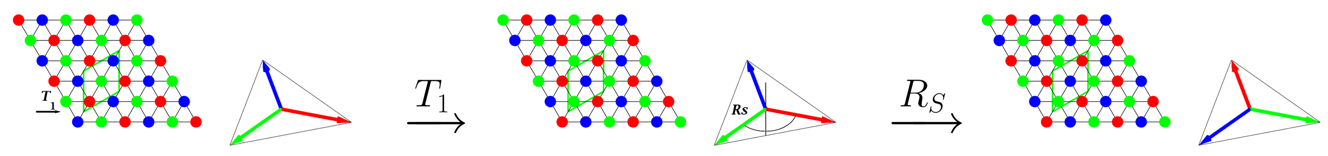


- The effect of a lattice symmetry $X\in S_L$ on a SL Ansatz can be compensated by a gauge transformation ϕ_X.[1] Several families of *projective symmetry groups* exist, corresponding to different SL's, labelled by their fluxes ϕ. $\phi_{12\dots 2n}$ = phase of $\mathcal{A}_{12}(-\mathcal{A}_{23}^*)\cdots(-\mathcal{A}_{2n,1}^*)$



Classical order — Quantum Ansatz

- A classical order is called *regular* if a lattice symmetry $X\in S_L$ can be compensated by a global spin transformation G_X. Thus this orders respects the lattice symmetries. Example:



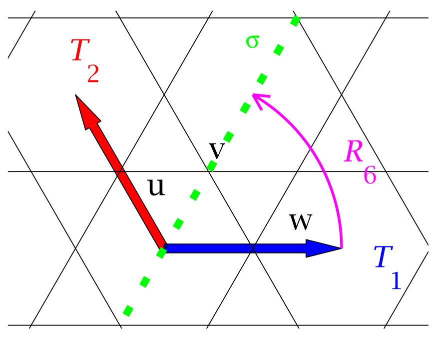
- No general method exists to find the ground state of a classical model except for Heisenberg models on Bravais lattice:

$$H=\sum_{\substack{\mathbf{v},\mathbf{x}\\ i,j=1,\dots,m}} J_{ij}(\mathbf{v})\,\mathbf{S}_{\mathbf{x}i}\cdot\mathbf{S}_{\mathbf{x}+\mathbf{v}j}=\sum_{\substack{\mathbf{q}\in BZ\\ i,j=1,\dots,m}} J_{ij}(\mathbf{q})\,\mathbf{S}_{-\mathbf{q}i}\cdot\mathbf{S}_{\mathbf{q}j}$$

(m =number of site in the lattice unit cell), where spiral states reach the lower energy bound: the lowest eigenvalue $\min_{\{\mathbf{q}\}}\left(J_{\mathbf{q}}^{\min}\right)$.

How to find all regular orders ?

- To find all RO's, the research consists in two steps.[2]
  (1) Find the ϕ_X from S_S respecting the algebraic structure of S_R,
  (2) For each set of ϕ_X, find the compatible states.
- Example of research of regular states on the kagome lattice



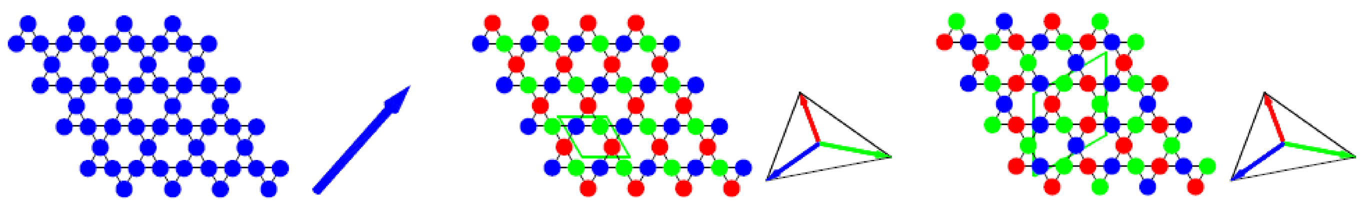
Kagome with Heisenberg spins

(1) Algebraic constraints on ϕ_X from O_3 :

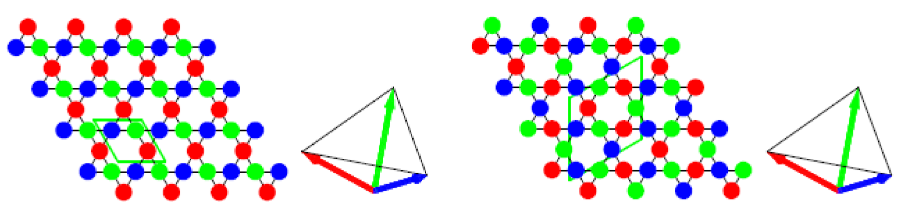
$$\phi_{T_1}\phi_{T_2}=\phi_{T_2}\phi_{T_1} \qquad \phi_{T_1}\phi_{R_6}\phi_{T_2}=\phi_{R_6}$$
$$\phi_{R_6}\phi_\sigma\phi_{R_6}=\phi_\sigma \qquad \phi_{T_1}\phi_\sigma=\phi_\sigma\phi_{T_2}$$
$$\phi_{R_6}^6=I \qquad \phi_\sigma^2=I$$
$$\phi_{R_6}\phi_{T_1}\phi_{T_2}=\phi_{T_2}\phi_{R_6}.$$

(2) Constraints on the spin state:

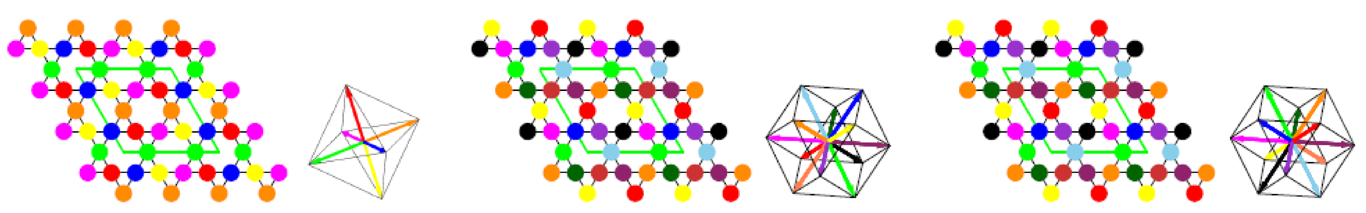
$$\phi_\sigma\mathbf{S}_v=\mathbf{S}_v,\qquad \phi_{T_1}\phi_{T_2}\phi_R^3\mathbf{S}_v=\mathbf{S}_v.$$



Ferromagnetic state — $\mathbf{q}=\mathbf{0}$ state — $\sqrt{3}\times\sqrt{3}$ state



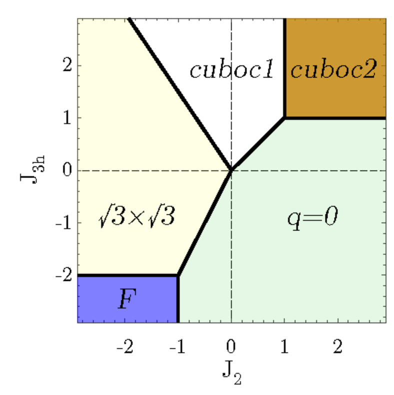
$\mathbf{q}=\mathbf{0}$ umbrella state — $\sqrt{3}\times\sqrt{3}$ umbrella state



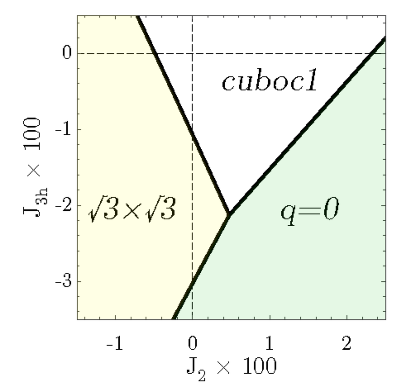
Octahedral state — Cuboc1 state — Cuboc2 state

Two applications on the kagome lattice

The antiferromagnet $J_1=1$[3]:

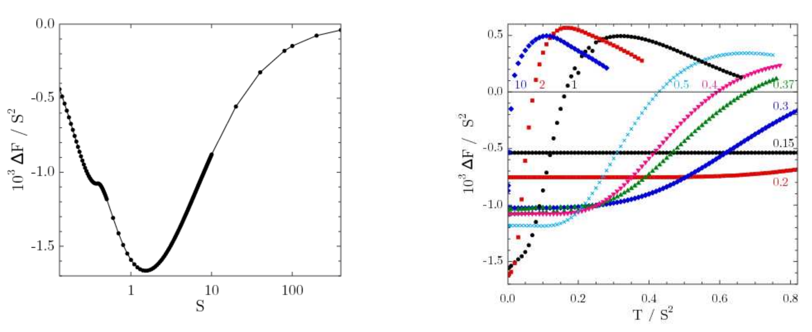


Classical phase diagram

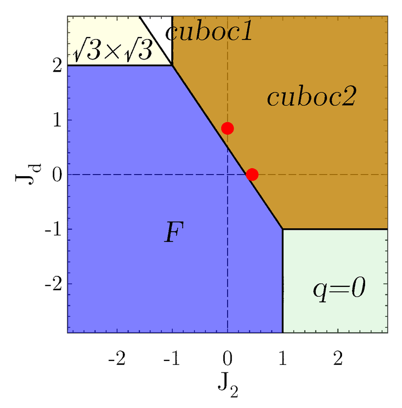


SBMFT, $\kappa=1$

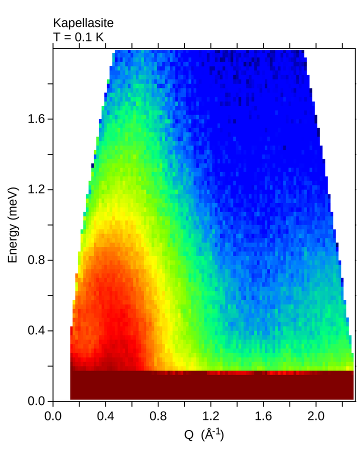
- Using the RO's energy properties, the exact classical phase diagram is known for $J_1=1$ as the lower energy bound is reached everywhere by a RO.
- The $J_1=1$, $J_2=J_{3h}=0$ point is classicaly infinitely degenerated and adjacent to 3 RO's in the J_2-J_{3h} phase diagram.
- SBMFT had only been applied to non chiral states, leading to propose the $\sqrt{3}\times\sqrt{3}$ SL as the ground state at this point.
- We obtain that the chiral *cuboc*1 SL has a lower MF energy and thus propose this chiral SL as the ground state.
- Effect of κ and of T: at a finite temperature, the $\sqrt{3}\times\sqrt{3}$ phase has a lower free energy than the *cuboc*1 phase, because of order by disorder.



Kapellasite $J_1=-1$[4]:



- A large value of J_{3h} is possible on kapellasite according to DFT computations.[5]
- Fits of high temperature series expansion to experimental data suggest a cuboc2 order.
- Neutron scattering results confirm this hypothesis with intensity at an unusual wave vector.



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